Economics, Law and Ethics

Part IB CST

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Lecture 4: Auction theory and game theory

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Overview

- Auctions:
  - Types of auctions
  - Equivalence
  - What goes wrong
  - Advertising auctions
- Game theory:
  - Cooperation or conflict
  - Strategies
  - Types of games
  - Broader implications

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Auctions

- Around for millennia; standard way of selling livestock, fine art, mineral rights, bonds…
- Many other sales from corporate takeovers to house sales are also really auctions
- Auctions are a big success of the Internet, from eBay to Google
- Some unpleasant side-effects
- Rapidly growing interest in theoretical computer science: auction resources in distributed systems
- Many issues of asymmetric info, signaling, strategic play… – plus some solid theory!

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Types of auction

- English, or ascending-bid: start at reserve price and raise till a winner is left (art, antiques)
- Dutch, or descending-bid: start high and cut till somebody bids (flowers)
- First-price sealed-bid auction: one bid per bidder (government contracts)
- Second-price sealed-bid auction, or Vickrey auction: highest bidder wins and pays second-highest bid (postage stamps)
- All-pay auction: everyone pays at every round until one remaining bidder gets the goods (war, litigation, winner-takes-all market race)

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The Aalsmeer flower auction

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Strategic equivalence

- A Dutch auction and a first-price sealed-bid auction give the same result: the highest bidder gets the goods at his reservation price
- They are ‘strategically equivalent’
- Ditto the English auction and the second-price sealed-bid auction (modulo the bid increment)
- But the two pairs are not strategically equivalent!
  - in a second-price auction it’s best to bid truthfully
  - in a Dutch / first-price auction, you should bid low if you think your valuation is much higher than everybody else’s

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Revenue equivalence

- This is weaker – not 'who will win' but 'how much money on average'
- According to the revenue equivalence theorem, you get the same revenue from any well-behaved auction under ideal conditions
- These include risk-neutral bidders, no collusion, Pareto efficiency (highest value bidder gets goods), reserve price, independent valuations, …
- Then bidders adjust their strategies and the English, Dutch and all-pay auction yield the same
- So when you design an auction, you must focus on any ways the conditions aren't ideal

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What goes wrong (1)

- In a 'private-value auction', each bidder's value v_i is exogenous (think: sculpture). In a second-price auction, everything you buy is a bargain
- In a 'public-value auction', each item has a true price which bidders estimate at $v + \varepsilon_I$ (think mineral leases; spectrum auctions). The buyer is the sucker who overestimated the most!
- This is called 'the winner's curse'
- Many real auctions lie somewhere between these two extremes

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What goes wrong (2)

- Bidding rings – bidders collude to buy low, have a private auction later, split the proceeds
- First-price auctions are harder to rig; with second-price, New Zealand bids of $7m and $5000
- Entry detection / deterrence: an early (1991) ITV franchise auction required bidders to draw up a detailed programming plan. In Midlands & Central Scotland, industry knew there was no competition; bids under 1p per head (vs £9–16 elsewhere)
- Predation: 'we'll top any other bid' in takeovers
- Sniping and other boundary effects

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What goes wrong (3)

- Risk aversion: if you prefer a certain profit of £1 to a 50% chance of £2, you'll bid higher at a first-price auction
- Signaling games: show aggression by a price hike
- E.g. in simultaneous auctions, as in the USA, signal "we want SF, LA, SD and if you compete with us there we'll push prices up in your patch")
- Budget constraints: if bidders are cash-limited, all-pay auctions are more profitable
- Externalities between bidders – e.g. arms sales

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Combinatorial auctions

- Externalities lead to preferences for particular bundles of goods: landing slots at airports, spectrum, mineral rights…
- Bid ($x for A+B+C) or ($y for A+D+E) or…
- Critical app for CS: routing in presence of congestion (bid for AB and BC, or AD and DC…)
- The allocation problem is NP-complete; practical algorithms work up to a few thousand objects
- Also: how can we make the auction strategy-proof (i.e. truth-telling is the best strategy)?
- New field of 'algorithmic mechanism design'

