

Kahoot!

- Open Kahoot.it
- Enter Game PIN nr...

 Sign in



Stop and search

🕒 This article is more than 3 years old

Met police 'disproportionately' use stop and search powers on black people

**London's minority black population
targeted more than white population in
2018 - official figures**

The Guardian

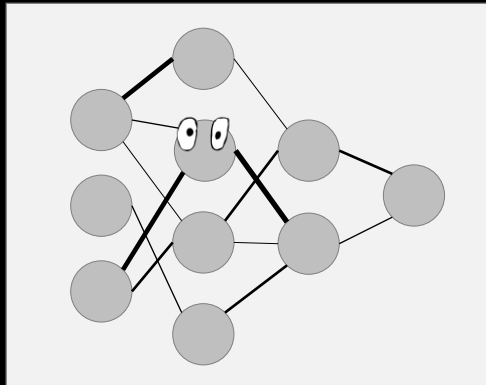
News website of the year

Do I trust this finding?

- ❖ Proportionate to what?
- ❖ Is it cherry picking?
- ❖ Why the scare quotes?

force	Date	LatLng	Object of search	Gender	Age range	Officer-defined ethnicity	Outcome
cambridgeshire	2023-08-31 15:44:04+00:00	(52.43,-0.142)	Controlled drugs				A no further action disposal
cambridgeshire	2023-08-31 15:35:41+00:00	(52.43,-0.142)	Firearms	Male	25-34	White	Khat or Cannabis warning
cambridgeshire	2023-08-31 14:44:04+00:00	(52.43,-0.142)	Firearms	Male	25-34	White	Khat or Cannabis warning
cambridgeshire	2023-08-31 03:44:14+00:00	(52.58,-0.244)	Offensive weapons	Male		Other	A no further action disposal
cambridgeshire	2023-08-31 02:34:16+00:00	(52.59,-0.247)	Controlled drugs	Male	25-34	White	Arrest
cambridgeshire	2023-08-31 02:27:10+00:00	(52.21,0.124)	Controlled drugs	Male	18-24	White	A no further action disposal
cambridgeshire	2023-08-30 22:28:13+00:00	(52.45,-0.117)	Controlled drugs	Female	over 34	White	A no further action disposal
cambridgeshire	2023-08-30 20:24:13+00:00	(52.32,-0.0708)	Controlled drugs	Male	10-17	White	Summons / charged by post
cambridgeshire	2023-08-30 14:26:58+00:00	(52.57,-0.24)	Controlled drugs	Male	over 34	Asian	A no further action disposal
cambridgeshire	2023-08-30 14:13:45+00:00	(52.57,-0.24)	Controlled drugs	Male	25-34	Black	Arrest

In a dataset of police stop-and-search records, is there evidence of ethnic bias? What about gender bias? If so, do these biases *intersect*, or is the net bias simply additive?



I can predict exactly what will happen to a person when they're stopped by the police!

Just tell me their gender. And ethnicity. And location. And whether they're left or right handed. And whether they have a pet cat or a dog. And what their pet is called. ...

What was the question again?

ML mindset

We just want to make good predictions, we don't care about the parameters

ML

building
models
from data

data
science

Science mindset

We have questions in mind, and we can answer them by looking at our model's parameters

 Sign in



Stop and search

🕒 This article is more than 3 years old

Met police 'disproportionately' use stop and search powers on black people

**London's minority black population
targeted more than white population in
2018 - official figures**

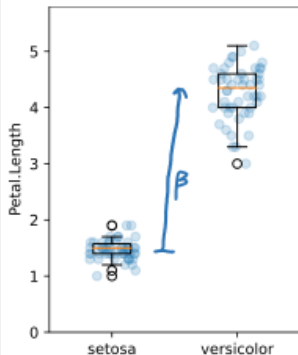
The Guardian

News website of the year

Can I set up a model with
a parameter that
measures the quantity
I'm interested in?

COMPARING GROUPS

§2.2



Measurements for condition A: $a = [a_1, a_2, \dots, a_m]$

Measurements for condition B: $b = [b_1, b_2, \dots, b_n]$

Can we use a linear model to compare A and B?

$$\vec{x} \approx \alpha_A \mathbf{1}_{\text{cond} = A} + \alpha_B \mathbf{1}_{\text{cond} = B}$$

Or

$$\vec{x} = \alpha + \beta \mathbf{1}_{\text{cond} = B}$$

For a person of type A, $x \approx \alpha$
For a person of type B, $x \approx \alpha + \beta$

β measures the difference between the two groups.

cond	x
A	a_1
A	\vdots
A	a_m
B	b_1
B	b_2
\vdots	\vdots
B	b_n

We know how to compare groups!

Let y_i be the response in row i , $y_i = \begin{cases} 1 & \text{if the police found something} \\ 0 & \text{if the police found nothing} \end{cases}$

The average response in ethnic group k is

$$\text{avg } y \text{ for ethnicity } k = \frac{\sum_{i: \text{eth}_i = k} y_i}{|\{i: \text{eth}_i = k\}|} = \frac{\# \text{finds}}{\# \text{stops}} = \mathbb{P}(\text{find something})$$

Let's fit the model $y_i \approx \alpha + \beta_{\text{eth}_i}$

i.e. $\mathbb{P}(\text{find something}) \approx \alpha + \beta_k$ for a person in ethnic group k

If $\beta_k < 0$, that means $\mathbb{P}(\text{find something})$ is low compared to other ethnic groups, i.e. the police are stopping relatively more innocent people.

Let's fit a model using officer-defined ethnicity as the predictor,

$$y \approx \alpha + \beta_{\text{eth}}$$

Writing it as a linear model with one-hot coding,

$$y \approx \alpha + \underbrace{\beta_{\text{As}} 1_{\text{eth}=\text{As}}}_{\text{Asian}} + \underbrace{\beta_{\text{Bl}} 1_{\text{eth}=\text{Bl}}}_{\text{Black}} + \underbrace{\beta_{\text{Mi}} 1_{\text{eth}=\text{Mi}}}_{\text{Mixed}} + \underbrace{\beta_{\text{Oth}} 1_{\text{eth}=\text{Oth}}}_{\text{Other}}$$

```
1 ethnicity_levels = np.unique(eth)
2 eth_onehot = [np.where(eth==k,1,0) for k in ethnicity_levels]
3
4 model = sklearn.linear_model.LinearRegression()
5 model.fit(np.column_stack(eth_onehot), y)
6
7 alpha, betas = model.intercept_, model.coef_
8
9 print(f'alpha = {alpha}')
10 for k, beta in zip(ethnicity_levels, betas):
11     print(f'beta[{k}] = {beta}')
```



$\alpha = -34037792910.00365$
 $\beta[\text{Asian}] = 34037792910.26522$
 $\beta[\text{Black}] = 34037792910.265717$
 $\beta[\text{Mixed}] = 34037792910.2939$
 $\beta[\text{Other}] = 34037792910.2604$
 $\beta[\text{White}] = 34037792910.261$



§2.5 The geometry of linear models

NST Maths A, Michaelmas

1.7.2 Shortest distance of a point from a plane

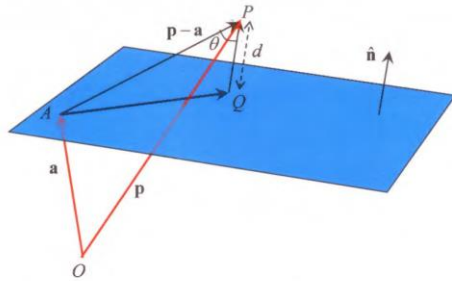


Figure 13: The shortest distance between the point P and the plane defined by the point A and normal \hat{n} .

