Lecture 3
Live variable analysis

Data-flow analysis

```
MOV t32,arg1
MOV t33,arg2
ADD t34,t32,t33
MOV t35,arg3
MOV t36,arg4
ADD t37,t35,t36
MUL res1,t34,t37
```

Discovering information about how *data* (i.e. variables and their values) may move through a program.

Motivation

Programs may contain

- code which gets executed but which has no useful effect on the program's overall result;
- occurrences of variables being used before they are defined; and
- many variables which need to be allocated registers and/or memory locations for compilation.

The concept of *variable liveness* is useful in dealing with all three of these situations.

Liveness

Liveness is a data-flow property of variables:
“Is the value of this variable needed?” (cf. dead code)

```
int f(int x, int y) {
  int z = x * y;
  ⋮
```

?

?

?

Liveness

At each instruction, each variable in the program is either live or dead.

We therefore usually consider liveness from an instruction's perspective: each instruction (or node of the flowgraph) has an associated *set* of live variables.

```
⋮
```

n: $\boxed{\texttt{int z = x * y;}}$ *live*(*n*) = { `s`, `t`, `x`, `y` }

```
return s + t;
```

Semantic vs. syntactic

There are two kinds of variable liveness:

- Semantic liveness
- Syntactic liveness

Semantic vs. syntactic

A variable *x* is *semantically* live at a node *n* if there is *some execution sequence* starting at *n* whose (externally observable) behaviour can be affected by changing the value of *x*.

```
int x = y * z;        x LIVE
⋮
return x;
```

Semantic vs. syntactic

A variable *x* is *semantically* live at a node *n* if there is *some execution sequence* starting at *n* whose (externally observable) behaviour can be affected by changing the value of *x*.

```
int x = y * z;
⋮
x = a + b;
⋮
return x;
```

x DEAD

Semantic vs. syntactic

Semantic liveness is concerned with
the *execution behaviour* of the program.

This is undecidable in general.
(e.g. Control flow may depend upon arithmetic.)

Semantic vs. syntactic

A variable is *syntactically* live at a node if there is a path to the exit of the flowgraph along which its value may be used before it is redefined.

Syntactic liveness is concerned with properties of the *syntactic structure* of the program.

Of course, this *is* decidable.

So what's the difference?

Semantic vs. syntactic

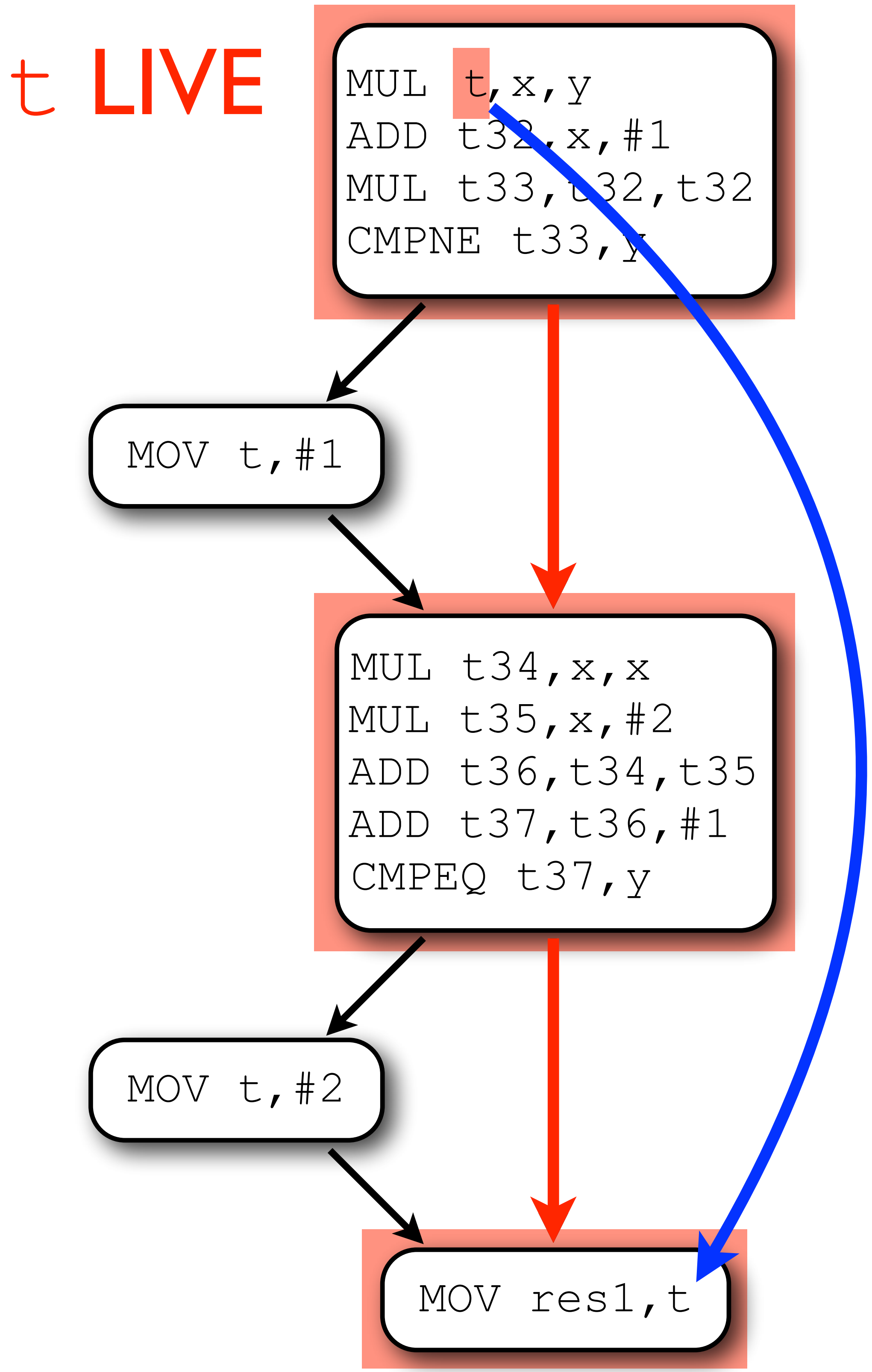
```
int t = x * y;  t DEAD
if ((x+1)*(x+1) == y) {
  t = 1;
}
if (x*x + 2*x + 1 != y) {
  t = 2;
}
return t;
```

