Lecture 4 - Auctions and game theory

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From market failures to strategic allocation

Lecture 4 overview

- Last time: we explored why markets sometimes fail.
- Next: Different allocation mechanisms (**auctions**) and strategic considerations (**game theory**)

Auctions

- Introduction to auctions and their properties.
- Why auctions may also fail.
- Auction examples for computer industries.

Game Theory

- Define games and Nash equilibrium.
- Present classical games.
- Map games to computer industries.

Where auctions appear in computer science

Applications of auction mechanisms

- **Online platforms & advertising** (Google Ads, Meta Ads)
- **Cloud computing resources** (AWS Spot Instances, Google Preemptible VMs)
- **Network allocation & spectrum** (telecom frequency auctions, congestion pricing)
- **Mechanism design & blockchain** (VCG mechanisms, Ethereum gas auctions, NFT sales)
- **Digital markets & games** (eBay, Steam Marketplace, in-game auctions in World of Warcraft)

Types of auctions

Timing of bids

- **English (Ascending-bid):** Start at a reserve price, raise bids until one remains (art, antiques).
- **Dutch (Descending-bid):** Start high and lower until someone accepts (flowers).
- **Sealed-bid:** All bids privately simultaneously (government contracts).

Pricing and payment rules

- **First-price auction:** Highest bidder wins, pays their own bid.
- **Second-price auction:** Highest bidder wins, pays second-highest bid.
- **All-pay auction:** Everyone pays their bid, only one wins (wars, litigation, market races).

Strategic equivalence in auctions

Dutch = First-price auction

- Outcome: highest bidder wins at reservation price.
- Strategy: bid below true value.

English = Second-price auction

- Outcome: Highest bidder wins, pays second-best bid.
- Strategy: bid truthfully.

But the two pairs are not strategically equivalent!

- First-price/Dutch $\Rightarrow$ bid shading (strategic misrepresentation).
- Second-price/English $\Rightarrow$ truthful bidding is optimal.
- Strategic incentives, not just outcomes, differ across formats.

Revenue equivalence in auctions

Revenue Equivalence Theorem

- In theory, all well-behaved auctions yield the same **expected revenue**.
- Conditions: risk-neutral bidders, independent private values, no collusion, Pareto efficiency, and common reserve price.
- Hence: English, Dutch, and first-price auctions raise the same average revenue.
- All-pay auction differs: everyone pays, winner or not, $\Rightarrow$ revenue higher.

Design implications

How to design auctions when conditions are not ideal?

Auctions and information problems

Winner's curse

- In **public-value auctions**, everyone estimates the same underlying value (mineral rights, spectrum).
- The winner tends to be the one who *overestimates* the most, the "curse".

Bidding rings

- Groups of bidders collude to buy low, then hold a private auction and share profits.
- Undermines competition and drives prices below fair value.
- Harder in first-price auctions; easier in second-price.

Auctions and strategic manipulation

Entry deterrence and predation

- Incumbents can bid aggressively to keep rivals out.
- Example: ITV franchise auctions, local monopolies bid almost nothing when no competition existed.
- “We’ll top any bid” tactics discourage opponents.

Signalling

- Bids across multiple auctions communicate intent.
- Example: U.S. spectrum auctions, “We’ll take SF, LA; stay out of our patch.”
- Indirect signals blur the line between strategy and collusion.

Auctions and behavioural/structural effects

Risk aversion

- Risk-averse bidders prefer certain small over uncertain high gains. $\Rightarrow$ higher bids in first-price auctions, lower efficiency.

Budget constraints and externalities

- Limited liquidity caps bidding.
- All-pay auctions are more profitable, but attract fewer bidders.
- Externalities matter (arms races).

Combinatorial Auctions

Key idea

Bidders have preferences for *bundles* of items due to externalities.

- Example: landing slots at airports, spectrum, mineral rights.
- Bid on bundles: \$x for A+B+C, \$y for A+D+E.
- Critical CS application: routing under congestion.
- One part of a bundle is useless without the others $\rightarrow$ combinatorial complexity.
- Allocation problem is NP-complete.
- Emerging field: **algorithmic mechanism design** studies how to make combinatorial auctions *strategy-proof*.

Generalised second-price auction (Google/Meta)

Main idea

Ads are allocated via a *generalized second-price (GSP) auction*.

- Each advertiser submits a bid and has a quality score.
- Ad rank = bid × quality score $\rightarrow$ determines slot assignment.
- Highest ad rank gets top slot, second-highest $\rightarrow$ next slot, etc.
- Price per click = minimum bid needed to maintain your position.

