



# GENERATIVE MODELLING

Param Hanji • Nov 2023

# Outline

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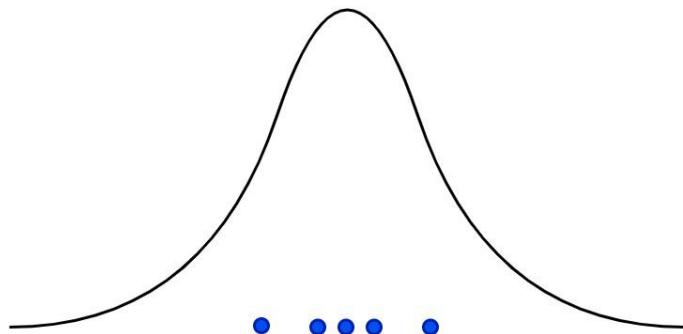
- Basic operations
- Problems relevant to machine perception
- Commonly used models
  - VAE
  - GAN
  - Normalizing flows
  - Diffusion models
- Evaluation

# Images as samples

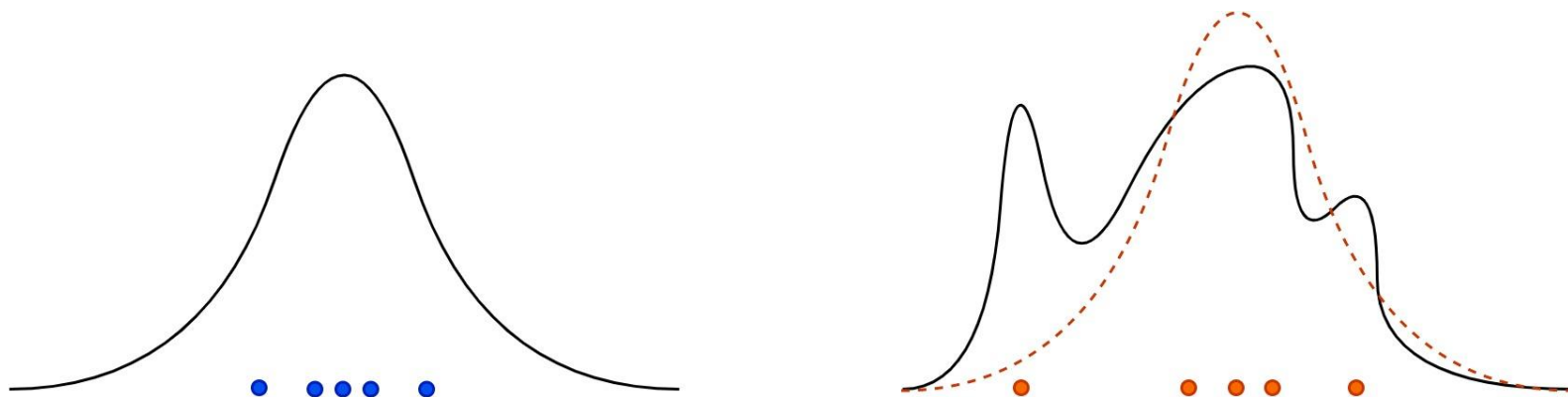


# Operation 1: Sample generation

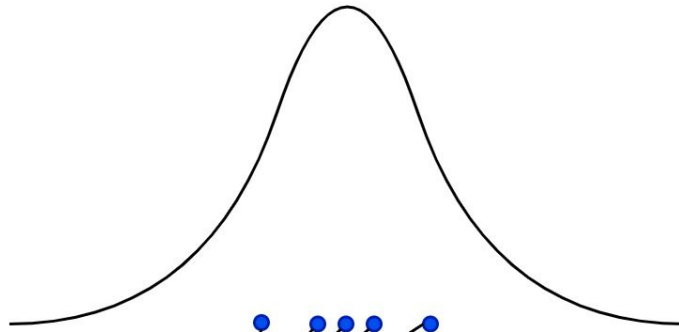
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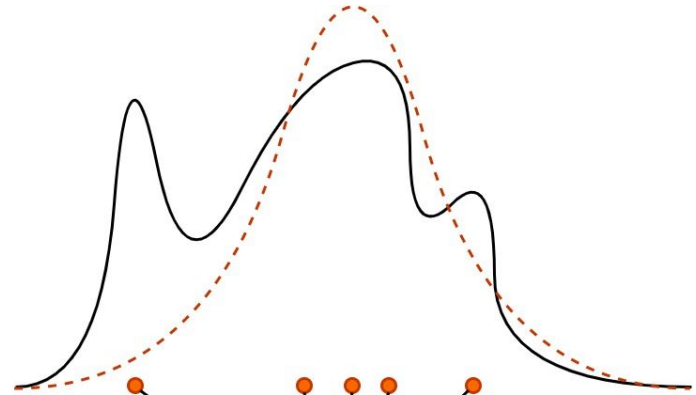
# Operation 1: Sample generation



