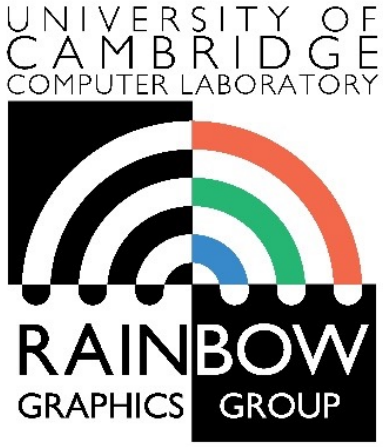

Advanced Graphics & Image Processing

Advanced image processing

Part 1/2 – edge stopping filters

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Edge stopping filters

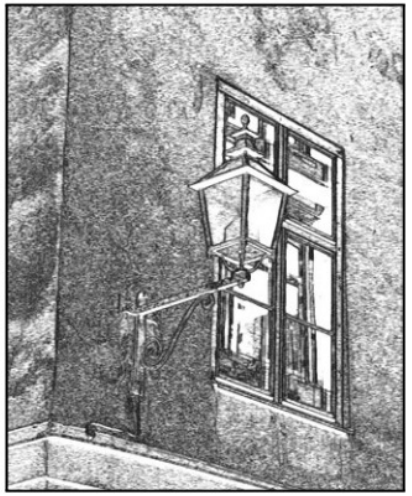
Original

Edge-aware smoothing

Detail enhancement

Stylization

Recoloring



Pencil drawing

Depth-of-field

Examples from [Gastal & Oliveira 2011]

Nonlinear filters: Bilateral filter

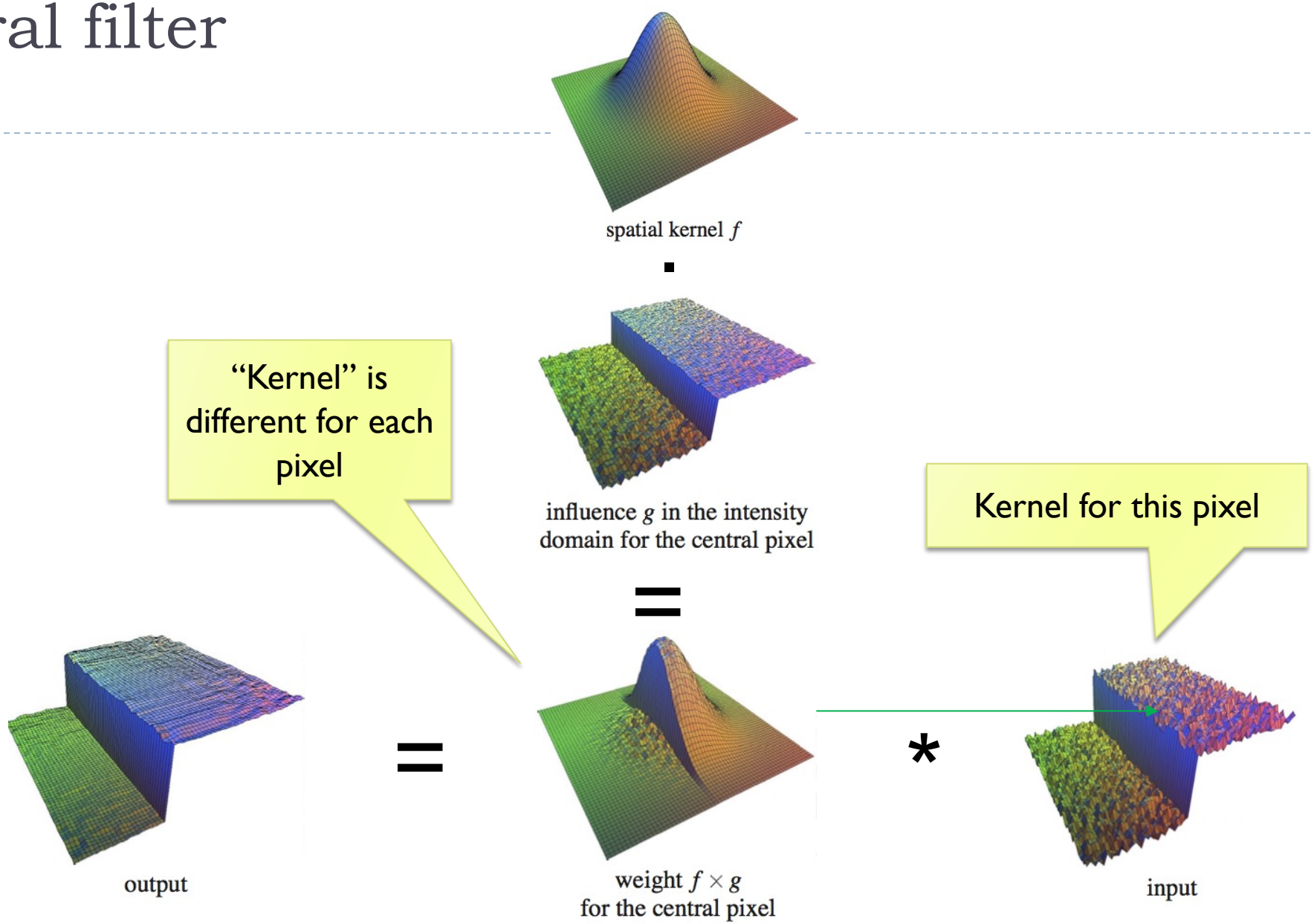
- Goal: Smooth out an image without blurring edges