Consider the plane that passes through point A (given by position vector \mathbf{a}) and that has unit normal \hat{n} . From (1.1), the equation for the plane is defined by the points \mathbf{r} satisfying

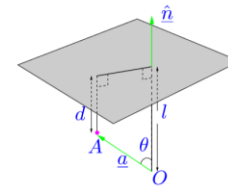
$$(\mathbf{r} - \mathbf{a}) \cdot \hat{n} = 0.$$

The closest point on the plane to a point P , given by position vector \mathbf{p} , is the point Q , where \overrightarrow{QP} is normal to the plane and $|\overrightarrow{QP}| = d$.

NST Maths B, Michaelmas

Example: Distance of point from plane

- What is distance of point A with position vector \underline{a} from plane $\underline{r} \cdot \underline{\hat{n}} = l$?



- Line containing A and point of closest approach of plane to A must be $\parallel \hat{n}$; has equation

$$\underline{r} = \underline{a} + \lambda \hat{n}$$

- Line meets plane where $\underline{r} \cdot \hat{n} = l$, i.e. where

$$l = \underline{a} \cdot \hat{n} + \lambda$$

- λ is distance along line from \underline{a} so required distance is $|\underline{a} \cdot \hat{n} - l|$

NST Maths A, Easter

Orthogonality - 2/3

- The **vectors** $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ are a **basis** of **orthonormal vectors** in \mathbb{R}^3 :

$$\mathbf{e}_i \cdot \mathbf{e}_j = \delta_{ij}. \quad (2)$$

- We use the **orthogonality properties** (2) to **calculate** the components of \mathbf{a} :

$$\mathbf{e}_1 \cdot \mathbf{a} = a_1 \times 1 + a_2 \times 0 + a_3 \times 0 = a_1.$$

In general

$$a_i = \mathbf{e}_i \cdot \mathbf{a}, \quad \text{for } i = 1, 2, 3. \quad (3)$$

- The above **generalises** to Euclidean space \mathbb{R}^n with

$$\mathbf{a} = a_i \mathbf{e}_i, \quad \text{for } i = 1, \dots, n.$$

the **components** a_i are **evaluated** in the **same way** as in the case with $n = 3$ because (2) and (3) **still hold**, but with i, j now in the **range** 1 to n .

NST Maths B, Easter

Definition. V is called a **vector space over** K , and the elements of V are called **vectors**, if the following **axioms** hold:

A1 For any vectors $u, v, w \in V$, $(u + v) + w = u + (v + w)$. (**Associativity.**)

A2 For any vectors $u, v \in V$, $u + v = v + u$. (**Commutativity.**)

A3 There is a vector in V denoted 0 , called the **zero vector** for which $u + 0 = u$ $\forall u \in V$.

A4 For each vector $u \in V$ there is a vector in V denoted $-u$ for which $u + (-u) = 0$. (**Inverse.**)

A5 For any $a \in K$ and any $u, v \in V$, $a(u + v) = au + av$.

A6 For any $a, b \in K$ and any $u \in V$, $(a + b)u = au + bu$.

A7 For any $a, b \in K$ and any $u \in V$, $(ab)u = a(bu)$.

A8 For the unit scalar $1 \in K$ and any $u \in V$, $1u = u$.

The *subspace spanned* by a collection of vectors $\{e_1, \dots, e_K\}$ is the set of all linear combinations

$$\mathcal{S} = \{\lambda_1 \mathbf{e}_1 + \cdots + \lambda_K \mathbf{e}_K : \lambda_k \in \mathbb{R} \text{ for all } k\}$$

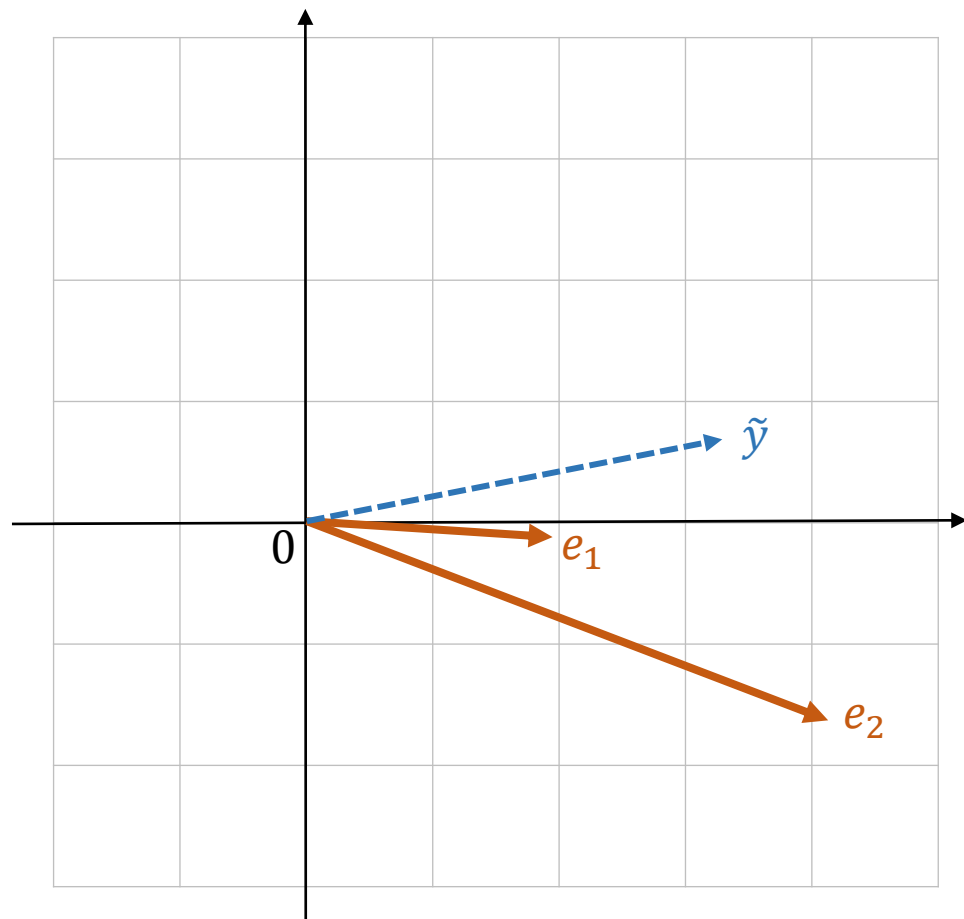
The vectors are *linearly dependent* if at least one of the e_k can be written as a linear combination of the others, i.e. there is some set of real numbers $(\lambda_1, \dots, \lambda_K)$ not all equal to zero such that

$$\lambda_1 e_1 + \dots + \lambda_K e_K = 0$$

If not, they are *linearly independent*, and

$$\lambda_1 \mathbf{e}_1 + \dots + \lambda_K \mathbf{e}_K = \mathbf{0} \quad \Rightarrow \quad \lambda_1 = \dots = \lambda_K = 0$$