# Operation 2: Density estimation



$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp - \frac{(x - \mu)^2}{2\sigma^2}$$



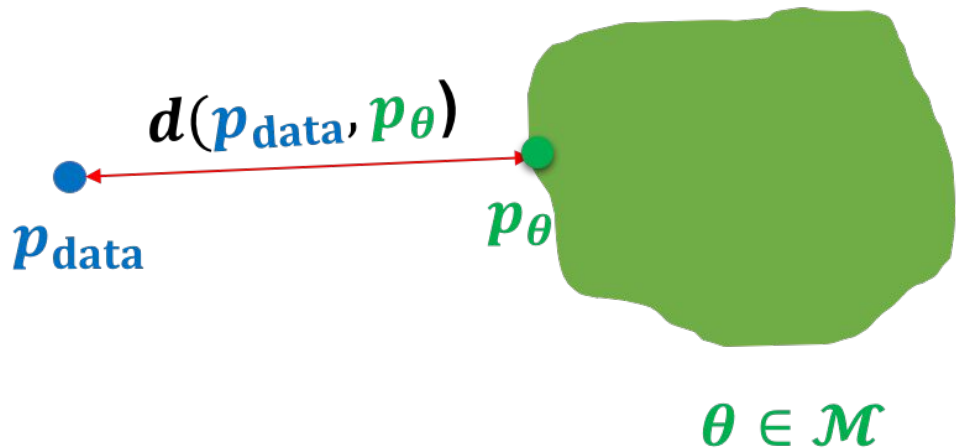
$$p(x) = ?$$

$$q_{\theta}(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp - \frac{(x - \mu)^2}{2\sigma^2}$$

# Training



$$\mathbf{x}^{(j)} \sim p_{\text{data}} \\ j = 1, 2, \dots, |\mathcal{D}|$$



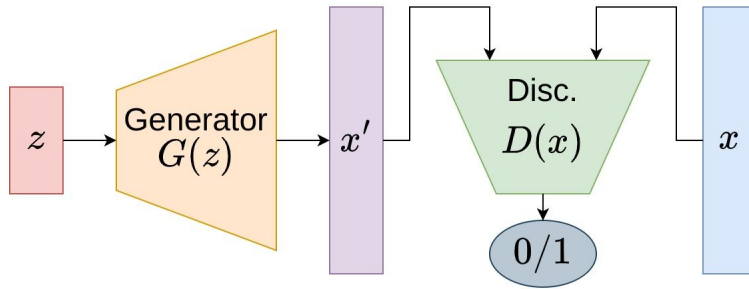
**Model family**

# Domains

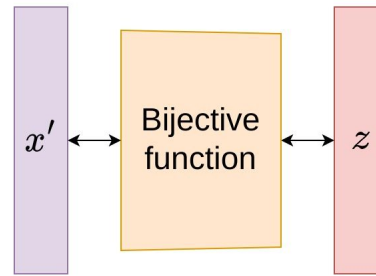
- Computer vision
- Computer graphics
- Text generation
- Medical imaging
- Audio synthesis
- Astrophysics