Semantically: one of the conditions will be true, so on every execution path `t` is redefined before it is returned. The value assigned by the first instruction is never used.

Semantic vs. syntactic

```
       MUL t,x,y
       ADD t32,x,#1
       MUL t33,t32,t32
       CMPNE t33,y,lab1
       MOV t,#1
lab1:  MUL t34,x,x
       MUL t35,x,#2
       ADD t36,t34,t35
       ADD t37,t36,#1
       CMPEQ t37,y,lab2
       MOV t,#2
lab2:  MOV res1,t
```

Semantic vs. syntactic



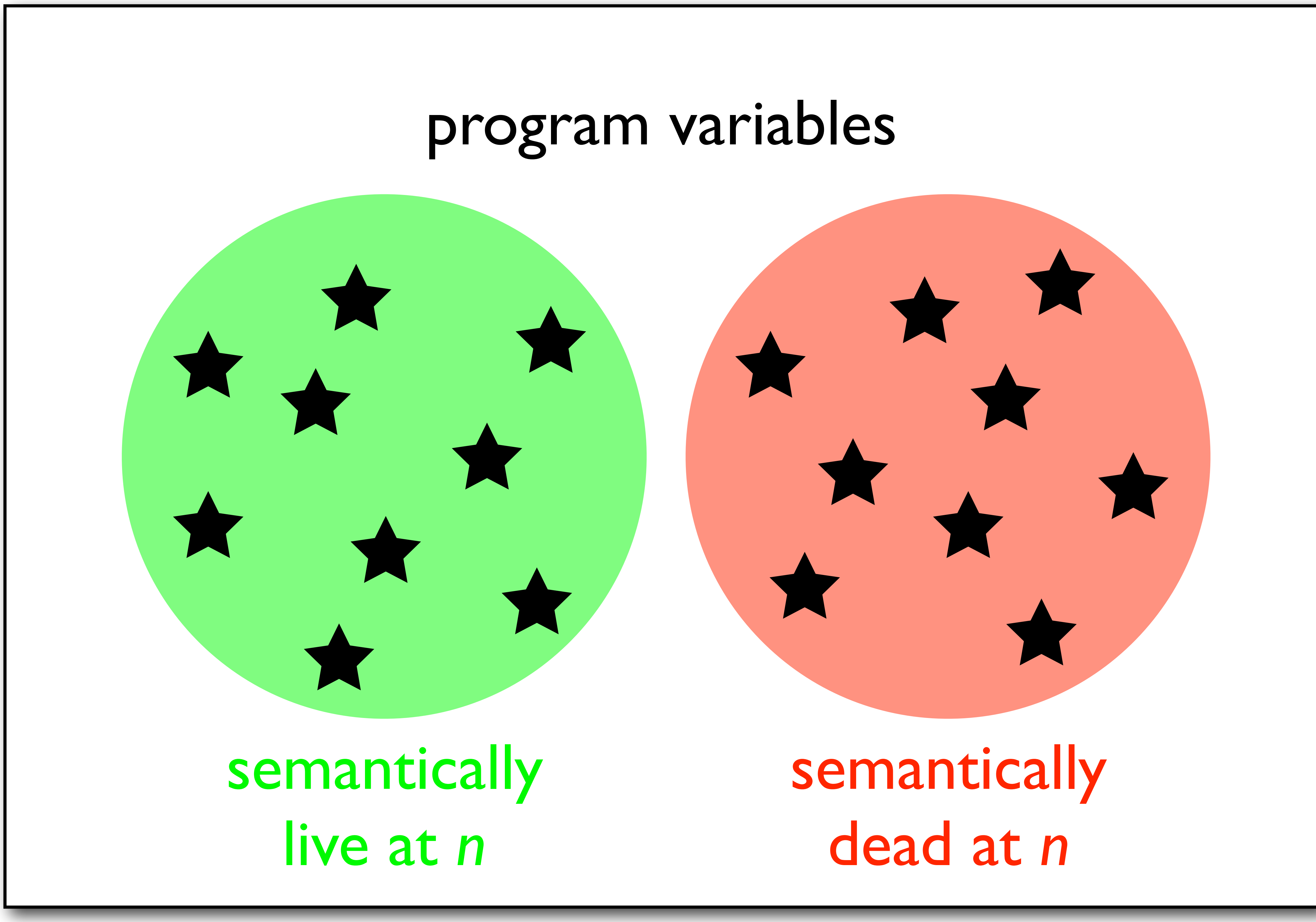
On *this* path through the flowgraph, `t` is not redefined before it's used, so `t` is *syntactically* live at the first instruction.

Note that this path never actually occurs during execution.

