

Optimising Compilers Exercise Sheet 3

The purpose of this exercise is to gain familiarity with *constraint-based analyses*, particularly *0CFA* (zeroth-order control-flow analysis).

Questions

1. (a) What is a higher-order function?
- (b) How do higher-order functions make it harder to predict control flow within a program?
- (c) How does the 0CFA help to predict control flow?
- (d) Do object-oriented programs have analysis issues related to higher-order functions?

Consider the following simple λ -calculus like language, call it $\mathcal{L}$:

$$e ::= v \mid c \mid \lambda v.e \mid e_1 e_2 \mid \mathbf{let} \ v = e_1 \ \mathbf{in} \ e_2 \mid \mathbf{if} \ e_1 \ \mathbf{then} \ e_2 \ \mathbf{else} \ e_3 \mid e_1 \oplus e_2$$

where v ranges over variables, c ranges over integer constants, and $\oplus$ ranges over binary operations.

0CFA computes information about control flow in a program by computing a subset of a program's data flow: the flow of functions (or function pointers). In the following, the data flow of integer constants will also be tracked to aid understanding.

2. (a) Define informally the notion of a *binding site* and *use site* and indicate the binding and use sites in the syntax of $\mathcal{L}$.
- (b) The following expression has a *single program point* labelling the formal parameter x of f :

$$\mathbf{let} \ f = (\lambda x^0.x + x) \ \mathbf{in} \ f \ 2 + f \ 3$$

Label the remaining program points (it may help to write the expression as a tree).

- (c) Given *flow variables* α_i associating sets to each program point, what is the value of set α_0 following a 0CFA? What integer values flow out of the body of the λ ?
- (d) Write down and explain the rule for generating constraints for **let**-bindings and variables v .
- (e) Consider the following expression with a partial labelling of program points:

$$\mathbf{let} \ f = (\lambda x.x^1 \ 0) \ \mathbf{in} \ (\mathbf{let} \ g = (\lambda y^0.y + 1) \ \mathbf{in} \ (f \ g) + (g \ 1))$$

Compute the flow sets for α_1 and α_0 .

3. (a) Calculate a full 0CFA (tracking just function values, not integer values) for the following expression:
- $$\mathbf{let} \ f = (\lambda x.x \ 0) \ \mathbf{in} \ (f (\lambda y.y * 3)) + (f (\lambda z.z + 1))$$
- (b) Write down and explain the rule for generating constraints for functions and function application.

Suggested past exam questions[2004 Paper 9 Question 3](#)[2007 Paper 9 Question 16](#)**Relevant past exam questions**

This section contains links to all past exam questions relevant to the topics covered in this supervision sheet. Note that some questions appear under multiple headings and / or on multiple exercise sheets when they cover more than one topic.

- [2015 Paper 9 Question 9](#)
- [2007 Paper 9 Question 16](#)
- [2004 Paper 9 Question 3](#)
- [1998 Paper 9 Question 7](#)