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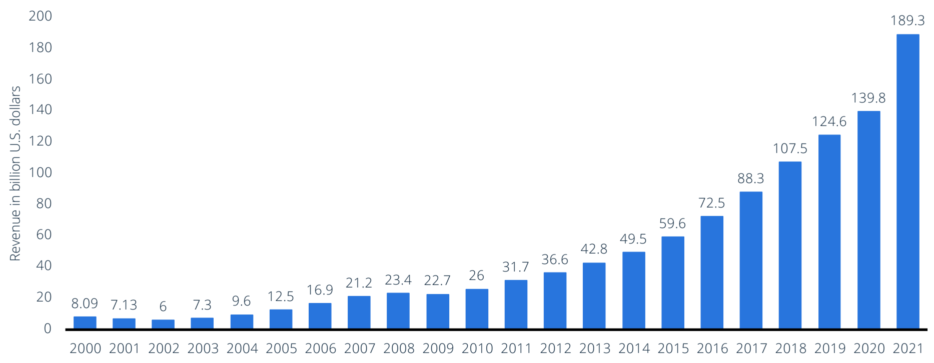
Ad auctions

Online advertising revenue in the United States 2000 to 2021 (in US$bn)



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Note(s): United States; 2000 to 2021
Source(s): PwC; IAB (U.S.); ID 183816

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Ad auctions (2)

- Pioneered by Google
- Basic idea: second-price auction mechanism but tweaked to optimise platform revenue
- Bidders bid prices p_i, platform estimates ad quality e_i, and then ad rank $a_i = p_i.e_i$
- Ad quality e_i = relevance . clickthrough rate
- So how do we work out who wins the auction and how much they pay?

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Ad auctions (3)

Advertiser	Quality Score	Bid	**Ad Rank**	Rank	CPC
Jerry	4	$2.00	8	1	$1.50
Elaine	2	$3.00	6	2	$2.00
George	1	$4.00	4	3	$3.00
Kramer	3	$1.00	3	4	$0.70

- Here George bids $4 and Jerry $2 but Jerry wins the auction because of higher ad quality – the platform expects he'll get four times the clicks
- Jerry pays a cost per click of only $1.50 (bid times competitor ad rank / own ad rank, $2.00*6/8)

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Ethical aspects of ad auctions

- Translated to social media, ad ‘quality’ can easily segue into ‘virality’
- Then if your ads are good clickbait, and your followers follow them, you pay less
- See Martinez ‘How Trump conquered Facebook – without Russian ads’ (web page)
- Many sites tend to serve ever more provocative and extreme content...

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Cooperation or conflict

- One way of getting what you want is to make it, or make something else of value and trade for it – ‘Economics’
- Another way is to just take it, whether by force or via the ballot box – ‘Politics’
- Choices between cooperation and conflict are made at all sorts of levels all the time
- They can evolve in complex combinations
- The main tool we use to analyse them is game theory

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Game theory

- The study of problems of cooperation and conflict among independent decision-makers
- We focus on games of strategy, rather than chance
- We abstract to players, choices, payoffs, strategies
- There are
  - games of perfect information (such as chess and go)
  - games of imperfect information (which are often more interesting to analyse)

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Nash equilibrium

- Definition: A **Nash Equilibrium** is a situation in a game where no player has an incentive to unilaterally change their strategy, assuming the other players stick to their strategies
- In other words, everyone is playing their best response to what others are doing
- It may not be unique, but it always exists

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Strategic form

- Example: matching pennies. Alice and Bob throw H or T. If they're different, Alice gets Bob's penny; else he gets hers. The strategic form is

Bob

Alice

	H	T
H	-1, 1	1, -1
T	1, -1	-1, 1

- This is an example of a zero-sum game: Alice's gain = Bob's loss

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Dominant strategy equlibrium

- In the following game, Bob's better off playing left; similarly Alice is always better off playing bottom

Bob

Alice

	Left	Right
Top	1, 2	0, 1
Bottom	2, 1	1, 0

- A strategy is an algorithm: input state, output play
- Here, each player's optimal play is a constant
- The is called a 'dominant strategy equilibrium'

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Battle of the sexes

- Consider this game:

Bob

Alice

	Left	Right
Top	2, 1	0, 0
Bottom	0, 0	1, 2

- Each player's optimal strategy depends on what they think the other will do
- Two strategies are in Nash equilibrium when A's choice is optimal given B's, and vice versa
- Here there are two: top left and bottom right

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Pure v mixed strategies

- With deterministic algorithms, some games have no Nash equilibrium

Bob

Alice

	scissors	paper	stone
scissors	0,0	1, -1	-1, 1
paper	-1, 1	0,0	1, -1
stone	1, -1	-1, 1	0,0

- Alice plays scissors → Bob will play stone → Alice will play paper …
- Fix: randomised algorithm. Called a 'mixed' strategy; deterministic algorithms are called 'pure'

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Prisoners' dilemma

- Two prisoners are arrested on suspicion of planning a robbery. The police tell them separately: if neither confesses, one year each for gun possession; if one confesses he goes free and the other gets 6 years; if both confess then each will get 3 years

Benjy

Alfie

	confess	deny
confess	-3, -3	0, -6
deny	-6, 0	-1, -1

- (confess, confess) is the dominant strategy equilibrium
- It's obviously not optimal for the villains!
- Is this a problem? If so, what's the solution?