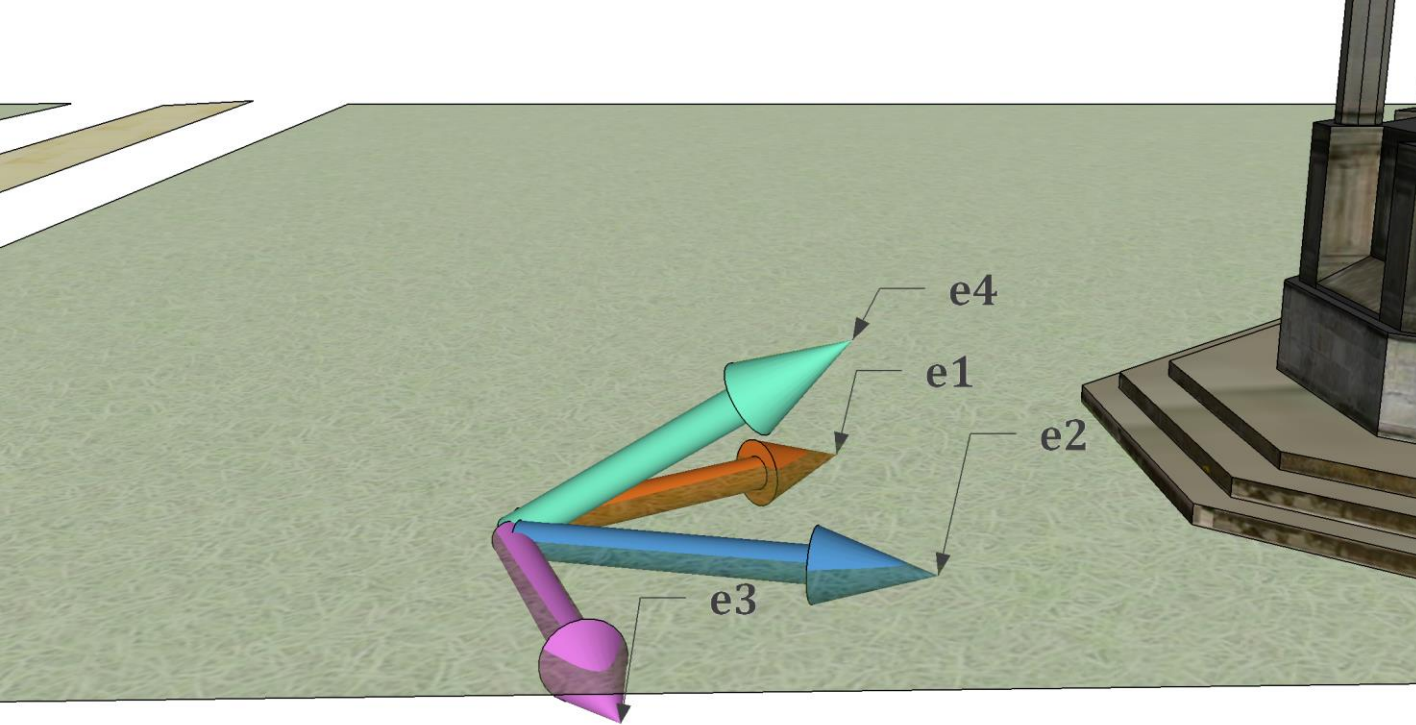
`np.linalg.matrix_rank(np.column_stack([e_1, \dots, e_K]))` is $< K$ if linearly dependent
 $= K$ if linearly independent



The subspace spanned by $\{e_1, e_2\}$ is \mathbb{R}^2

Any $\tilde{y} \in \mathbb{R}^2$ can be written as a linear combination of e_1 and e_2

❖ by eye, $\tilde{y} = 2.5e_1 - 0.3e_2$



The subspace spanned by $\{e_1, e_2, e_3, e_4\}$ is \mathbb{R}^3

Are $\{e_1, e_2, e_3, e_4\}$ linearly independent? *No.*

If we discarded e_2 ...

Are $\{e_1, e_3, e_4\}$ linearly independent? What's the span?

They are linearly ind, span is \mathbb{R}^3

If we discarded e_1 ...

Are $\{e_2, e_3, e_4\}$ linearly independent? What's the span?

They are linearly ind, span is \mathbb{R}^3 .

Exercise 2.5.2

Are the following five vectors linearly independent? If not, find a subset that is.

$$e_1 = [1, 1, 1, 1]$$

$$e_2 = [0, 1, 1, 0]$$

$$e_3 = [1, 0, 0, 1]$$

$$e_4 = [1, 1, 1, 0]$$

$$e_5 = [0, 0, 0, 1]$$

$$e_1 = e_2 + e_3$$

$$e_1 = e_4 + e_5$$

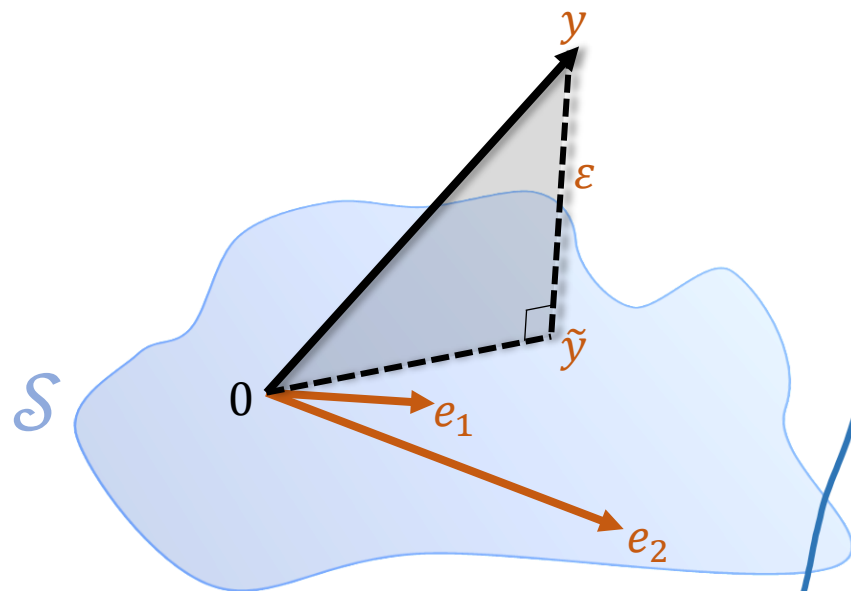
$$\begin{aligned} \text{So } \text{span}(\{e_1, e_2, e_3, e_4, e_5\}) \\ &= \text{span}(\{e_1, e_2, e_3, e_4\}) \text{ since } e_5 = e_1 - e_4 \\ &= \text{span}(\{e_1, e_2, e_4\}) \text{ since } e_3 = e_1 - e_2. \end{aligned}$$

Q. Are $\{e_1, e_2, e_4\}$ linearly independent?

$$\text{Suppose } \lambda_1 e_1 + \lambda_2 e_2 + \lambda_4 e_4 = 0$$

$$\Rightarrow \lambda_1 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \lambda_2 \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} + \lambda_4 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \left. \begin{aligned} \lambda_1 + \lambda_4 &= 0 \\ \lambda_1 + \lambda_2 + \lambda_4 &= 0 \\ \lambda_1 + \lambda_2 + \lambda_4 &= 0 \\ \lambda_1 &= 0 \end{aligned} \right\} \Rightarrow \begin{aligned} \lambda_1 &= 0 \\ \lambda_4 &= 0 \\ \lambda_2 &= 0 \end{aligned} \Rightarrow \text{they are linearly independent.}$$



What does "closest" mean?