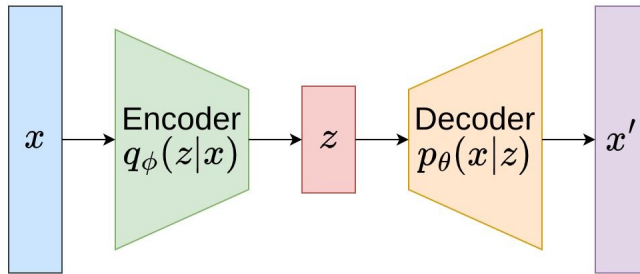
# Deep generative modelling



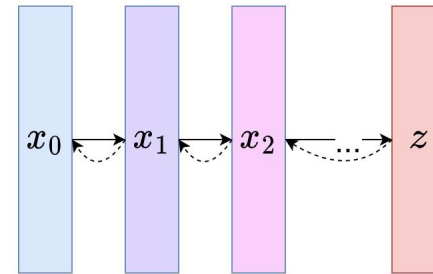
Generative adversarial network



Normalizing flow



Variational autoencoder



Diffusion method

# Inverse problems



Input

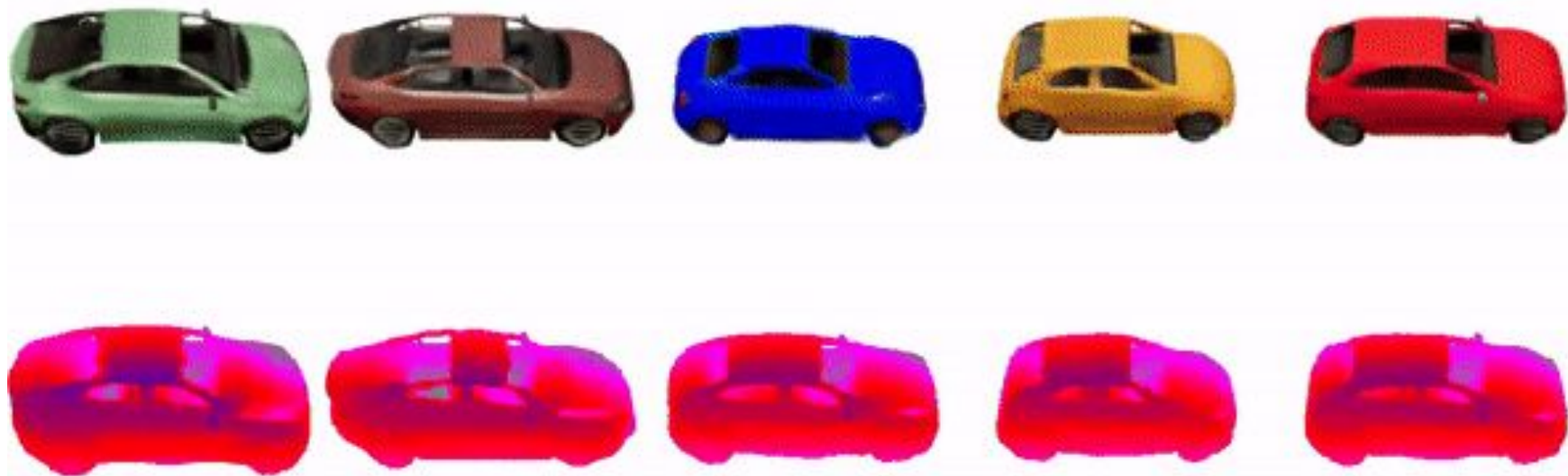


Super-Resolution

# Inverse imaging

- Superresolution
- Denoising
- Colorization
- Inpainting
- Uncropping
- Deblurring
- Single-image HDR

# Inverse graphics



# Text-conditional generation



A dragon fruit wearing karate belt in the snow.



Android Mascot made from bamboo.



A bald eagle made of chocolate powder, mango, and whipped cream.