Gaussian filter

Unsharp masking

Bilateral filter

Bilateral filter



Bilateral filter

Input image

$$y(\boldsymbol{p}) = \frac{\sum_{\boldsymbol{q}\in\Omega} x(\boldsymbol{q})w(\boldsymbol{p},\boldsymbol{q})}{\sum_{\boldsymbol{q}\in\Omega} w(\boldsymbol{p},\boldsymbol{q})}$$

Pixel coordinates $\boldsymbol{p} = (i, j)$

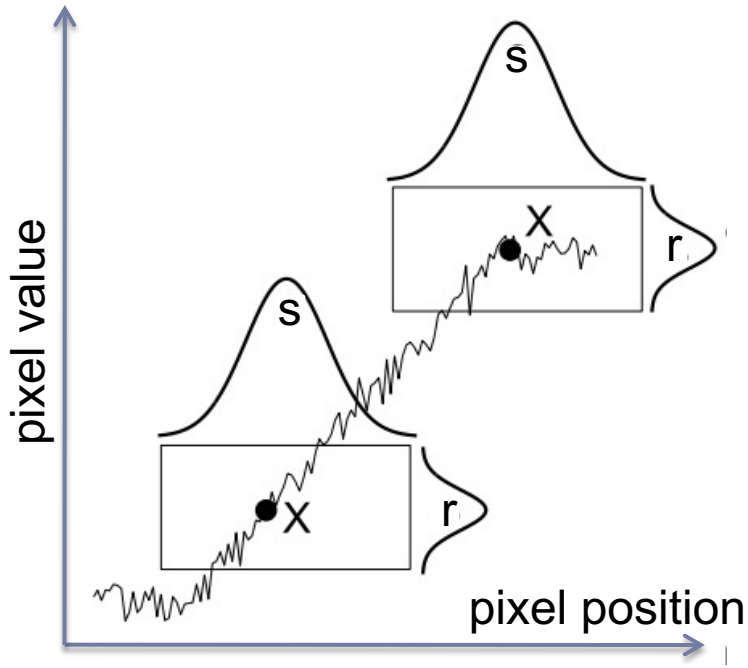
Neighborhood of the pixel **p**

$$w(\boldsymbol{p},\boldsymbol{q}) = g_s(\boldsymbol{p}-\boldsymbol{q})\, g_r(x(\boldsymbol{p}) - x(\boldsymbol{q}))$$

distance in the spatial position (x,y)

distance (difference) in pixel values

$$g_s(\boldsymbol{d}) = \exp\left(\frac{-\|\boldsymbol{d}\|_2}{2\sigma_s^2}\right) \qquad g_r(d) = \exp\left(\frac{-d^2}{2\sigma_r^2}\right)$$



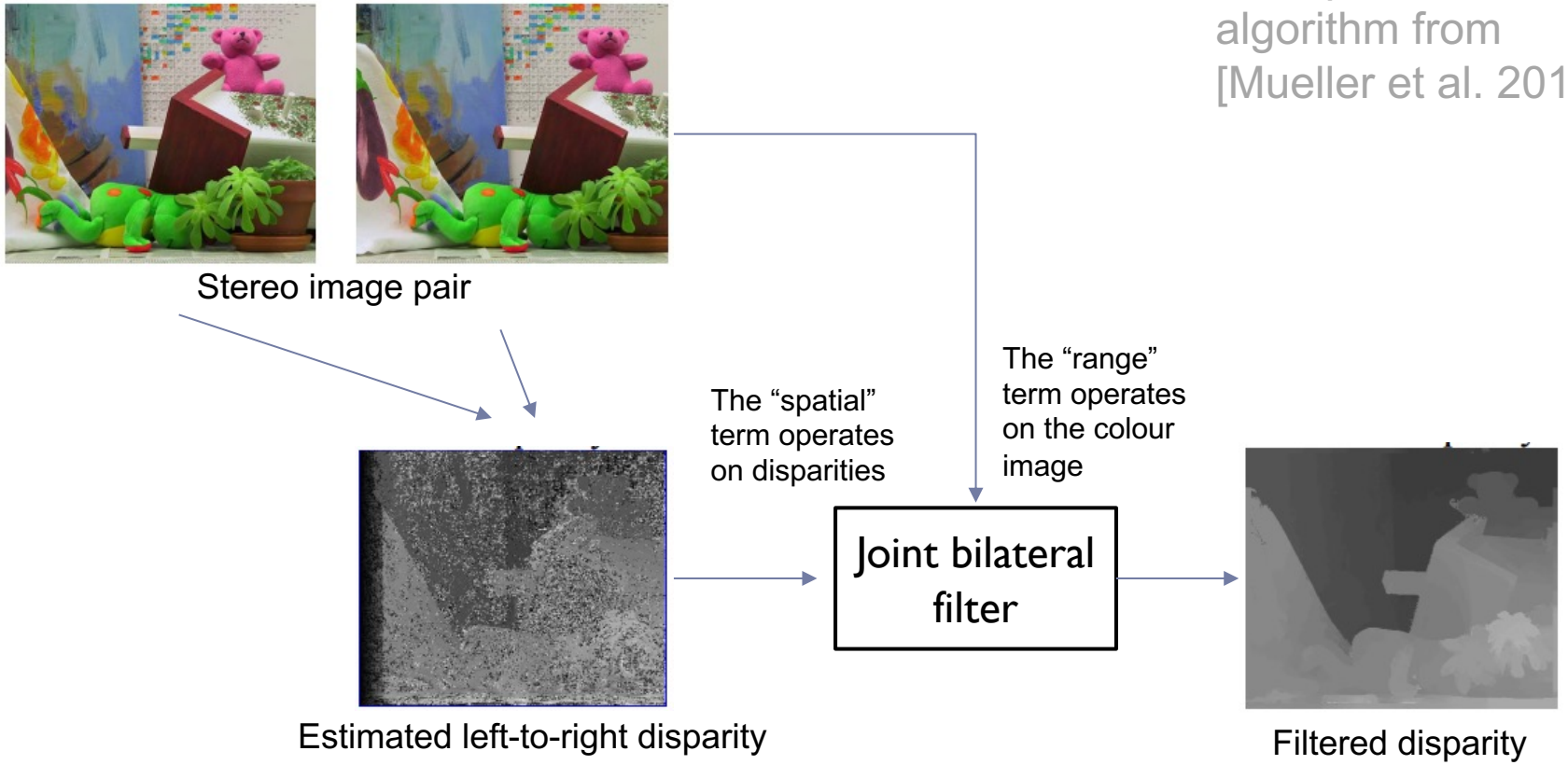
How to make the bilateral filter fast?