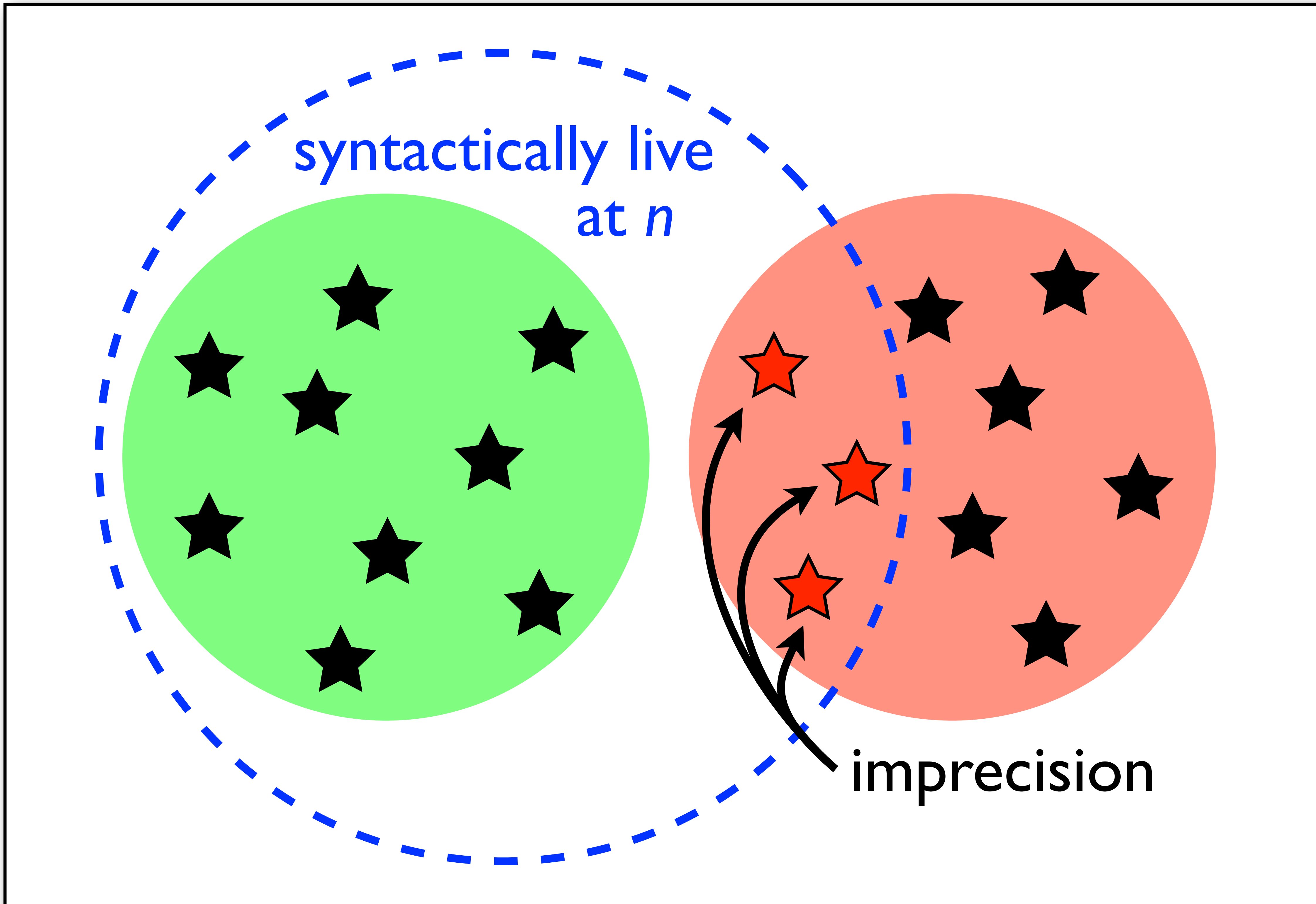
Semantic vs. syntactic

So, as we've seen before, *syntactic* liveness is a computable approximation of *semantic* liveness.

Semantic vs. syntactic



Semantic vs. syntactic



Semantic vs. syntactic

$$sem\text{-}live(n) \subseteq syn\text{-}live(n)$$

Using syntactic methods, we *safely overestimate* liveness.

Live variable analysis

LVA is a *backwards* data-flow analysis: usage information from *future* instructions must be propagated backwards through the program to discover which variables are live.

```
int f(int x, int y) {
  int z = x * y;
  ⋮
int a = z*2;          print z;
          if (z > 5) {
```

Live variable analysis

Variable liveness flows (backwards) through the program in a continuous stream.

Each instruction has an effect on the liveness information as it flows past.

Live variable analysis

An instruction makes a variable live
when it *references* (uses) it.

Live variable analysis

{ b, c, e, f }

```
a = b * c;
```
REFERENCE b, c

{ e, f }

```
d = e + 1;
```
REFERENCE e

{ f }

```
print f;
```
REFERENCE f

{ }

Live variable analysis

An instruction makes a variable dead
when it *defines* (assigns to) it.

Live variable analysis

{ }

`a = 7;` DEFINE `a`

{ a }

`b = 11;` DEFINE `b`

{ a, b }

`c = 13;` DEFINE `c`

{ a, b, c }

Live variable analysis

We can devise functions *ref*(*n*) and *def*(*n*) which give the sets of variables referenced and defined by the instruction at node *n*.

ref(`x = 3`) = { }

def(`x = 3`) = { `x` }

ref(`print x`) = { `x` }

def(`print x`) = { }

ref(`x = x + y`) = { `x`, `y` }

def(`x = x + y`) = { `x` }

Live variable analysis

As liveness flows backwards past an instruction, we want to modify the liveness information by *adding* any variables which it references (they become live) and *removing* any which it defines (they become dead).

{ x, y }

ref(`print x`) = { x }

{ y }

{ y }

def(`x = 3`) = { x }

{ x, y }

Live variable analysis

If an instruction both references and defines variables, we must remove the defined variables *before* adding the referenced ones.