It means: find $\tilde{y} \in S$ to minimize $\|\vec{y} - \tilde{y}\|$

i.e. to minimize $\sqrt{\sum \epsilon_i^2}$ where $\vec{\epsilon} = \vec{y} - \tilde{y}$

The minimization is over all $\tilde{y} \in S$.

Every $\tilde{y} \in S$ can be written as a linear combination of e_1, \dots, e_K .

What does "best approx." mean?
It means: find β_1, \dots, β_K to minimize

$\frac{1}{\text{\#datapoints}} \sum \epsilon_i^2$ where

$$\vec{\epsilon} = \vec{y} - (\beta_1 \vec{e}_1 + \dots + \beta_K \vec{e}_K)$$

GEOMETRY

Let S be the span of $\{e_1, \dots, e_K\}$.

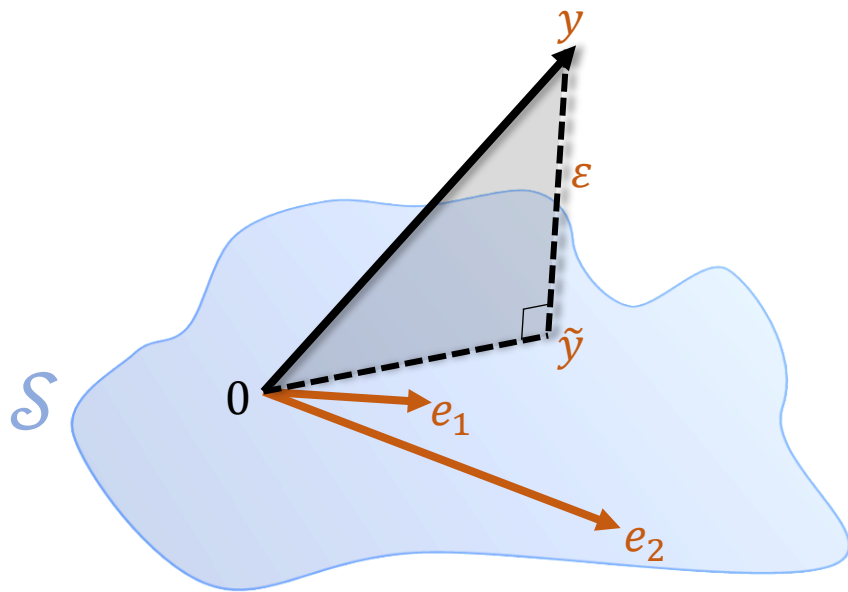
What's the closest we can get to y , while staying in S ?

LINEAR MODELLING / LEAST SQUARES ESTIMATION

Given features $\{e_1, \dots, e_K\}$ let's approximate $y \approx \beta_1 e_1 + \dots + \beta_K e_K$.

What parameters give us the best approximation?

The span of $\{e_1, \dots, e_K\}$ is called the *feature space*



Geometric intuition can help us understand what Least Squares Estimation is doing ...

GEOMETRY

Q. Is there a unique way to write \tilde{y} as a linear combination of $\{e_1, \dots, e_K\}$?

A. Unique way to write $\tilde{y} \Leftrightarrow \{e_1, \dots, e_K\}$ linearly independent

LINEAR MODELLING / LEAST SQUARES ESTIMATION

Q. When we run least squares estimation, does it always return the same parameter estimates?

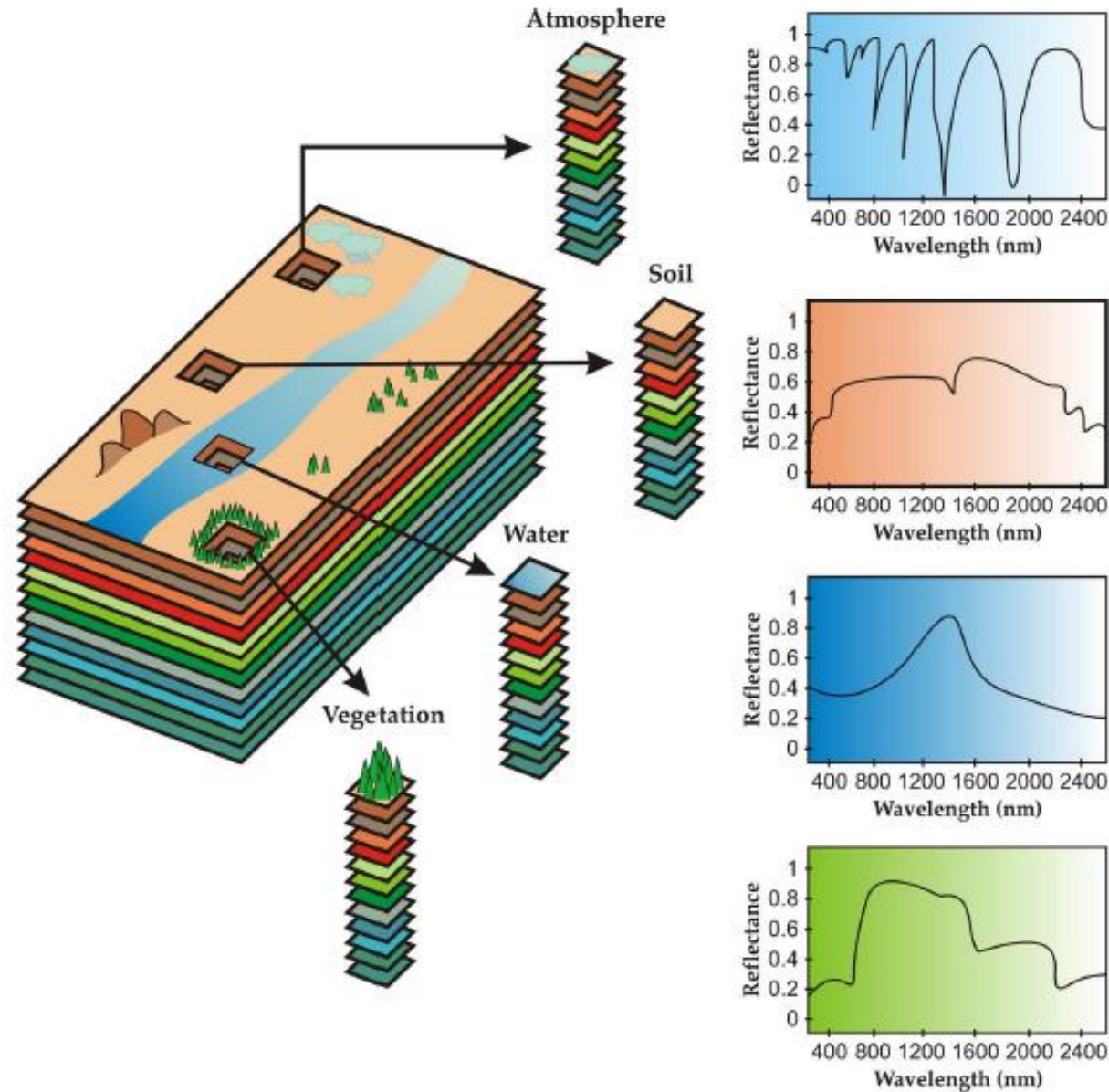
A. If the features are NOT linearly independent, different runs might give different parameters (and if some other data scientist reports different parameter estimates, our audience will be confused!)