- A number of approximations have been proposed
  - Combination of linear filters [Durand & Dorsey 2002, Yang et al. 2009]
  - Bilateral grid [Chen et al. 2007]
  - Permutohedral lattice [Adams et al. 2010]
  - Domain transform [Gastal & Oliveira 2011]

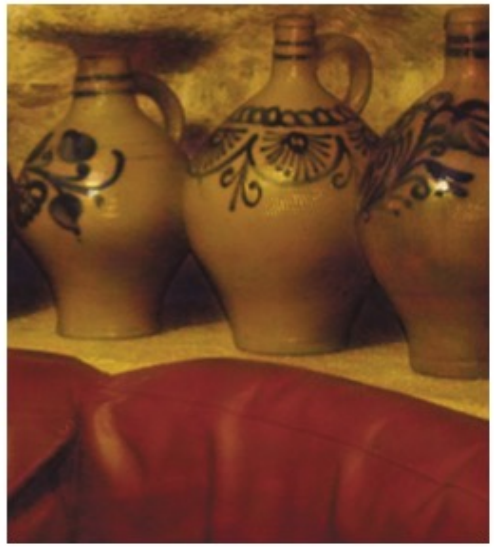
Joint-bilateral filter (a.k.a guided/cross b.f.)

- The "range" term does not need to operate in the same domain as the filter output
  - Example:

A simplified algorithm from [Mueller et al. 2010]



Joint bilateral filter: Flash / no-flash

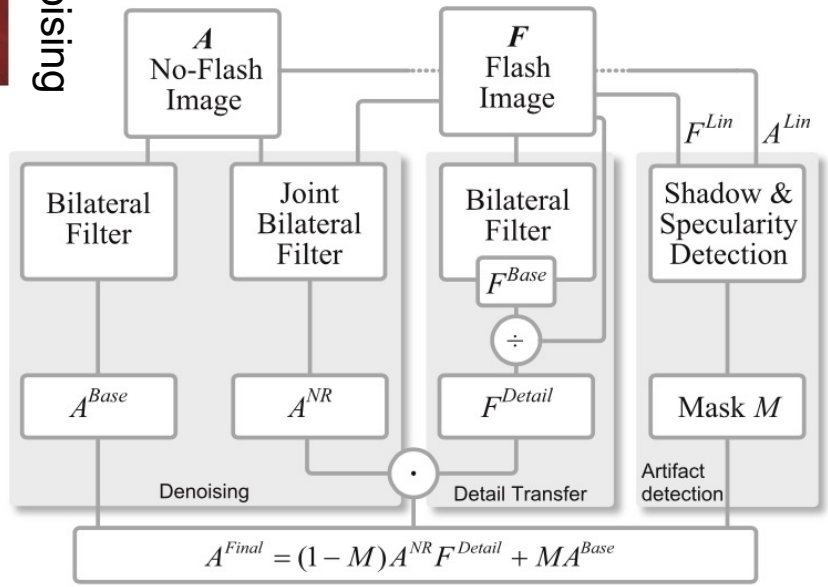


Flash

No-flash

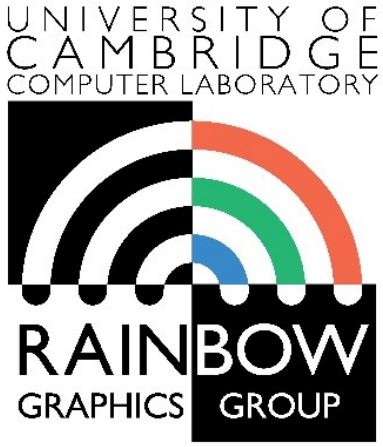
- Preserve colour and illumination from the no-flash image
- Use flash image to remove noise and add details
- [Petshnigg et al. 2004]

Detail transfer with denoising



Example of edge preserving filtering

- Domain Transform for Edge-Aware Image and Video Processing
- Video:
  - https://youtu.be/UI1xh1IQrTY?t=4m10s
  - From: http://inf.ufrgs.br/~eslgastal/DomainTransform/



Advanced Graphics & Image Processing

Advanced image processing

Part 1/2 – processing by optimization

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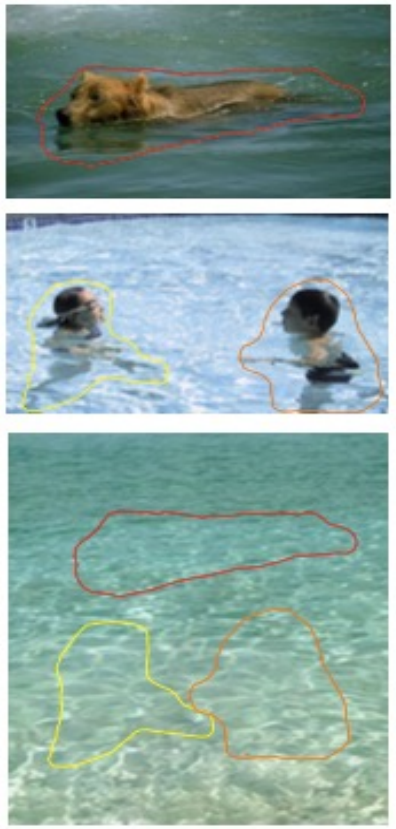
Optimization-based methods



