$S \subseteq N$ is recursively enumerable (r.e.) if

S is empty, or

$$S = \{ f(n) \mid n \in N \}$$

for some total $f : N \to N$ that

is Register machine computable.

$S_0 = \{ e \mid \phi_e(0)\downarrow \}$ is r.e.

We need a RM computable f s.t.

$S_0 = \{ f(n) \mid n \in N \}$:

(1) $i := 0$, $c := 0$. input: n

(2) decode i to $i = \langle\langle e, t \rangle\rangle$

(3) if $\phi_e(0)$ halts in t steps, then $c := c + 1$

(4) if $c = n$ the output e and HALT else $i := i + 1$; goto(2)

$S_1 = \{ e \mid \phi_e \text{ is total} \}$ is <u>not</u> r.e.

<u>Proof:</u> Suppose $\exists$ total $F : N \to N$ s.t.

$$S_1 = \{ F(n) \mid n \in N \}.$$

Define $g(i) = \phi_{F(i)}(i) + 1$ } is total!

So g is represented by $e \in S_1$: $g = \phi_{F(e)}$

But. $g(e) = \phi_{F(e)}(e) \neq \phi_{F(e)}(e) + 1 = g(e)$