{ x, y, z }

```
x = x + y
```

{ x, z }

def(x = x + y) = { x }

ref(x = x + y) = { x, y }

Live variable analysis

So, if we consider *in-live*(*n*) and *out-live*(*n*), the sets of variables which are live immediately *before* and immediately *after* a node, the following equation must hold:

$$in\text{-}live(n) = \Big(out\text{-}live(n) \setminus def(n)\Big) \cup ref(n)$$

Live variable analysis

$$\textit{in-live}(n) = \Big(\textit{out-live}(n) \setminus \textit{def}(n)\Big) \cup \textit{ref}(n)$$

$\textit{in-live}(n) = \big(\textit{out-live}(n) \setminus \textit{def}(n)\big) \cup \textit{ref}(n)$

$= (\{\ \texttt{x}, \texttt{z}\ \} \setminus \{\ \texttt{x}\ \}) \cup \{\ \texttt{x}, \texttt{y}\ \}$

$= \{\ \texttt{z}\ \} \cup \{\ \texttt{x}, \texttt{y}\ \} = \{\ \texttt{x}, \texttt{y}, \texttt{z}\ \}$

n: $\boxed{\texttt{x = x + y}}$

$\textit{out-live}(n) = \{\ \texttt{x}, \texttt{z}\ \}$

$\textit{def}(n) = \{\ \texttt{x}\ \}$ $\textit{ref}(n) = \{\ \texttt{x}, \texttt{y}\ \}$

Live variable analysis

So we know how to calculate *in-live*(*n*) from the values of *def*(*n*), *ref*(*n*) and *out-live*(*n*).