sources/destinations

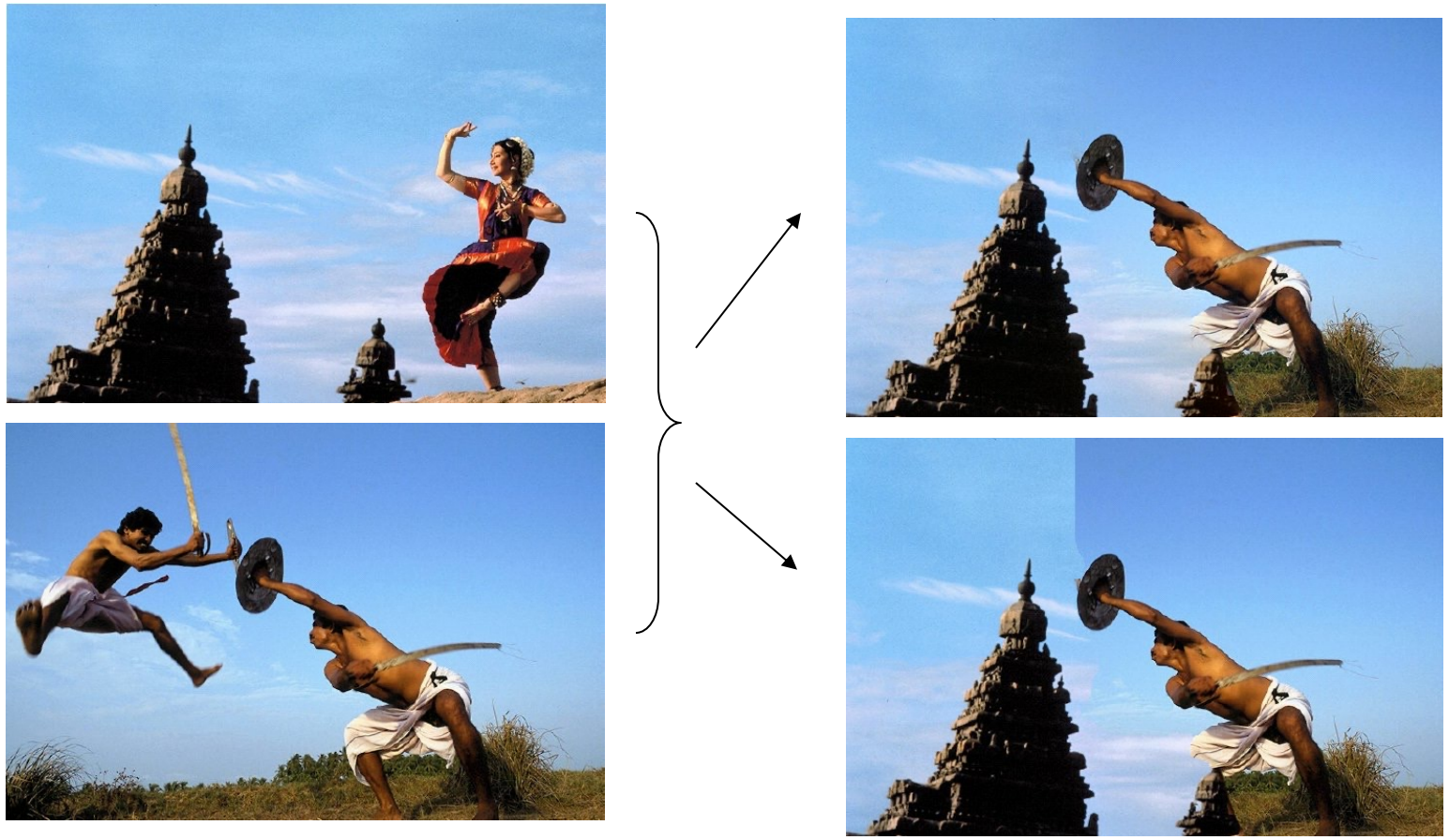
cloning

seamless cloning

Poisson image editing [Perez et al. 2003]