§2.6 Interpreting parameters

- Write out the predicted response for a few typical / representative datapoints.
This helps see what the parameters mean.
- Write out the features.
If two models have different features but the same feature space, then (once fitted) they make the same predictions on the dataset.
- Check if the features are linearly dependent.
If so, the parameters have no intrinsic meaning.
We say the features are *confounded*, and the parameters are *non-identifiable*.

Land cover/land use in a nutshell

- Land classes are associated with spectral signatures of the signals acquired by satellites
- Given I spectral channels, we could associate different land classes to different I -dimensional spectral trends (signatures)



Land cover/land use in a nutshell

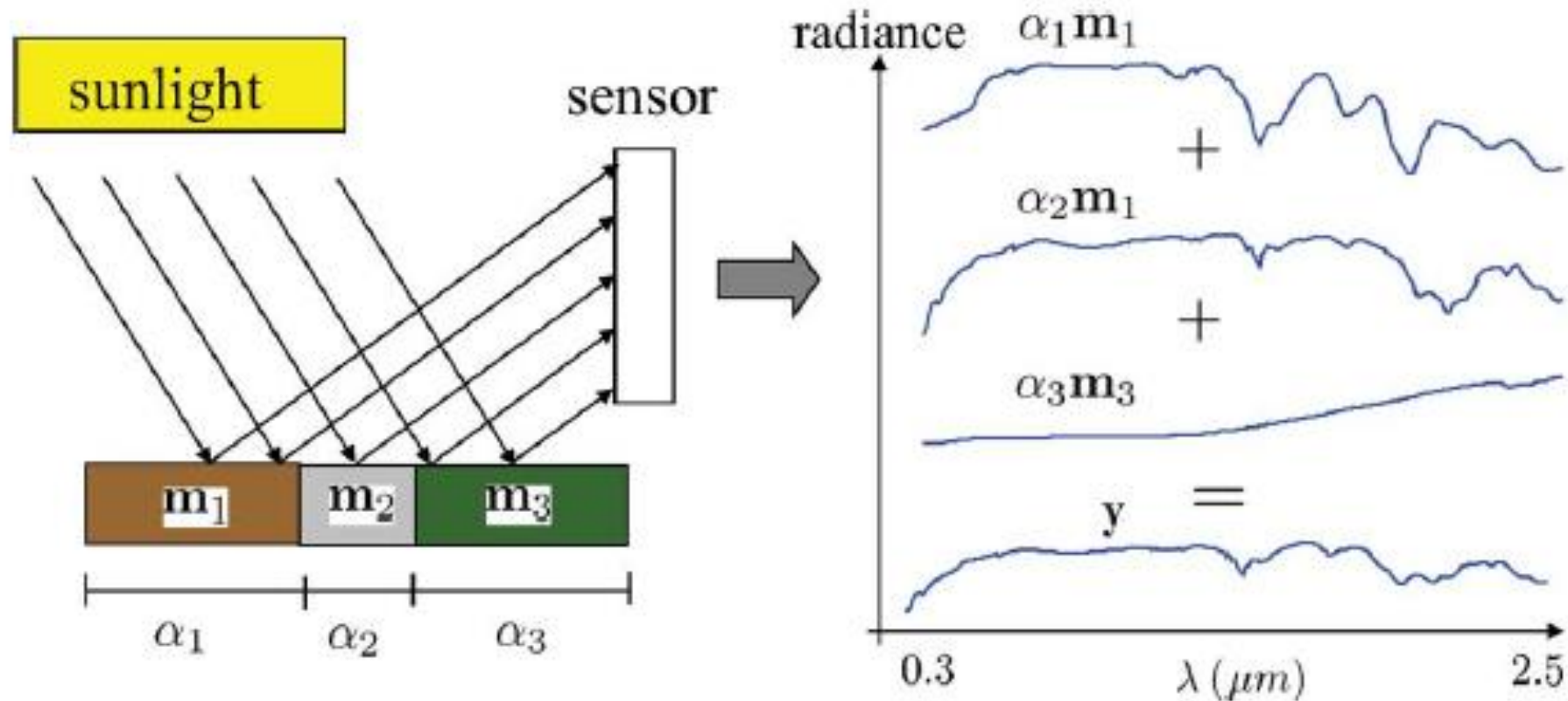


Fig. 2. Linear mixing. The measured radiance at a pixel is a weighted average of the radiances of the materials present at the pixel.

Problem: given the various nonidealities that the system induces, several spectral signatures might be “noisy” (i.e., affected by undesired effects)

Why?

J Bioucas-Dias, et al., “Hyperspectral Unmixing Overview: Geometrical, Statistical, and Sparse Regression-Based Approaches,” IEEE Journal Of Selected Topics In Applied Earth Observations And Remote Sensing, Vol. 5, No. 2, April 2012