But how do we calculate *out-live*(*n*)?

$$\textit{in-live}(n) = \big(\textit{out-live}(n) \setminus \textit{def}(n)\big) \cup \textit{ref}(n)$$

n: `x = x + y`

$$\textit{out-live}(n) = ?$$

Live variable analysis

In straight-line code each node has a unique successor, and the variables live at the exit of a node are exactly those variables live at the entry of its successor.

Live variable analysis

l:

$\mathit{out\text{-}live}(l) = \{\ \mathtt{s}, \mathtt{t}, \mathtt{x}, \mathtt{y}\ \}$

$\mathit{in\text{-}live}(m) = \big(\mathit{out\text{-}live}(m) \setminus \mathit{def}(m)\big) \cup \mathit{ref}(m)$

m: `z = x * y;`

$\mathit{out\text{-}live}(m) = \{\ \mathtt{s}, \mathtt{t}, \mathtt{z}\ \}$

$\mathit{in\text{-}live}(n) = \big(\mathit{out\text{-}live}(n) \setminus \mathit{def}(n)\big) \cup \mathit{ref}(n)$

n: `print s + t;`

$\mathit{out\text{-}live}(n) = \{\ \mathtt{z}\ \}$

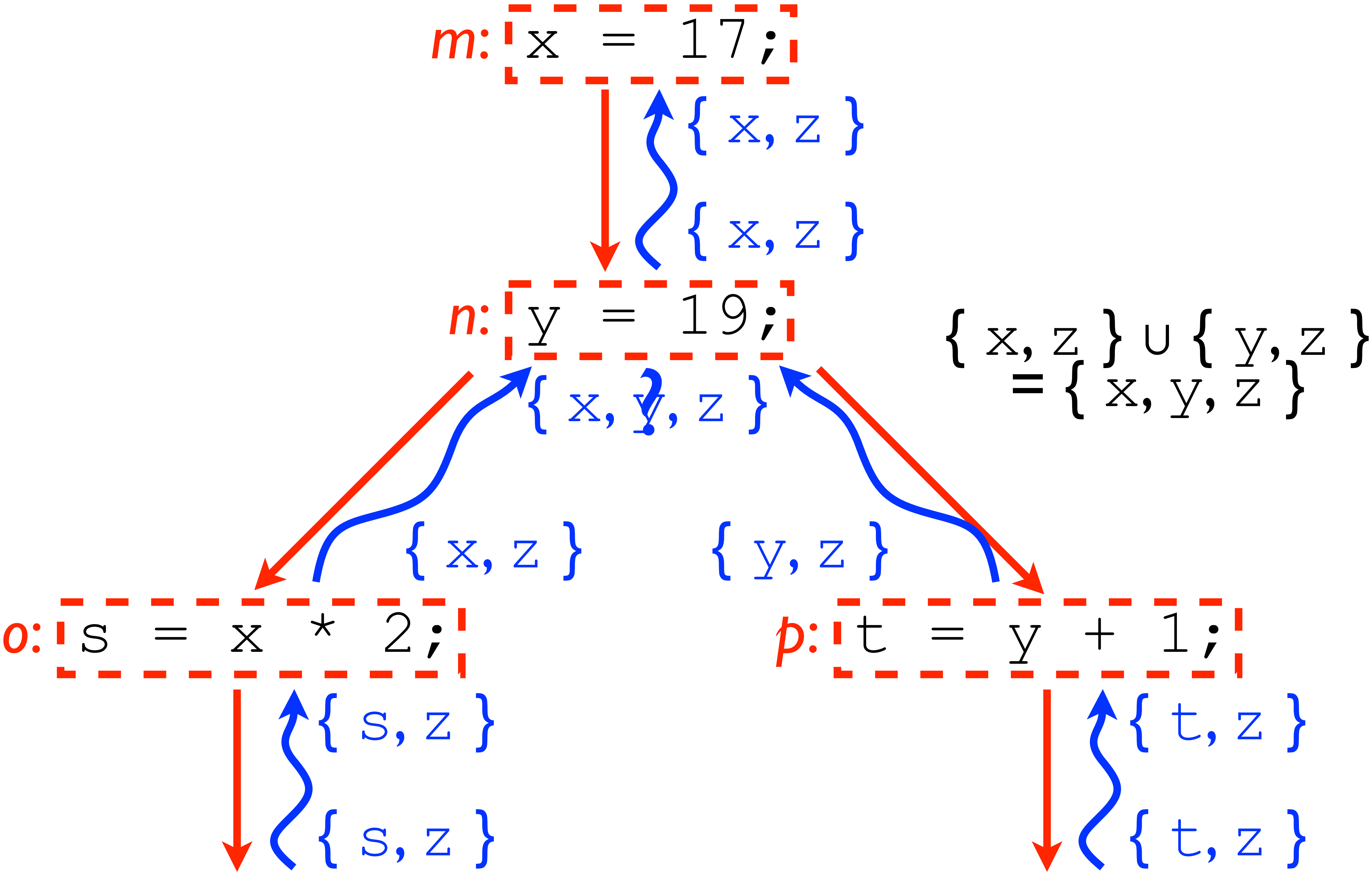
$\mathit{in\text{-}live}(o) = \big(\mathit{out\text{-}live}(o) \setminus \mathit{def}(o)\big) \cup \mathit{ref}(o)$