Gradient Domain compositing

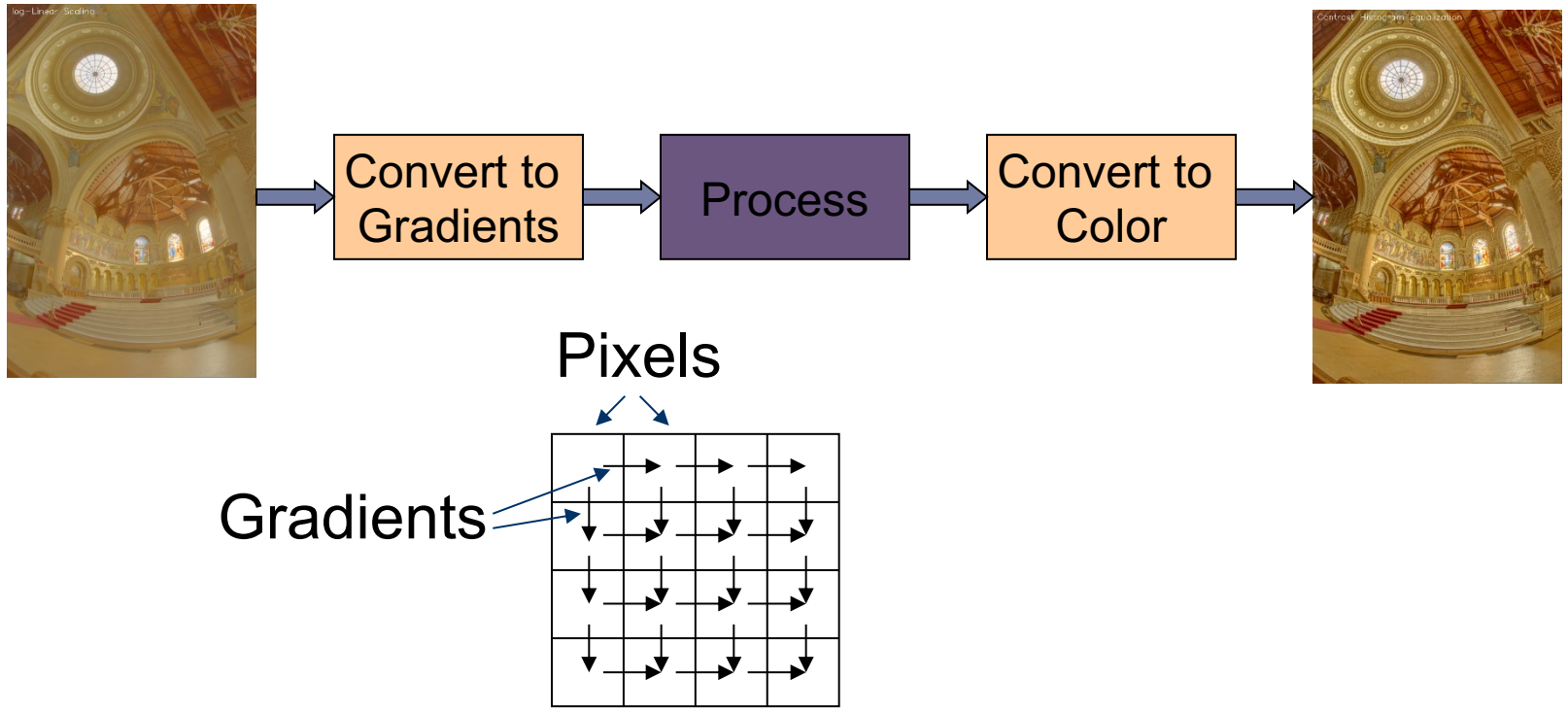
- Compositing [Wang et al. 2004]



images from [Drori at al. 2004]

Gradient domain methods

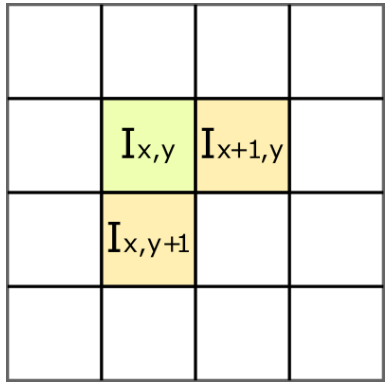
- Operate on pixel gradients instead of pixel values



Forward Transformation

- Forward Transformation
  - Compute gradients as forward differences between a pixel and its two neighboors

$$\nabla I_{x,y} = \begin{bmatrix} I_{x+1,y} - I_{x,y} \\ I_{x,y+1} - I_{x,y} \end{bmatrix}$$



  - Result: 2D gradient map (2 x more values than the number of pixels)

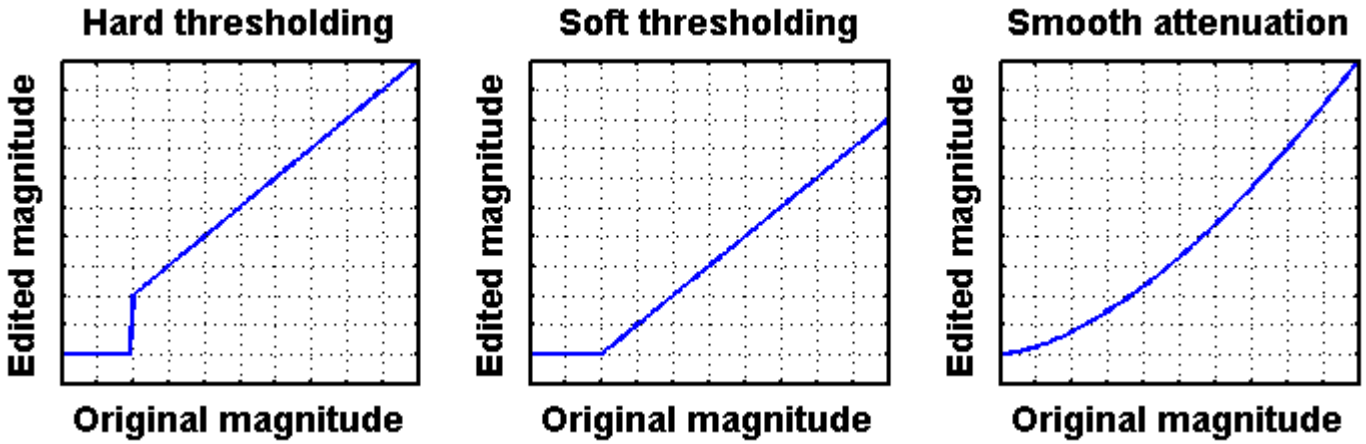
Processing gradient field

- Typically, gradient magnitudes are modified while the gradient direction (angle) remains the same

$$G_{x,y} = \nabla I_{x,y} \cdot \frac{f\left(||\nabla I_{x,y}||\right)}{||\nabla I_{x,y}||}$$