Land cover/land use in a nutshell

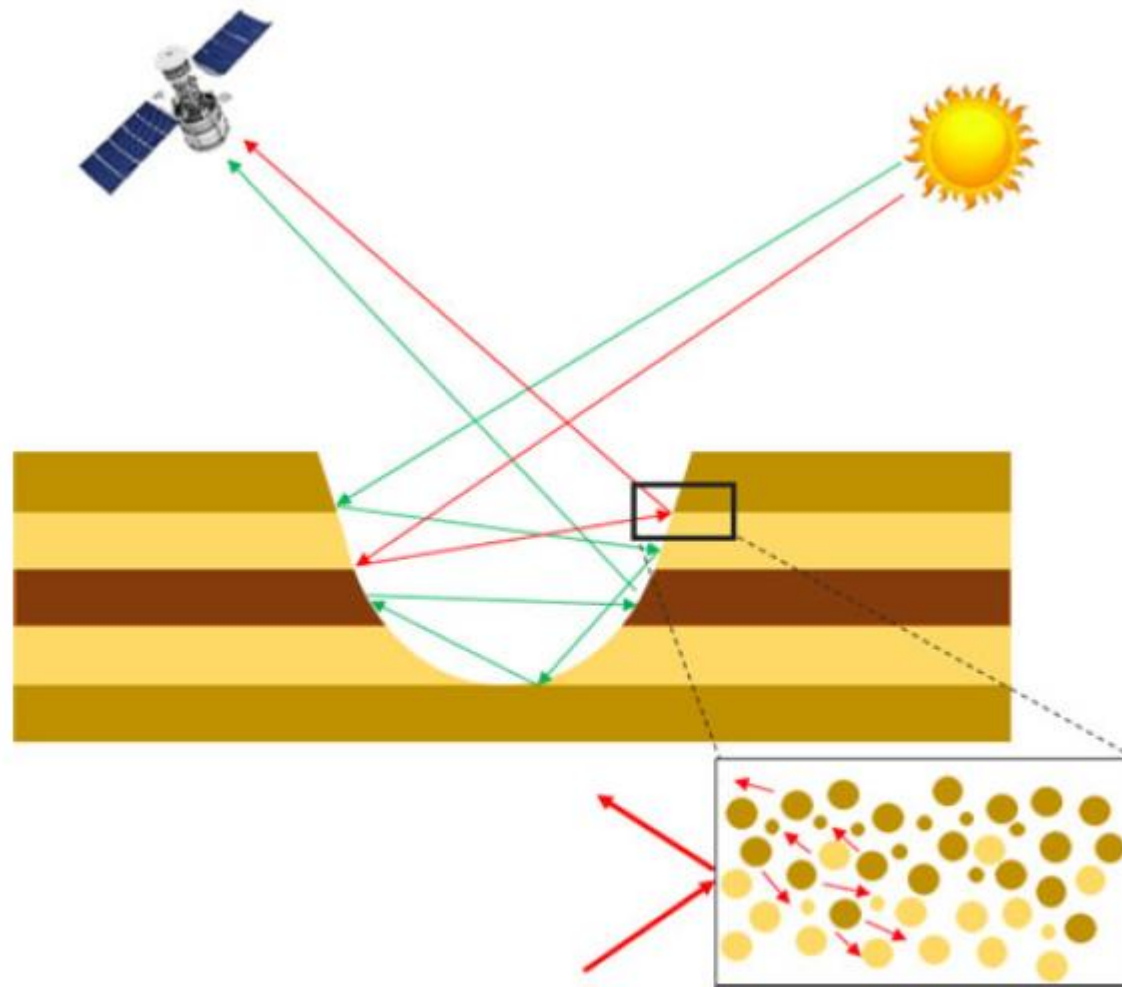


Fig. 1. Sketch of nonlinear mixture of the reflectances recorded by the sensor, where the contributions results from multiple scatterings and interferences also at a finer spatial resolution than what can be captured by the sensor.

