Algebraic Techniques for Programming: Exercise Sheet

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Lent 2026

Exercise 1. Define two different monoids whose carrier is the natural numbers.

Exercise 2. Add a property to the definition of monoid to make it into a *commutative* monoid.

Exercise 3. What are the initial and final objects in Poset, the category of partially ordered sets and monotone functions?

Exercise 4. What are the initial and final objects in CMon, the category of commutative monoids and monoid homomorphisms?

Exercise 5. What do products in Rel, the category of sets and relations, look like? (*Hint.* The product of A and B is not the cartesian product of sets!)

Exercise 6. The signature for `INDUCTIVE` has a comment saying that `out` is not strictly necessary. Show that you can implement `out` using `fold`, `into` and `F.map`. Why did we include it in the API nonetheless?

Exercise 7. Prove that $[\![F]\!]$ defines a functor for all F.

Exercise 8. Recall that if an object A and a family of maps $f_n : A \to X_n$ forming a cone over the projective diagram, the mediating map $\vec{f}$ can be explicitly given as:

$$\vec{f}\,(a) = n \mapsto f_n(a)$$

Verify that if there is any other $h : A \to \lim X_i$ such that $f_i = h; \pi_i$ for every i, then $h = \vec{f}$.

Exercise 9. The *Levenshtein distance*, or *edit distance*, between two strings be naively computed as follows (writing $\langle\rangle$ for the empty string, and $c \cdot s$ for the string starting with the character c and continuing with s):

$$\begin{array}{lcl} \mathsf{lev}(s_1, \langle\rangle) & = & |s_1| \\ \mathsf{lev}(\langle\rangle, s_2) & = & |s_2| \\ \mathsf{lev}(c \cdot s_1', c \cdot s_2') & = & \mathsf{lev}(s_1', s_2') \\ \mathsf{lev}(c \cdot s_1', d \cdot s_2') & = & 1 + \min(\mathsf{lev}(s_1, s_2'), \mathsf{lev}(s_1', s_2), \mathsf{lev}(s_1', s_2')) \end{array}$$

Formulate this algorithm as a coalgera-to-algebra morphism, and then solve it with dynamic programming.