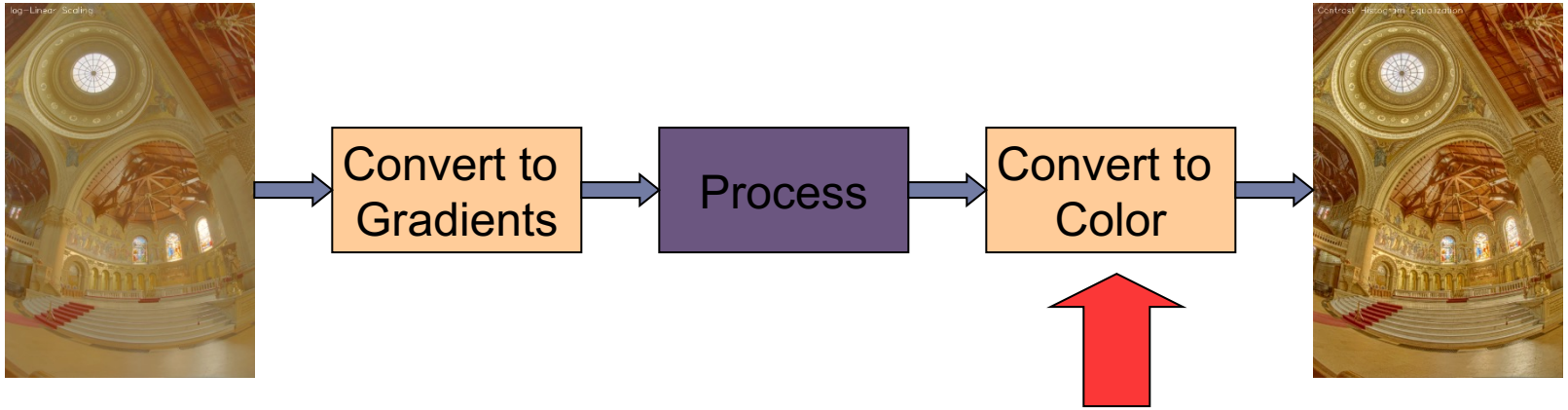
Gradient editing function

- Examples of gradient editing functions:



Inverse transform: the difficult part

- There is no straightforward transformation from gradients to luminance



- Instead, a minimization problem is solved:

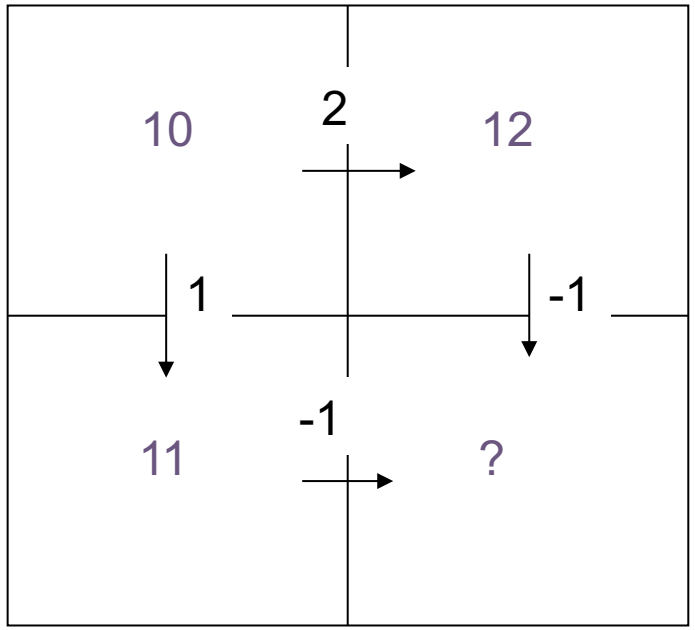
$$\arg\min_{I} \sum_{x,y} \left[\left(I_{x+1,y} - I_{x,y} - G^{(x)}_{x,y} \right)^2 + \left(I_{x,y+1} - I_{x,y} - G^{(y)}_{x,y} \right)^2 \right]$$

Image Pixels

Target gradients

Inverse transformation

- Convert modified gradients to pixel values
  - Not trivial!
  - Most gradient fields are inconsistent - do not produce valid images
  - If no accurate solution is available, take the best possible solution
  - Analogy: system of springs



Gradient field reconstruction: derivation

- The minimization problem is given by:

$$\arg\min_{I} \sum_{x,y} \left[\left(I_{x+1,y} - I_{x,y} - G^{(x)}_{x,y} \right)^2 + \left(I_{x,y+1} - I_{x,y} - G^{(y)}_{x,y} \right)^2 \right]$$

- After equating derivatives over pixel values to 0 we get:
  - Derivation done in the lecture

$$I_{x-1,y} + I_{x+1,y} + I_{x,y-1} + I_{x,y+1} - 4I_{x,y} = G^{(x)}_{x,y} - G^{(x)}_{x-1,y} + G^{(y)}_{x,y} - G^{(y)}_{x,y-1}$$

  - In matrix notation:

Laplace operator (NxN matrix)

$$\nabla^2 I = \mathrm{div} G$$

Divergence of a vector field (Nx1 vector)

$$\nabla^2 I_{x,y} = I_{x-1,y} + I_{x+1,y} + I_{x,y-1} + I_{x,y+1} - 4I_{x,y}$$

Image as a column vector

$$\begin{bmatrix} I_{1,1} \\ I_{2,1} \\ \dots \\ I_{N,M} \end{bmatrix}$$

$$\mathrm{div} G_{x,y} = G^{(x)}_{x,y} - G^{(x)}_{x-1,y} + G^{(y)}_{x,y} - G^{(y)}_{x,y-1}$$

Laplace operator for 3x3 image

$$\nabla^2 = \begin{bmatrix} -2 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -3 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -3 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -3 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -2 \end{bmatrix}$$