Land cover/land use in a nutshell

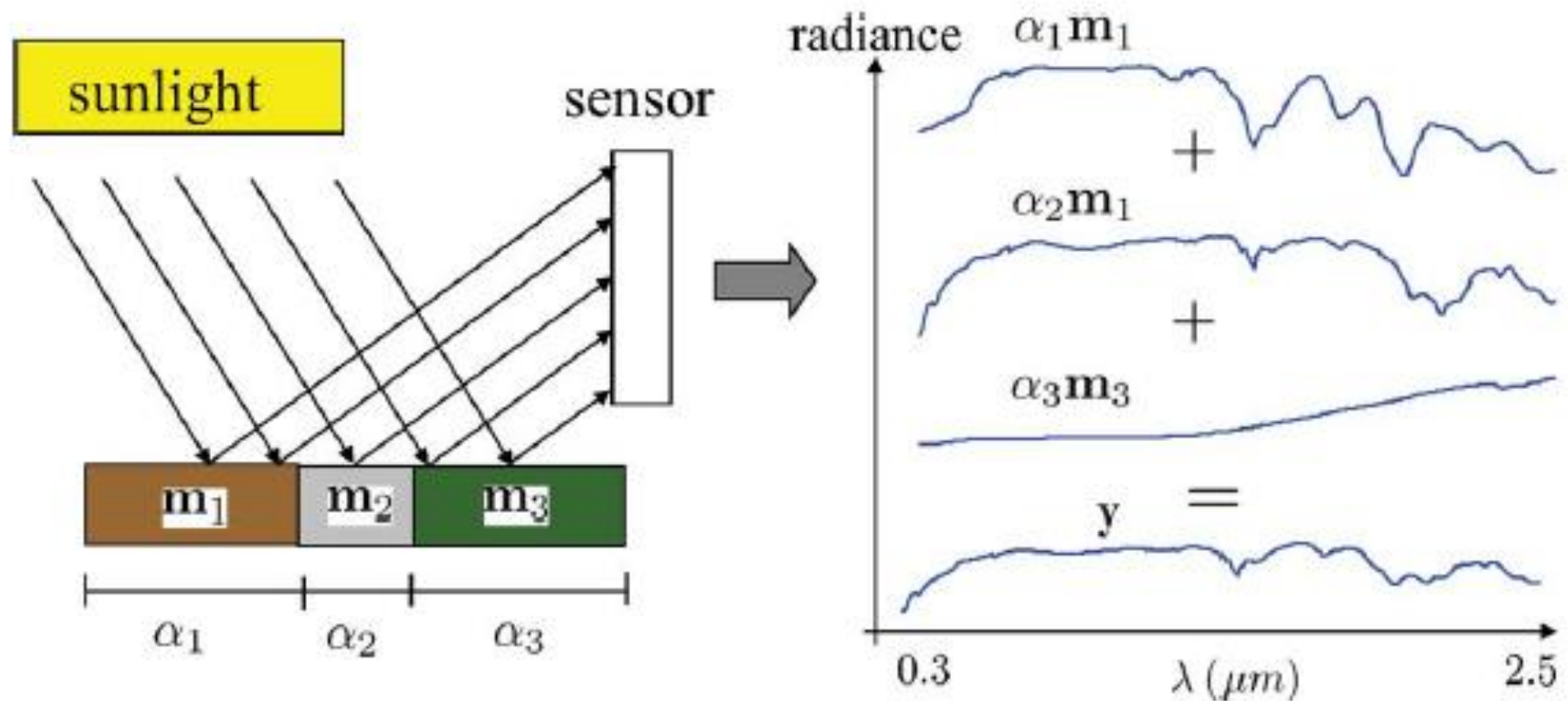


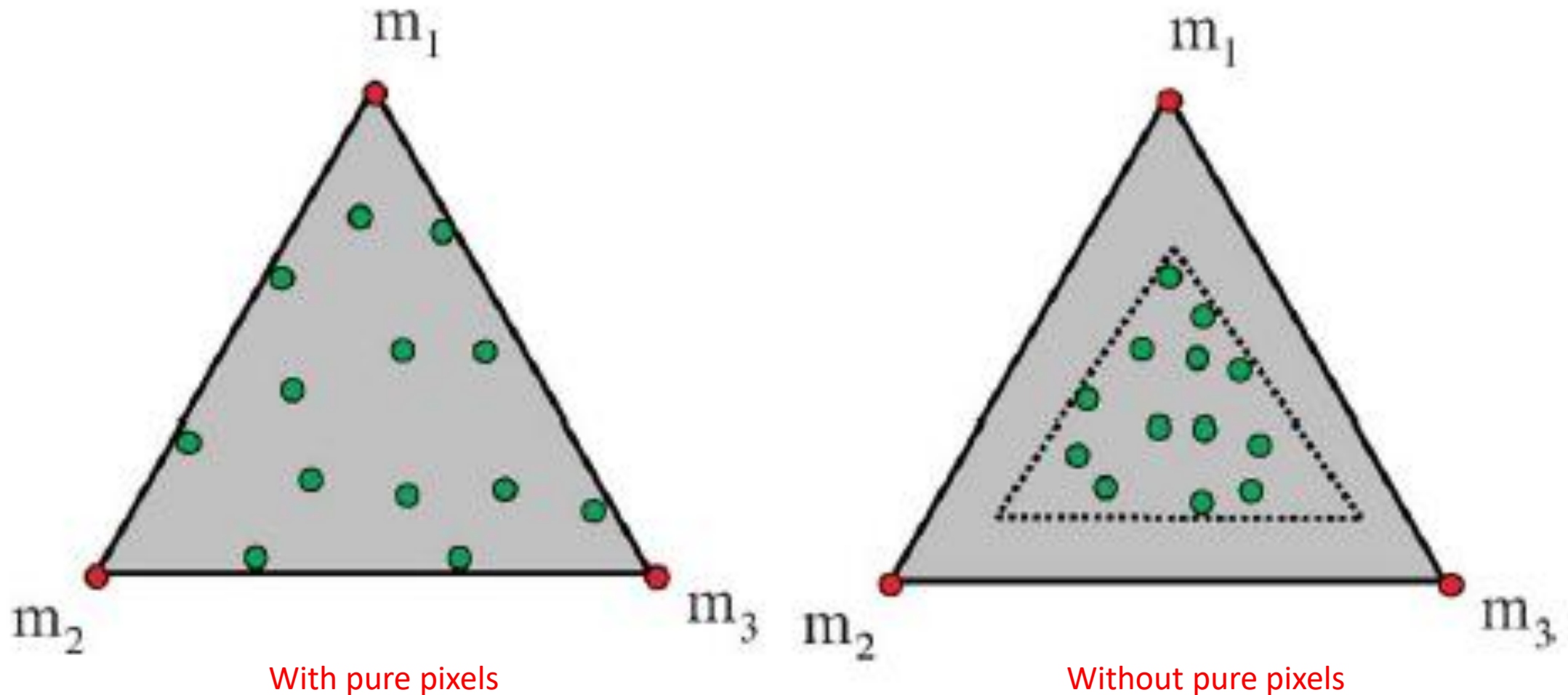
Fig. 2. Linear mixing. The measured radiance at a pixel is a weighted average of the radiances of the materials present at the pixel.

Problem: given the various nonidealities that the system induces, several spectral signatures might be "noisy" (i.e., affected by undesired effects)

What does this imply?

J Bioucas-Dias, et al., "Hyperspectral Unmixing Overview: Geometrical, Statistical, and Sparse Regression-Based Approaches," IEEE Journal Of Selected Topics In Applied Earth Observations And Remote Sensing, Vol. 5, No. 2, April 2012

Land cover/land use in a nutshell



Pure pixels = pixels whose spectral signature is exactly the same of one of the possible (known to the mankind) land classes

J Bioucas-Dias, et al., "Hyperspectral Unmixing Overview: Geometrical, Statistical, and Sparse Regression-Based Approaches," IEEE Journal Of Selected Topics In Applied Earth Observations And Remote Sensing, Vol. 5, No. 2, April 2012

Risks

When performing LC/LU analysis, we have two main design options in front of us:

- Consider number of features independently by the region of interest:
 - Risk = we might lose track of outliers
- Adaptively select number of features according to deeper investigation of the data associated with the given region
 - Risk = we might have “artificial features” (i.e., features that are not associated with any LC/LU class; e.g., features resulting from combinations of LC/LU classes)
 - loss of reliability

Risk = jumping to conclusions

- It's important to consider the capacity and limits of these algorithms, because the causes of nonidealities could be massive (think of analysing extraterrestrial bodies)
- Evaluating pros and cons, and uncertainties (cost/benefit analysis), could lead to big missions... also extraterrestrial!

Gloria Oladipo
and agency

Wed 28 Jun 2023 17.26
CEST



Nasa aims to mine resources on moon in next decade

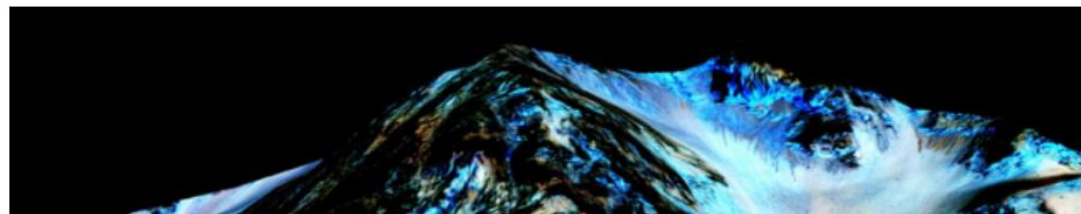
US space agency plans to send Americans to the moon by 2025, including the first women and person of color



Nasa scientists find evidence of flowing water on Mars

Researchers say discovery of stains from summertime flows down cliffs and crater walls increases chance of finding life on red planet

- **Water on the red planet - in pictures**
- **Water on Mars - an interactive guide**



Ian Sample
Science editor

🐦 @iansample
Mon 28 Sep 2015 17.00
CEST

**The
Guardian**

§2.6 Interpreting parameters

- Write out the predicted response for a few typical / representative datapoints.
This helps see what the parameters mean.
- Write out the features.
If two models have different features but the same feature space, then (once fitted) they make the same predictions on the dataset.
- Check if the features are linearly dependent.
If so, the parameters have no intrinsic meaning.
We say the features are *confounded*, and the parameters are *non-identifiable*.

These three models yielded very different estimates for α when considering the Cambridge climate dataset. Why?

Model 0: $\text{temp} \approx \alpha \mathbf{1} + \beta_1 \sin(2\pi \mathbf{t}) + \beta_2 \cos(2\pi \mathbf{t})$

$$\Rightarrow \hat{\alpha} = 10.6^\circ\text{C}$$

Model A: $\text{temp} \approx \alpha \mathbf{1} + \beta_1 \sin(2\pi \mathbf{t}) + \beta_2 \cos(2\pi \mathbf{t}) + \gamma \mathbf{t}$

$$\Rightarrow \hat{\alpha} = -60.2^\circ\text{C}$$

Model B: $\text{temp} \approx \alpha \mathbf{1} + \beta_1 \sin(2\pi \mathbf{t}) + \beta_2 \cos(2\pi \mathbf{t}) + \gamma (\mathbf{t} - 2000)$

$$\Rightarrow \hat{\alpha} = 10.5^\circ\text{C}$$



These three models yielded very different estimates for α when considering the Cambridge climate dataset. Why?

Model 0:	$\text{temp} \approx \alpha \mathbf{1} + \beta_1 \sin(2\pi \mathbf{t}) + \beta_2 \cos(2\pi \mathbf{t})$	$\Rightarrow \hat{\alpha} = 10.6^\circ\text{C}$
Model A:	$\text{temp} \approx \alpha \mathbf{1} + \beta_1 \sin(2\pi \mathbf{t}) + \beta_2 \cos(2\pi \mathbf{t}) + \gamma \mathbf{t}$	$\Rightarrow \hat{\alpha} = -60.2^\circ\text{C}$
Model B:	$\text{temp} \approx \alpha \mathbf{1} + \beta_1 \sin(2\pi \mathbf{t}) + \beta_2 \cos(2\pi \mathbf{t}) + \gamma (\mathbf{t} - 2000)$	$\Rightarrow \hat{\alpha} = 10.5^\circ\text{C}$

EXERCISE. Write out the predicted response for a few typical / representative datapoints.