Numerical illustration

	Sam	Mary	Jane	Pat
Quality Score	10	4	1	2
Max bid	$2	$4	$8	$6
Ad Rank	20	16	8	12

	Sam	Mary	Pat
Quality Score	10	4	2
Ad Rank	20	16	12
Cost Per Click	16/10 + .01 = $1.61	12/4 + .01 = $3.01	8/2 + .01 = $4.01

Calculating ad rank and price

- Ad rank $=$ quality $\times$ bid
- Cost per click $= \frac{\text{Ad rank of next advertiser}}{\text{Your quality}} + 0.01$

Ad auctions have unintended consequences

From ad quality to virality

- Ad rank depends on both bid and quality score.
- In social media, quality $\simeq$ virality: clicks, shares, engagement.
- High engagement reduces cost per click $\rightarrow$ incentives for provocative content.

Potential backfire

- Clickbait and sensationalism get rewarded financially.
- Can lead to echo chambers and extreme content.

From auctions to game theory

Observation

Auctions are structured games: each bidder strategises based on others' actions and information.

- Bidders anticipate competitors' moves.
- Game theory models such strategic interactions.
- Concepts like equilibrium, dominance, and signalling.
- Next: we introduce basics of game theory.

Core concepts in game theory

Game

A **game** is a model of strategic interaction where multiple agents (players) make decisions that affect each other's outcomes.

Strategy

A **strategy** is a complete plan of action describing how a Alicects in every possible situation of the game.

Nash Equilibrium (NE)

A **Nash equilibrium** is a set of strategies, one per player, such that no player can improve their payoff by unilaterally changing their own strategy. It almost always exists, but may not be unique.

Example: A simple 2×2 game

	Bob	
Alice	**Left**	**Right**
Up	(3, 3)	(0, 5)
Down	(5, 0)	(1, 1)

Interpretation

Each cell shows the payoffs **(Alice, Bob)**. Example: if A plays **Up** and B plays **Left**, both get 3.

Dominant strategy equilibrium

	Bob	
Alice	Left	Right
Top	(1, 2)	(0, 1)
Bottom	(2, 1)	(1, 0)

	Bob	
Alice	Left	Right
Top	(1, 2)	(0, 1)
Bottom	(2, 1)	(1, 0)

Iterated elimination of strictly dominated strategies

Each player can rule out strategies their opponent would never take, and narrow down (or even fully pin down) their decision.

Battle of the Sexes

Alice \ Bob	Football	Opera
Football	(2, 1)	(0, 0)
Opera	(0, 0)	(1, 2)

Alice \ Bob	Football	Opera
Football	(2, 1)	(0, 0)
Opera	(0, 0)	(1, 2)

Multiplicity of Nash equilibria

- Coordination: both prefer being together, but differ on where.
- Pure NE: (Football, Football) and (Opera, Opera).
- Mixed NE: Each player chooses their preferred activity with prob 2/3, expected payoffs 2/3 for each player.
- Correlated equilibrium: Flip a 50/50 coin and choose one of the pure NE accordingly, expected payoffs of 3/2 for each player.

Matching pennies & Rock-Paper-Scissors

Matching Pennies

	Bob	
Alice	H	T
H	(-1, 1)	(1, -1)
T	(1, -1)	(-1, 1)

Rock-Paper-Scissors

	Bob		
Alice	Scissors	Paper	Stone
Scissors	(0, 0)	(1, -1)	(-1, 1)
Paper	(-1, 1)	(0, 0)	(1, -1)
Stone	(1, -1)	(-1, 1)	(0, 0)

Observation

- Both games are zero-sum, I win if you lose and vice versa.
- Both games have no pure strategy NE.
- Only mixed strategy NE exist.
- In Rock-Paper-Scissors, empirically, players often stick with a winning move and change losing moves.

Prisoner's Dilemma

	Prisoner B	
Prisoner A	Confess	Deny
Confess	(-3, -3)	(0, -6)
Deny	(-6, 0)	(-1, -1)

Observation

- (Confess, Confess) is the dominant strategy equilibrium.
- Not Pareto efficient: both would be better off with (Deny, Deny).
- Question: How can cooperation be encouraged?

Evolutionary game theory

Tit-for-Tat Strategy

- Cooperate in the first round.
- In round n, do what the opponent did in round $n-1$.
- Encourages cooperation and punishes defection.
- Veritasium excellent video

Bob Axelrod (1981)

Tit-for-Tat's success comes from being nice, retaliatory, forgiving, and clear.