Solving sparse linear systems

- Just use "\" operator in Matlab / Octave:
  - x = A \ b;
- Great "cookbook":
  - TEUKOLSKY, S.A., FLANNERY, B.P., PRESS, W.H., AND VETTERLING, W.T. 1992. *Numerical recipes in C*. Cambridge University Press, Cambridge.
- Some general methods
  - Cosine-transform – fast but cannot work with weights (next slides) and may suffer from floating point precision errors
  - Multi-grid – fast, difficult to implement, not very flexible
  - Conjugate gradient / bi-conjugate gradient – general, memory efficient, iterative but fast converging
  - Cholesky decomposition – effective when working on sparse matrices

Pinching artefacts

- A common problem of gradient-based methods is that they may result in "pinching" artefacts (left image)
- Such artefacts can be avoided by introducing weights to the optimization problem

Weighted gradients

- The new objective function is:

$$\arg\min_{I} \sum_{x,y} \left[w_{x,y}^{(x)} \left(I_{x+1,y} - I_{x,y} - G_{x,y}^{(x)} \right)^2 + w_{x,y}^{(y)} \left(I_{x,y+1} - I_{x,y} - G_{x,y}^{(y)} \right)^2 \right]$$

- so that higher weights are assigned to low gradient magnitudes (in the original image).

$$w_{x,y}^{(x)} = w_{x,y}^{(y)} = \frac{1}{||G_{x,y}|| + \epsilon}$$

- The linear system can be derived again
  - but this is a lot of work and is error-prone

Weighted gradients - matrix notation (1)

- The objective function:

$$\arg\min_{I} \sum_{x,y} \left[w_{x,y}^{(x)} \left(I_{x+1,y} - I_{x,y} - G_{x,y}^{(x)} \right)^2 + w_{x,y}^{(y)} \left(I_{x,y+1} - I_{x,y} - G_{x,y}^{(y)} \right)^2 \right]$$

- In the matrix notation (without weights for now):

$$\arg\min_{I} \left|\left| \begin{bmatrix} \nabla_x \\ \nabla_y \end{bmatrix} I - \begin{bmatrix} G^{(x)} \\ G^{(y)} \end{bmatrix} \right|\right|^2$$

Note that I is a column vector with an image. It is not an identity matrix!

- Gradient operators (for 3x3 pixel image):

$$\nabla_x = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \nabla_y = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Weighted gradients - matrix notation (2)

- The objective function again:

$$\arg\min_{I} \left|\left| \begin{bmatrix} \nabla_x \\ \nabla_y \end{bmatrix} I - \begin{bmatrix} G^{(x)} \\ G^{(y)} \end{bmatrix} \right|\right|^2$$

- Such over-determined least-square problem can be solved using pseudo-inverse:

Matrix transpose

$$\begin{bmatrix} \nabla'_x & \nabla'_y \end{bmatrix} \begin{bmatrix} \nabla_x \\ \nabla_y \end{bmatrix} I = \begin{bmatrix} \nabla'_x & \nabla'_y \end{bmatrix} \begin{bmatrix} G^{(x)} \\ G^{(y)} \end{bmatrix}$$

- Or simply:

$$\left(\nabla'_x \nabla_x + \nabla'_y \nabla_y\right) I = \nabla'_x G^{(x)} + \nabla'_y G^{(y)}$$

- With weights:

$$\left(\nabla'_x W \nabla_x + \nabla'_y W \nabla_y\right) I = \nabla'_x W G^{(x)} + \nabla'_y W G^{(y)}$$

WLS filter: Edge stopping filter by optimization

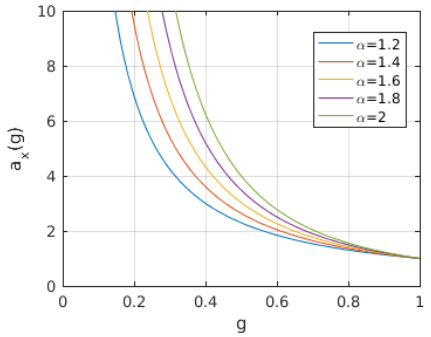
- Weighted-least-squares optimization

Make reconstructed image *u* possibly close to input *g*

Smooth out the image by making partial derivatives close to 0

$$\underset{\boldsymbol{u}}{\operatorname{argmin}} \sum_p \left((u_p - g_p)^2 + \lambda \left(a_{x,p}(g) \left(\frac{\partial u}{\partial x} \right)_p^2 + a_{y,p}(g) \left(\frac{\partial u}{\partial y} \right)_p^2 \right) \right)$$

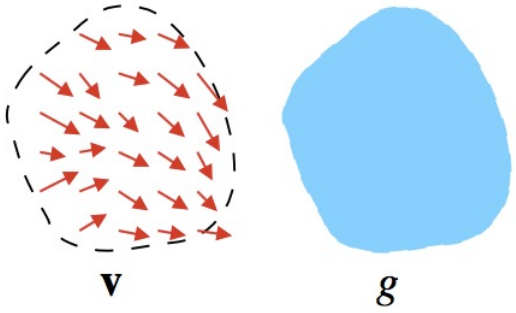
Spatially varying smoothing – less smoothing near the edges

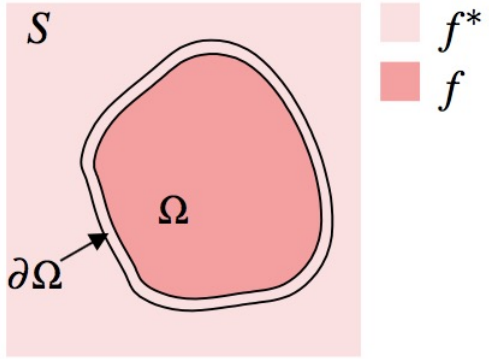


$$a_{x,p}(g) = \frac{1}{\left| \frac{\partial u}{\partial x}(g) \right|^{\alpha} + \epsilon}$$

- [Farbman, Z., Fattal, R., Lischinski, D., & Szeliski, R. (2008). Edge-preserving decompositions for multi-scale tone and detail manipulation. *ACM SIGGRAPH 2008*, 1–10.]