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Prisoners' dilemma (2)

- You might answer 'serves them right'!
- But this can't apply to all instances of the dilemma
  - Defence spending
  - Fishing quotas
  - Free riders in file-sharing systems
  - Reducing carbon emissions
  - …
- Tough but inescapable conclusion: if the game is truly as described, there is no escape. Both will cheat rather than cooperate, with bad outcome
- To fix it, you need to change the game somehow!

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The evolution of cooperation

- If PD played repeatedly, there’s a fix!
- ‘Tit-for tat’: cooperate at round 1, then at round n do what the other guy did at n-1
- Simulation competitions run by Bob Axelrod played off many iterated-game strategies; tit-for-tat did consistently well
- In the presence of noise, tit-for-tat gets locked into (defect, defect). So: forgive the other guy occasionally
- People have realised in the last 30 years or so that strategy evolution explains a lot of behaviour

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Price-fixing

- If it costs $250 to fly someone LHR-JFK, do airlines compete and charge $255 or collude and charge $500?
- Competition laws forbid price-fixing cartels, but the same behaviour can arise implicitly
- Try charging $500 and see what other airlines do. If they ‘defect’ by competing, play tit-for-tat
- If you’re the regulator, how do you cope?

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Stag hunt

- People can hunt rabbits on their own, but have to work together to hunt a stag. If your buddy runs off after a rabbit, the stag will escape

Frank

Bernard

	chase hare	hunt stag
chase hare	2, 2	5, 0
hunt stag	0, 5	10, 10

- Difference from PD: (stag, stag) is now a Nash equilibrium
- You'll only chase a rabbit if you believe your buddy will defect
- Thus while PD is payoff-dominant, stag hunt is risk-dominant

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Chicken

- In 'Footloose', Ren (Kevin Bacon) and Chuck (Jim Youngs) drive cars against each other to see who will "chicken" first

Chuck

Ren

	jump	drive on
jump	2, 2	1, 3
drive on	3, 1	0, 0

- Here, (1,3) and (3,1) are Nash equilibria
- Bertrand Russell suggested this as a model of nuclear confrontation in the Cold War
- Can a player force the NE where they win? Yes, if they credibly commit to the other player that they will "drive on"!

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Commitment

https://www.youtube.com/watch?v=gsEjZwTfNT8

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Game theory and evolution

- John Maynard Smith proposed the 'Hawk-dove' game as a simple model of animal behaviour. Consider a mixed population of aggressive and docile individuals:

	Hawk	Dove
Hawk	(v-c)/2, (v-c)/2	v, 0
Dove	0, v	v/2, v/2

- Food v at each round; doves share; hawks take food from doves; hawks fight (with risk of death c)
- If v > c, whole population becomes hawk (dominant strategy)
- What happens if c > v?

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Game theory and evolution (2)

- If c > v, a small number of hawks will prosper as most interactions will be with doves. Equilibrium reached at hawk probability p setting hawk payoff = dove payoff

	Hawk	Dove
Hawk	(v-c)/2, (v-c)/2	v, 0
Dove	0, v	v/2, v/2

- I.e. p(v-c)/2 + (1-p)v = (1-p)v/2
  ⇔ pv - pc+ 2v -2pv = v - pv
  ⇔ -pc = -v
  ⇔ p = v/c

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Information applications (1)

- Matching pennies ⇔ attacker vs defender in cybersecurity
- Defender may not have the resources to patch all possible vulnerabilities
- Attacker may not know which vulnerabilities are undefended
- Example: network security or intrusion detection systems, attackers and defenders must continuously adapt and guess each other’s moves

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Information applications (2)

- Prisoner dilemma⇔ two organisations who both want a secure communication channel
- Cooperation (costly encryption or defection (saving encryption costs), determined whether communication can be compromised or is safe
- Examples: security standard agreements between competing companies, public and private sector cooperation in cybersecurity, and even in user adherence to safety protocols
- If repeated interaction, cooperation more likely to emerge

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Information applications (3)

- Battle of the sexes⇔ systems negotiating which communication protocol to use
- One system prefers more modern (IPv6), other system prefers legacy (IPv4)
- Both systems prefer to coordinate, but hard to agree on which one
- Examples: distributed computing and network protocols, where systems need to agree on standards or communication methods (such as TCP/IP vs. UDP, HTTP vs. HTTPS)

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