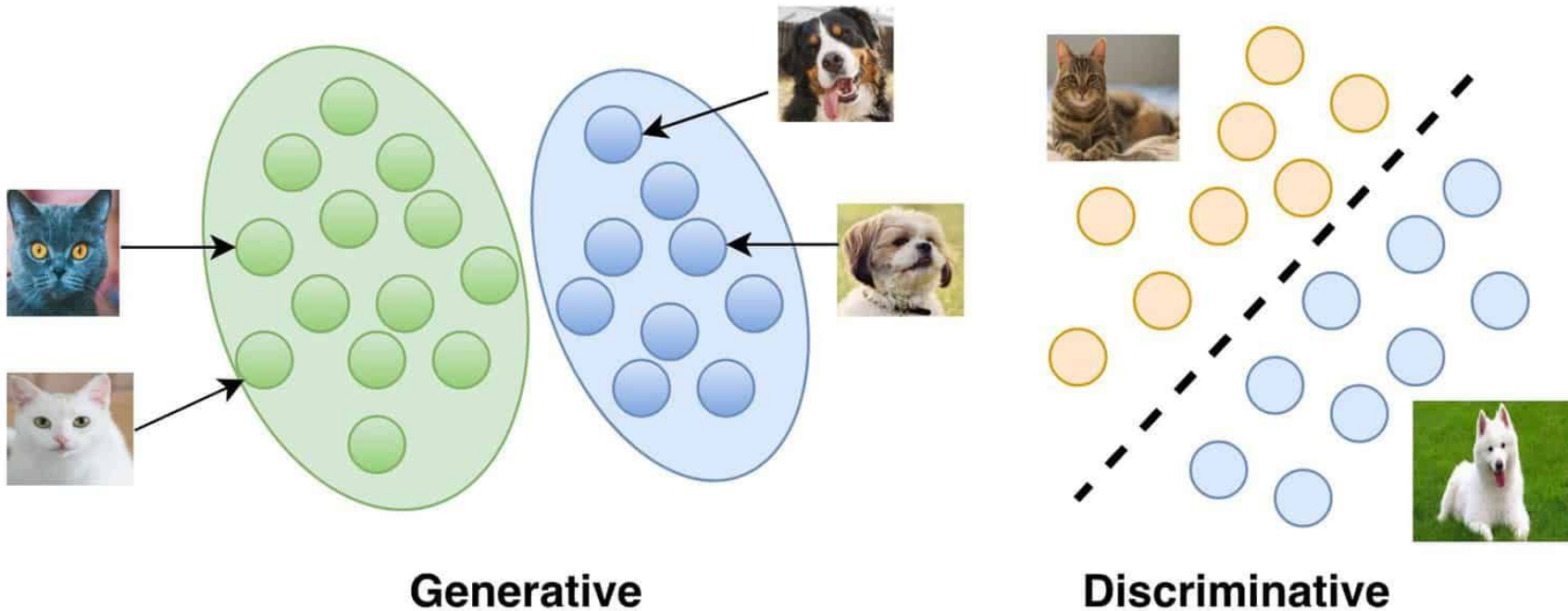


A strawberry mug filled with white sesame seeds. The mug is floating in a dark chocolate sea.

# Text-conditional 3D



# Classification



# Requirements

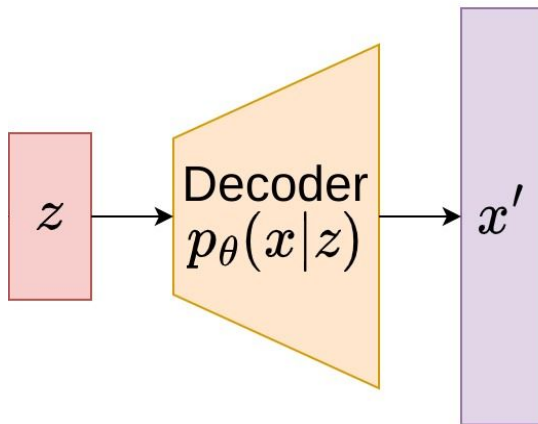
- Handle high dimensional data
- Fast, efficient sampling
- High sample quality
- Diverse samples
- [Optional] Density evaluation
- Low dimensional latent