Evolution of the Hawk-Dove game

	Hawk	Dove
Hawk	$\frac{v-c}{2}, \frac{v-c}{2}$	$v, 0$
Dove	$0, v$	$\frac{v}{2}, \frac{v}{2}$

- Models conflict between aggressive (Hawk) and peaceful (Dove) strategies.
- Food value v at each round; doves share; hawks take food from doves; hawks fight (risk of death c).
- Mixed strategy equilibrium: probability of Hawk $p = \frac{v}{c}$.
- If $v > c$, all-hawk population emerges (dominant strategy).
- If $c > v$, a mix of hawks and doves evolves.
- Mixed strategy can be interpreted as a population mixture.

Tit-for-Tat in airline pricing

Scenario

- Flight LHR-JFK costs $250 to operate.
- Airline A tries to charge $500.
- Other airlines may 'defect' by undercutting.
- Airline A responds by matching competitors $\rightarrow$ tit-for-tat.

Regulator perspective

- Hard to detect implicit collusion.
- Need monitoring, incentives, and competition enforcement.
- Tit-for-tat can sustain high prices without explicit agreement.

Stag Hunt

	Hunter B	
Hunter A	Hare	Stag
Hare	(2, 2)	(5, 0)
Stag	(0, 5)	(10, 10)

Observations

- Difference from prisoner's dilemma: (Stag, Stag) is also NE.
- You'll only chase a hare if you believe other hunter will defect.
- (Stag, Stag) is payoff-dominant, (Hare, Hare) is risk-dominant.

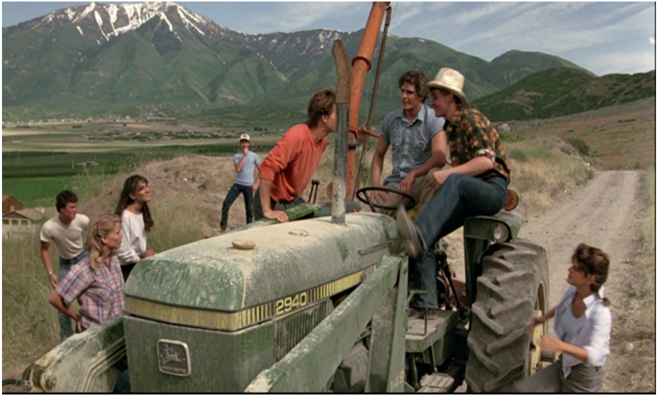
Chicken Game

	Chuck	
Ren	Jump	Drive on
Jump	(2, 2)	(1, 3)
Drive on	(3, 1)	(0, 0)

Observations

- Nash equilibria: (Jump, Drive on) and (Drive on, Jump).
- Bertrand Russell suggested this as a model of nuclear confrontation during the Cold War.
- A player can “win” if they credibly commit to drive on first.

Commitment in chicken game (Footloose, 1984)



Applications: Matching pennies

Attacker vs defender in cybersecurity

- Defender may not have the resources to patch all possible vulnerabilities.
- Attacker may not know which vulnerabilities are undefended.
- Example: network security or intrusion detection systems, attackers and defenders must continuously adapt and guess each other's moves.

Applications: Prisoner's Dilemma

Two organisations securing communication channels

- Cooperation (costly encryption) vs. defection (saving encryption costs) determines whether communication is secure or vulnerable.
- Examples: security standard agreements between competing companies, public-private sector cooperation in cybersecurity, and user adherence to safety protocols.
- If interactions are repeated, cooperation is more likely to emerge.

Applications: Battle of the Sexes

Negotiating communication protocols

- One system prefers a modern protocol (IPv6), the other prefers legacy (IPv4).
- Both systems prefer to coordinate, but agreement is hard to reach.
- Examples: distributed computing and network protocols where systems need to agree on standards or communication methods (TCP/IP vs. UDP, HTTP vs. HTTPS).

Lecture 4 Overview & Thanks

Topics Covered

- **Auctions:** types, strategic & revenue equivalence, winner's curse, bidding rings, auctions in digital industries.
- **Game Theory:** definitions, NE, dominant strategies, repeated games, Prisoner's Dilemma, Battle of the Sexes, Chicken, Stag Hunt, Hawk-Dove.
- **Applications in Computing and CS:** ad auctions, routing with congestion, cybersecurity scenarios (matching pennies, PD, BoS), mechanism design.

Enjoy the next lectures and good luck!