Model 0: at $t=0$, pred. temp = $\alpha + \beta_2$ \longrightarrow over a single sinusoid, α is avg. temperature.

Model A: at $t=0$, pred. temp. = $\alpha + \beta_2$ \longrightarrow over a single sinusoid in 1 BC, α is avg. temperature.
at $t=2000$, = $\alpha + \beta_2 + 2000\gamma$

Model B: at $t=0$, pred temp = $\alpha + \beta_2 - 2000\gamma$
at $t=2000$, = $\alpha + \beta_2$ \longrightarrow over a single sinusoid in 2000 AD, α is avg. temperature.

These three models yielded very different estimates for α when considering the Cambridge climate dataset. Why?

Model 0:	$\text{temp} \approx \alpha \mathbf{1} + \beta_1 \sin(2\pi \mathbf{t}) + \beta_2 \cos(2\pi \mathbf{t})$	$\Rightarrow \hat{\alpha} = 10.6^\circ\text{C}$
Model A:	$\text{temp} \approx \alpha \mathbf{1} + \beta_1 \sin(2\pi \mathbf{t}) + \beta_2 \cos(2\pi \mathbf{t}) + \gamma \mathbf{t}$	$\Rightarrow \hat{\alpha} = -60.2^\circ\text{C}$
Model B:	$\text{temp} \approx \alpha \mathbf{1} + \beta_1 \sin(2\pi \mathbf{t}) + \beta_2 \cos(2\pi \mathbf{t}) + \gamma (\mathbf{t} - 2000)$	$\Rightarrow \hat{\alpha} = 10.5^\circ\text{C}$

EXERCISE. Write out the features.

Model A: features $\{\mathbf{1}, \sin(2\pi \mathbf{t}), \cos(2\pi \mathbf{t}), \mathbf{t}\}$

Model B: features $\{\mathbf{1}, \sin(2\pi \mathbf{t}), \cos(2\pi \mathbf{t}), \underbrace{\mathbf{t} - 2000 \times \mathbf{1}}_{\downarrow}\}$

Model A and B have the same feature space.

This can be written as a linear combination of $\mathbf{1}$ and \mathbf{t}

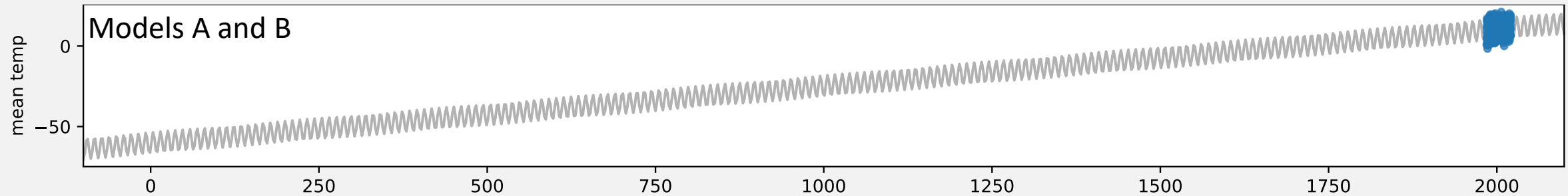
They're essentially the same model, just with different parameterization.

Model A asks: tell me the avg. temp in 18C

Model B asks: tell me the avg. temp in 2000 AD

These three models yielded very different estimates for α when considering the Cambridge climate dataset. Why?

Model 0:	$\text{temp} \approx \alpha \mathbf{1} + \beta_1 \sin(2\pi \mathbf{t}) + \beta_2 \cos(2\pi \mathbf{t})$	$\Rightarrow \hat{\alpha} = 10.6 \text{ }^{\circ}\text{C}$
Model A:	$\text{temp} \approx \alpha \mathbf{1} + \beta_1 \sin(2\pi \mathbf{t}) + \beta_2 \cos(2\pi \mathbf{t}) + \gamma \mathbf{t}$	$\Rightarrow \hat{\alpha} = -60.2 \text{ }^{\circ}\text{C}$
Model B:	$\text{temp} \approx \alpha \mathbf{1} + \beta_1 \sin(2\pi \mathbf{t}) + \beta_2 \cos(2\pi \mathbf{t}) + \gamma (\mathbf{t} - 2000)$	$\Rightarrow \hat{\alpha} = 10.5 \text{ }^{\circ}\text{C}$



Models A and B are essentially the same, because they have the same feature space.

But because they use different representations of the feature space, they report different readouts.

 Sign in



Stop and search

🕒 This article is more than 3 years old

Met police 'disproportionately' use stop and search powers on black people

**London's minority black population
targeted more than white population in
2018 - official figures**

The Guardian

News website of the year

Can I set up a model with
a parameter that
measures the quantity
I'm interested in?

Example 2.6.4

The UK Home Office makes available a dataset of police stop-and-search incidents. We wish to investigate whether there is racial bias in police decisions to stop-and-search. Consider the linear model

$$y_i \approx \alpha + \beta \text{eth}_i$$

where eth_i is the officer-defined ethnicity for record i , and y_i records the outcome: $y_i = 1$ if the police found something, 0 otherwise.

- Write this as a linear equation using one-hot coding.
- Are the parameters identifiable? If not, rewrite the model so that they are.
- Does the model suggest there is racial bias in policing actions?

(a)

$$y \approx \alpha \cdot 1 + \beta_{As} e_{As} + \beta_{Bl} e_{Bl} + \beta_{Mi} e_{Mi} + \beta_{oth} e_{oth} + \beta_{Wh} e_{Wh} \quad \text{where } e_k = 1_{\text{eth}=k}$$

ethnic groups

Asian
Black
Mixed
Other
White

(b) These are linearly dependent: $1 = e_{As} + e_{Bl} + e_{Mi} + e_{oth} + e_{Wh}$

So the parameters are not identifiable, i.e. we're likely to get silly answers out of linear_model fitting.

climate

x +

cl.cam.ac.uk/teaching/2223/DataSci/datasci/ex/climate.html

In [1]:

```
import numpy as np
import pandas
import matplotlib.pyplot as plt
import sklearn.linear_model
π = np.pi
```

Climate dataset challenge

- What is the rate of temperature increase in Cambridge?
- Are temperatures increasing at a constant rate, or has the increase accelerated?
- How do results compare across the whole of the UK?

Your task is to answer these questions using appropriate linear models, and to produce elegant plots to communicate your findings. Please submit a Jupyter notebook, or a pdf. Include explanations of what your models are, and of what your plots show.

The dataset is from <https://www.metoffice.gov.uk/pub/data/weather/uk/climate/>. Code for retrieving the dataset is given at the bottom.

Upload your answers to Moodle by Sunday

RICE CRUMB #3

- How can we write the distribution of the parameters estimated by MLE for $N \rightarrow +\infty$?