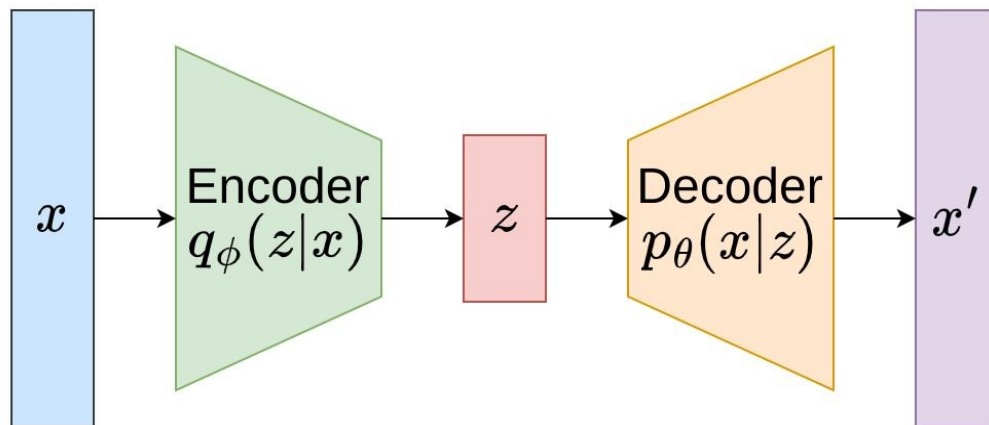
# Variational autoencoder

$$\theta^* = \arg \max_{\theta} \sum_{i=1}^n \log p_{\theta}(\mathbf{x}^{(i)})$$

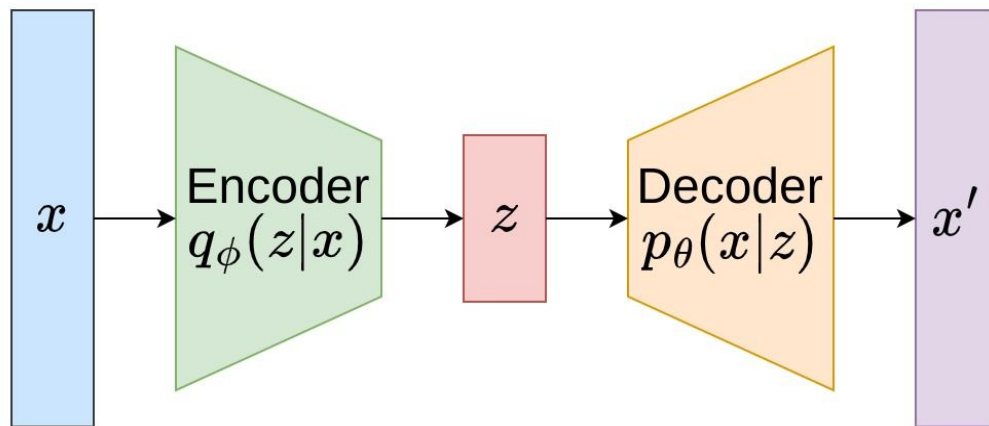
$$p_{\theta}(\mathbf{x}^{(i)}) = \int p_{\theta}(\mathbf{x}^{(i)} | \mathbf{z}) p_{\theta}(\mathbf{z}) d\mathbf{z}$$



# Variational autoencoder

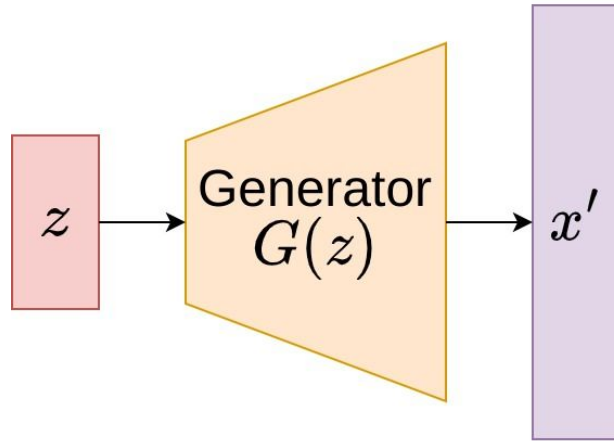


# Variational autoencoder

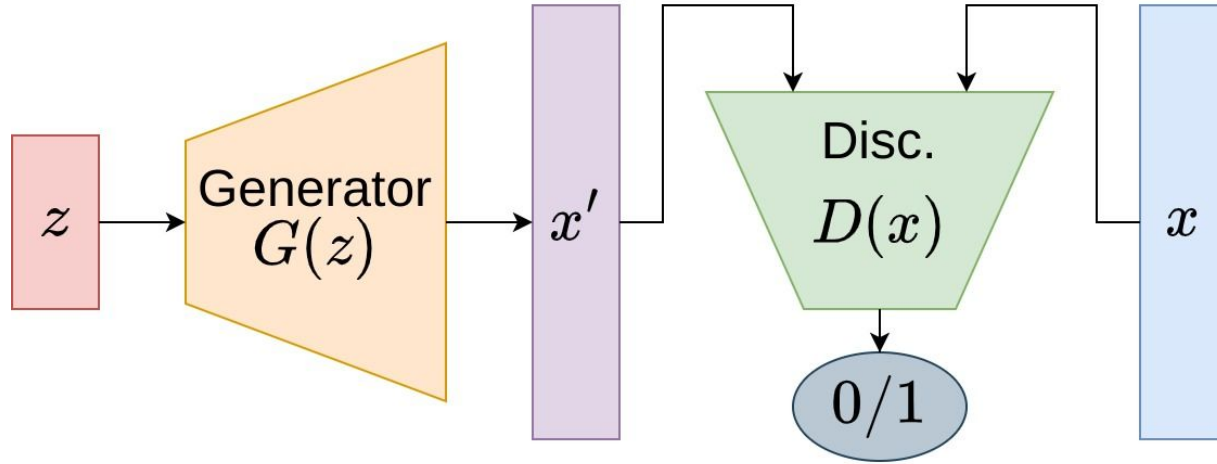


$$-L_{\text{VAE}} = \log p_{\theta}(\mathbf{x}) - D_{\text{KL}}(q_{\phi}(\mathbf{z}|\mathbf{x}) || p_{\theta}(\mathbf{z}|\mathbf{x})) \leq \log p_{\theta}(\mathbf{x})$$