o:

Live variable analysis

In general, however, each node has an arbitrary number of successors, and the variables live at the exit of a node are exactly those variables live at the entry of *any* of its successors.

Live variable analysis



Live variable analysis

So the following equation must also hold:

$$out\text{-}live(n) = \bigcup_{s \in succ(n)} in\text{-}live(s)$$

Data-flow equations

These are the *data-flow equations* for live variable analysis, and together they tell us everything we need to know about how to propagate liveness information through a program.

$$\mathit{in\text{-}live}(n) = \Big(\mathit{out\text{-}live}(n) \setminus \mathit{def}(n)\Big) \cup \mathit{ref}(n)$$

$$\mathit{out\text{-}live}(n) = \bigcup_{s \in \mathit{succ}(n)} \mathit{in\text{-}live}(s)$$

Data-flow equations

Each is expressed in terms of the other, so we can combine them to create one overall liveness equation.

$$live(n) = \left(\left(\bigcup_{s \in succ(n)} live(s) \right) \setminus def(n) \right) \cup ref(n)$$

Algorithm

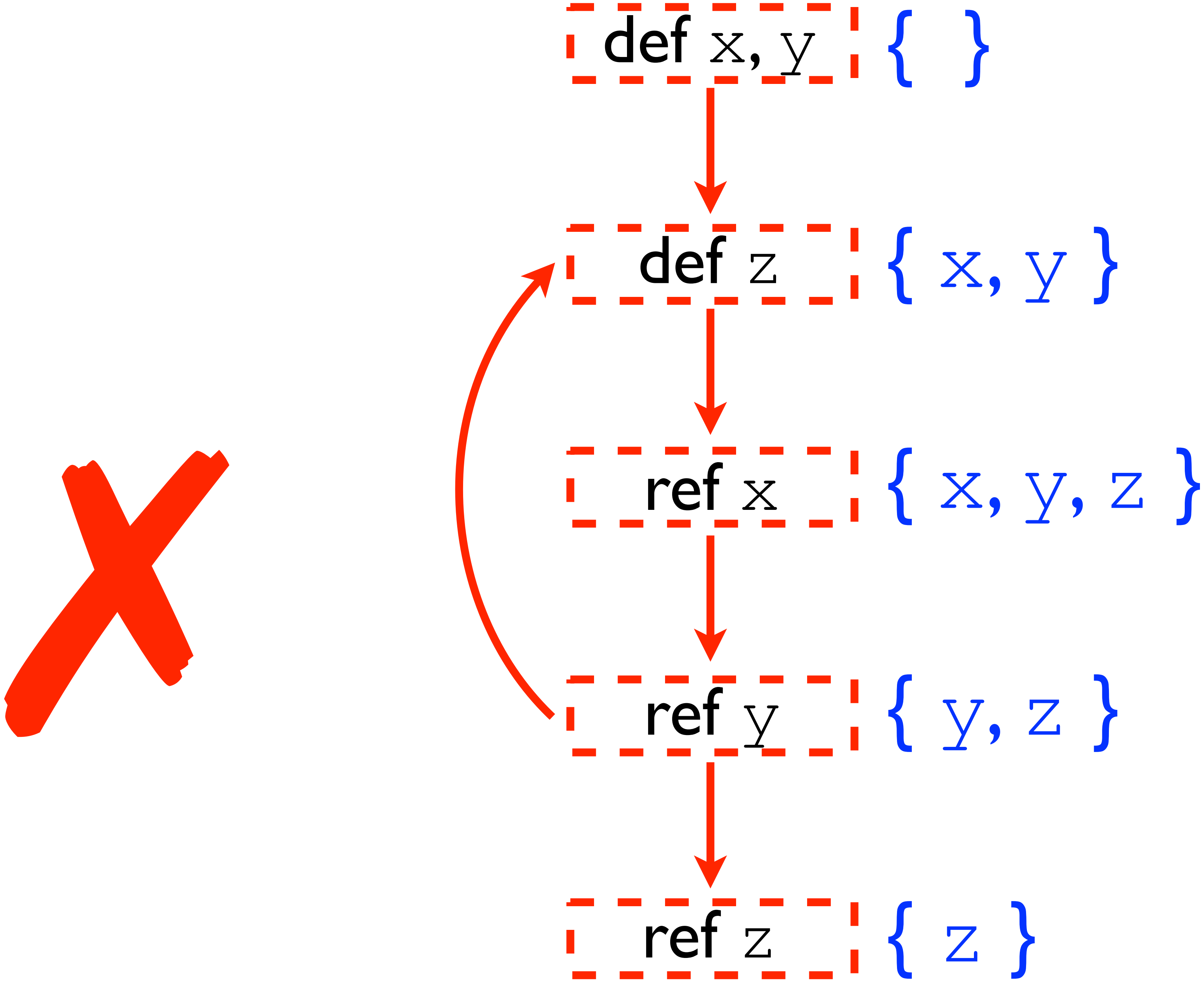
We now have a formal description of liveness, but we need an actual algorithm in order to do the analysis.