Poisson image editing





f^*

f

$$\min_f \iint_\Omega |\nabla f - \mathbf{v}|^2$$

subject to: $f|_{\partial\Omega} = f^*|_{\partial\Omega}$

- Reconstruct unknown values *f* given a source guidance gradient field *v* and the boundary conditions

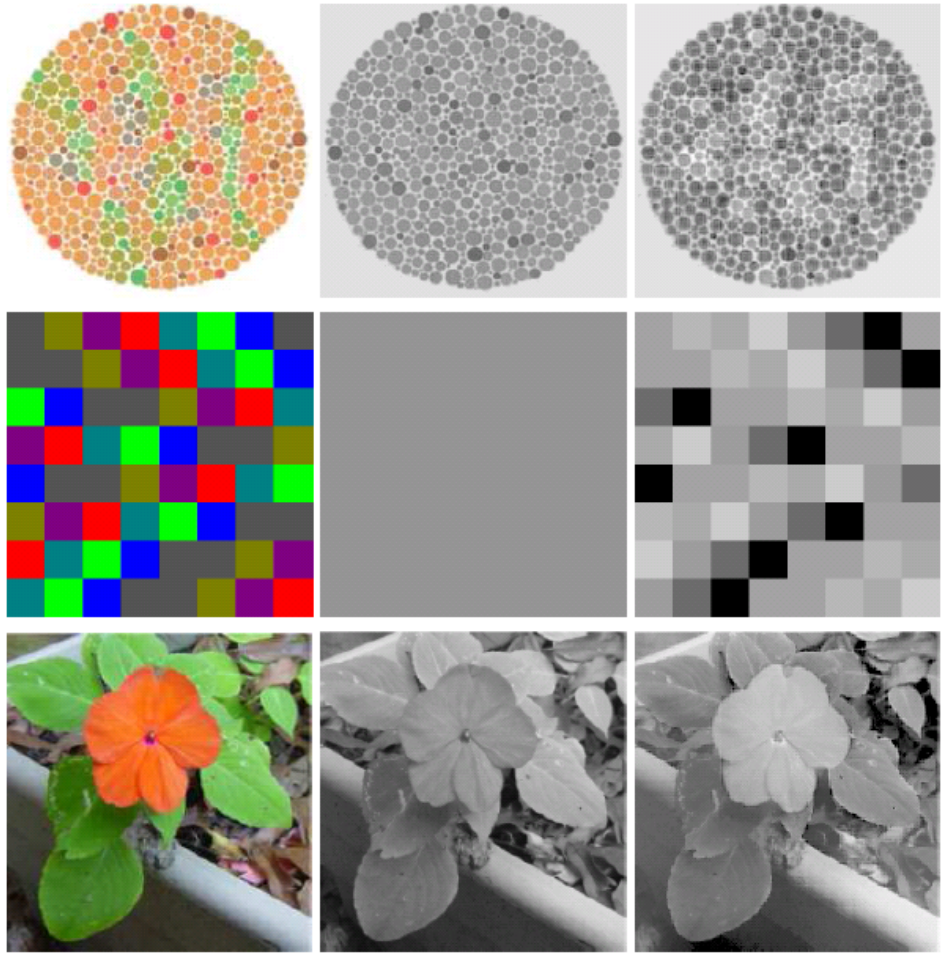
$$f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

- [Pérez, P., Michel Gangnet, & Blake, A. (2003). Poisson Image Editing. *ACM Transactions on Graphics*, *3*(22), 313–318. https://doi.org/10.1145/882262.882269]

Colour to Gray

- Transform colour images to greyscale
- Preserve colour saliency
  - When gradient in luminance close to 0
  - Replace it with gradient in chrominance
  - Reconstruct an image from gradients

- Gooch, A. A., Olsen, S. C., Tumblin, J., & Gooch, B. (2005). Color2Gray. *ACM Transactions on Graphics*, *24*(3), 634. https://doi.org/10.1145/1073204.1073241



Gradient domain – Applications

- More applications:
  - Lightness perception (Retinex) [Horn 1974]
  - Matting [Sun et al. 2004]
  - Color to gray mapping [Gooch et al. 2005]
  - Video Editing [Perez at al. 2003, Agarwala et al. 2004]
  - Photoshop's Healing Brush [Georgiev 2005]

References

- F. Durand and J. Dorsey, "Fast bilateral filtering for the display of high-dynamic-range images," *ACM Trans. Graph.*, vol. 21, no. 3, pp. 257–266, Jul. 2002.
- E. S. L. Gastal and M. M. Oliveira, "Domain transform for edge-aware image and video processing," *ACM Trans. Graph.*, vol. 30, no. 4, p. 1, Jul. 2011.
- Patrick Pérez, Michel Gangnet, and Andrew Blake. 2003. Poisson image editing. *ACM Trans. Graph.* 22, 3 (July 2003), 313-318. DOI: http://dx.doi.org/10.1145/882262.882269
- Zeev Farbman, Raanan Fattal, Dani Lischinski, and Richard Szeliski. 2008. Edge-preserving decompositions for multi-scale tone and detail manipulation. *ACM Trans. Graph.* 27, 3, Article 67 (August 2008), 10 pages. DOI: https://doi.org/10.1145/1360612.1360666