# Generative adversarial network

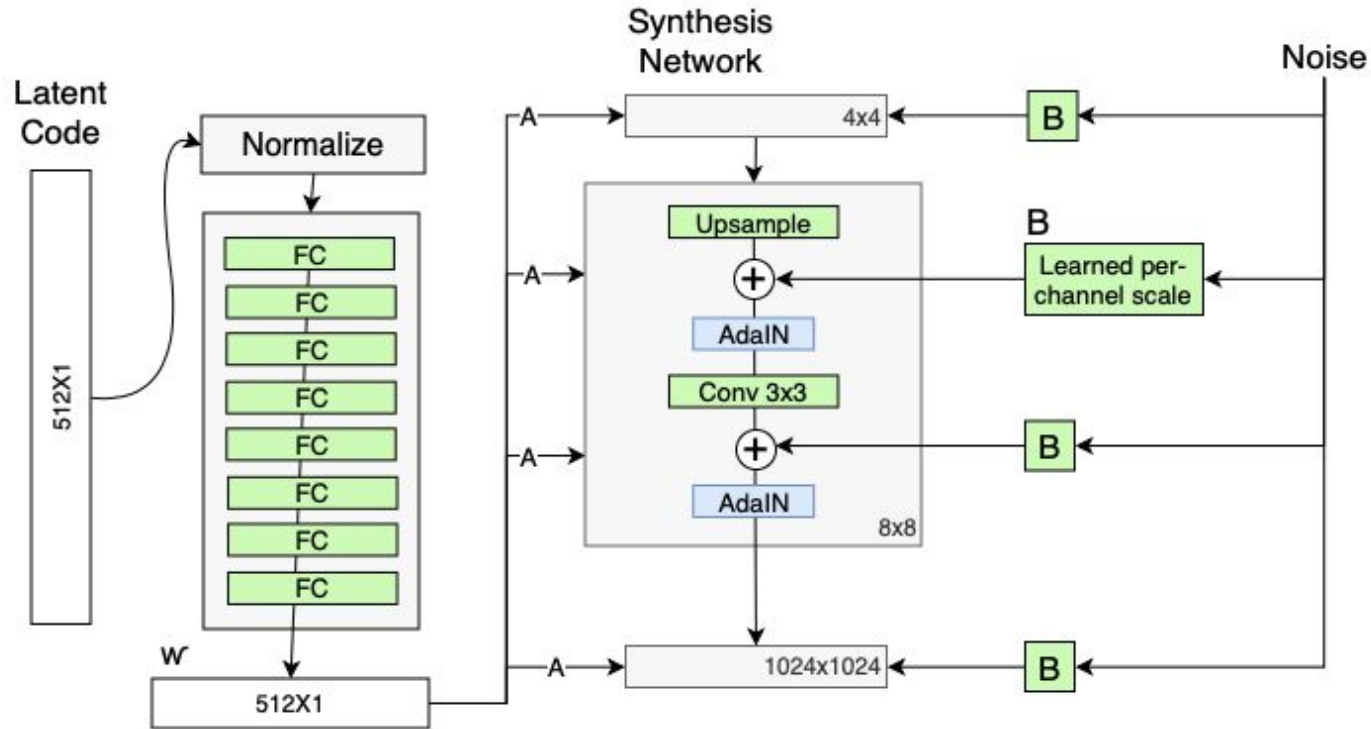


# Generative adversarial network

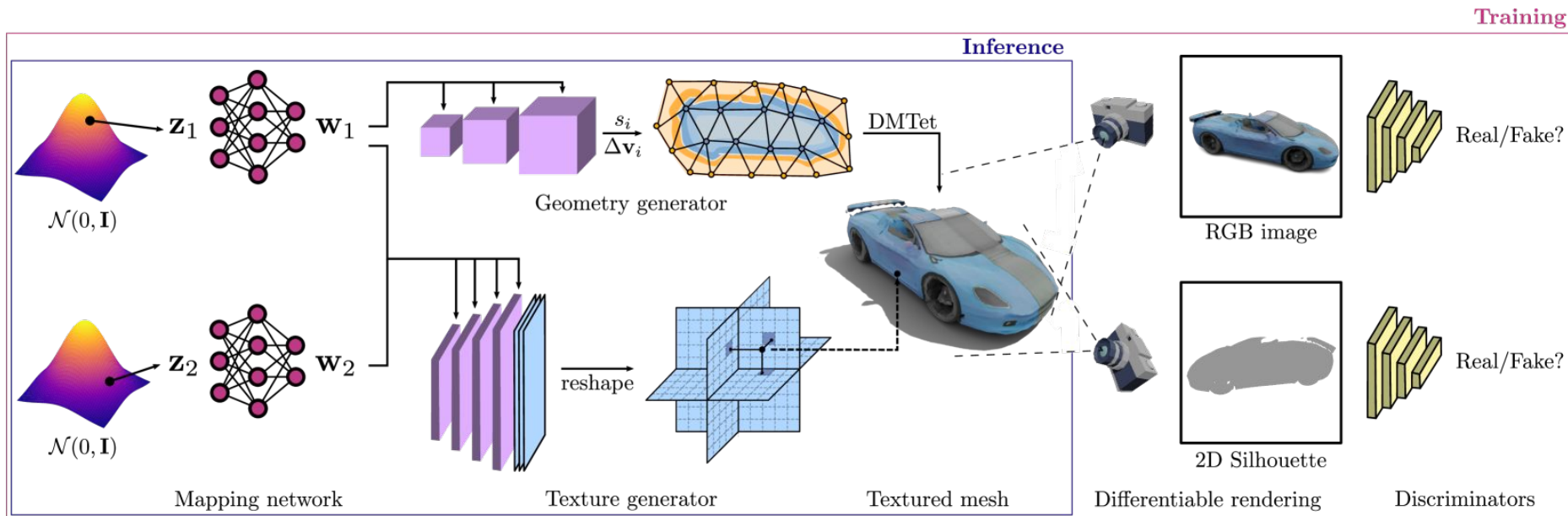


$$\min_G \max_D V(D, G) = \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z} \sim p_z(\mathbf{z})} [\log(1 - D(G(\mathbf{z})))]$$