Algorithm

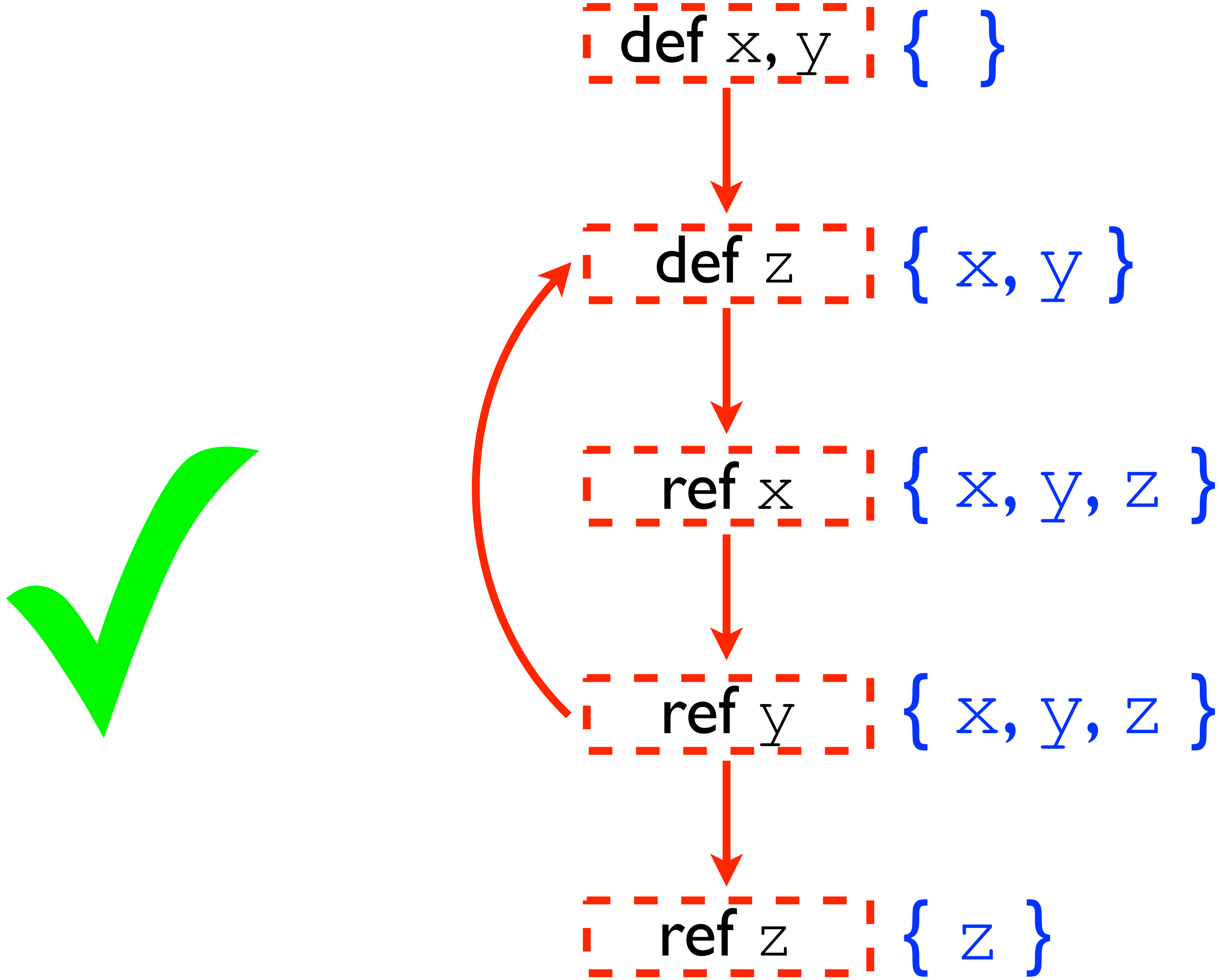
"Doing the analysis" consists of computing a value *live(n)* for each node *n* in a flowgraph such that the liveness data-flow equations are satisfied.

A simple way to solve the data-flow equations is to adopt an iterative strategy.

Algorithm



Algorithm



Algorithm

```
for i = 1 to n do live[i] := {}
while (live[] changes) do
   for i = 1 to n do
```

$$\texttt{live[i]} := \left(\left(\bigcup_{s \in succ(\texttt{i})} \texttt{live}[s]\right) \setminus def(\texttt{i})\right) \cup ref(\texttt{i})$$

Algorithm

This algorithm is guaranteed to terminate since there are a *finite* number of variables in each program and the effect of one iteration is *monotonic*.

Furthermore, although any solution to the data-flow equations is safe, this algorithm is guaranteed to give the *smallest* (and therefore most precise) solution.

(See the *Knaster-Tarski theorem* if you're interested.)

Algorithm

Implementation notes:

- If the program has *n* variables, we can implement each element of `live[]` as an *n*-bit value, with each bit representing the liveness of one variable.

- We can store liveness once per basic block and recompute inside a block when necessary. In this case, given a basic block *n* of instructions $i_1, ..., i_k$:

$$live(n) = \left(\bigcup_{s \in succ(n)} live(s) \right) \setminus def(i_k) \cup ref(i_k) \cdots \setminus def(i_1) \cup ref(i_1)$$

Safety of analysis

- Syntactic liveness safely overapproximates semantic liveness.
- The usual problem occurs in the presence of address-taken variables (cf. labels, procedures): *ambiguous* definitions and references. For safety we must
    - overestimate ambiguous references (assume all address-taken variables are referenced) and
    - underestimate ambiguous definitions (assume no variables are defined); this increases the size of the smallest solution.

Safety of analysis

```
MOV x,#1
MOV y,#2
MOV z,#3
MOV t32,#&x
MOV t33,#&y
MOV t34,#&z
:
m: STI t35,#7
:
n: LDI t36,t37
```

def(*m*) = { }
ref(*m*) = { t35 }

def(*n*) = { t36 }
ref(*n*) = { t37, x, y, z }

Summary

- Data-flow analysis collects information about how data moves through a program
- Variable liveness is a data-flow property
- Live variable analysis (LVA) is a backwards data-flow analysis for determining variable liveness
- LVA may be expressed as a pair of complementary data-flow equations, which can be combined
- A simple iterative algorithm can be used to find the smallest solution to the LVA data-flow equations