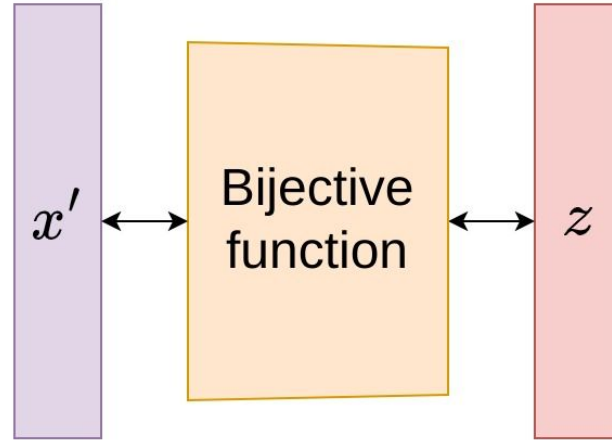
# StyleGAN



# GANs in complex systems

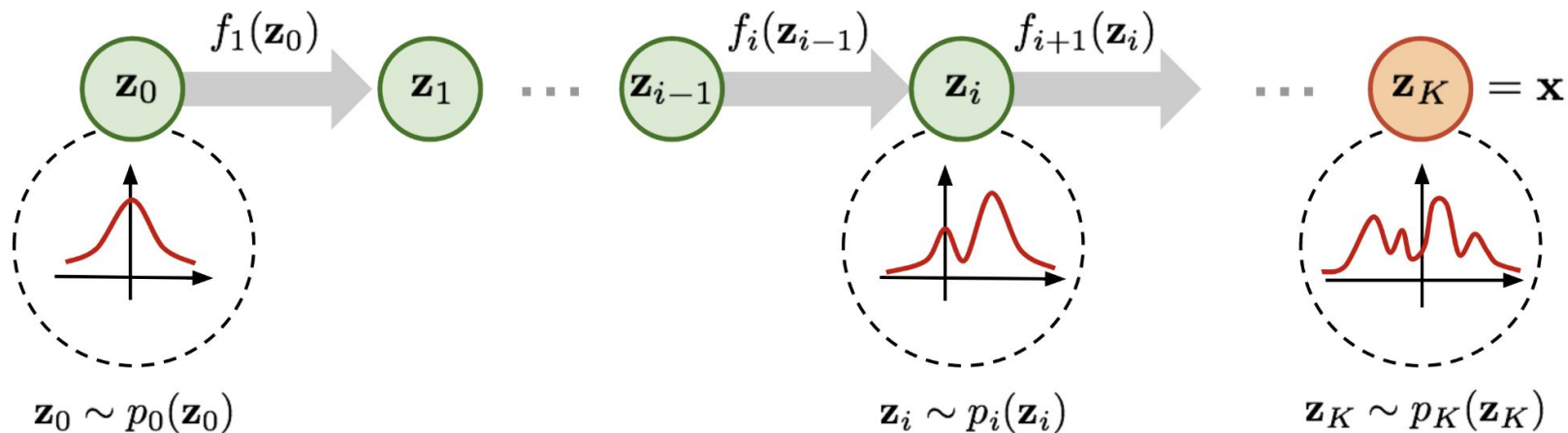


# Normalizing flows



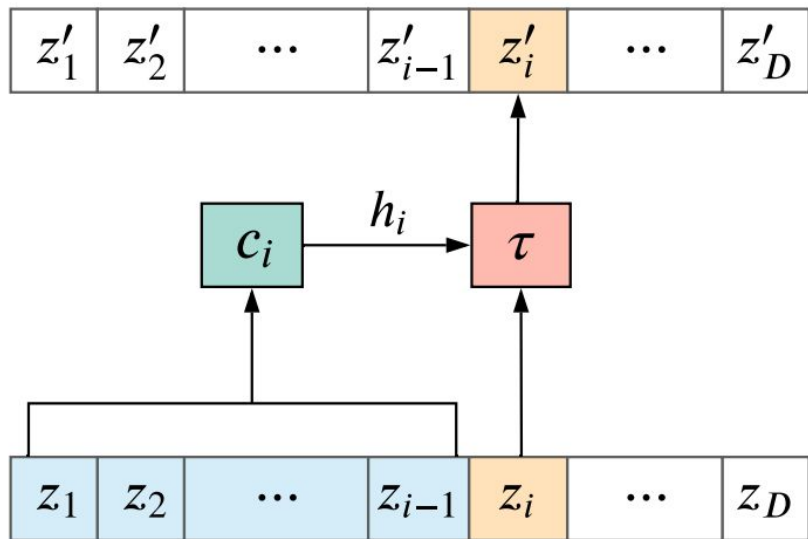


# Normalizing flows



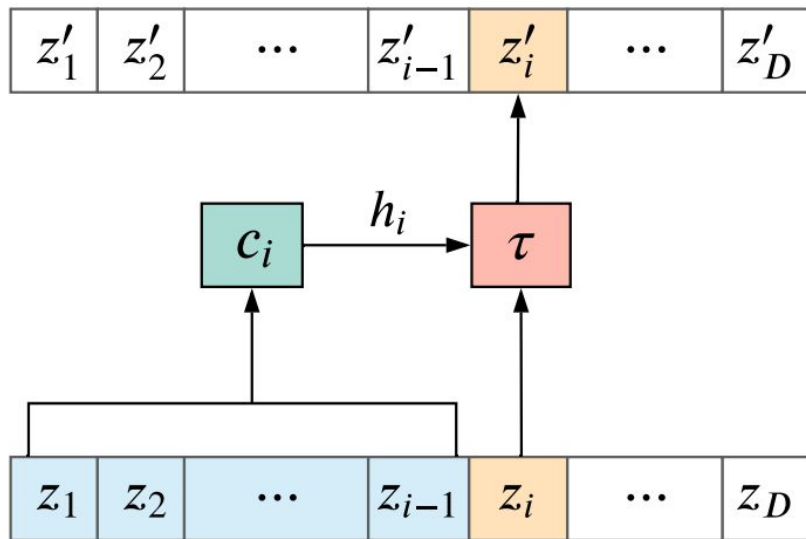
$$p_i(\mathbf{z}_i) = p_{i-1}(f_i^{-1}(\mathbf{z}_i)) \left| \det \frac{d f_i^{-1}}{d \mathbf{z}_i} \right|$$

# Normalizing flows - coupling

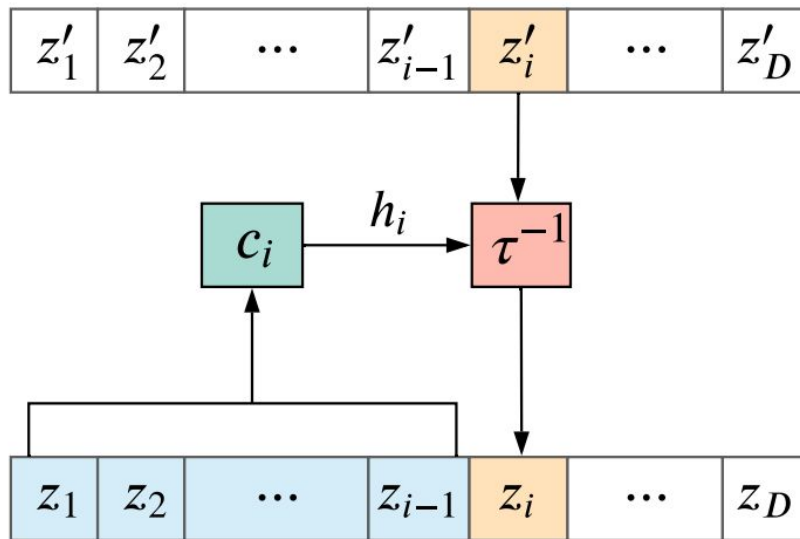


(a) Forward

# Normalizing flows - coupling

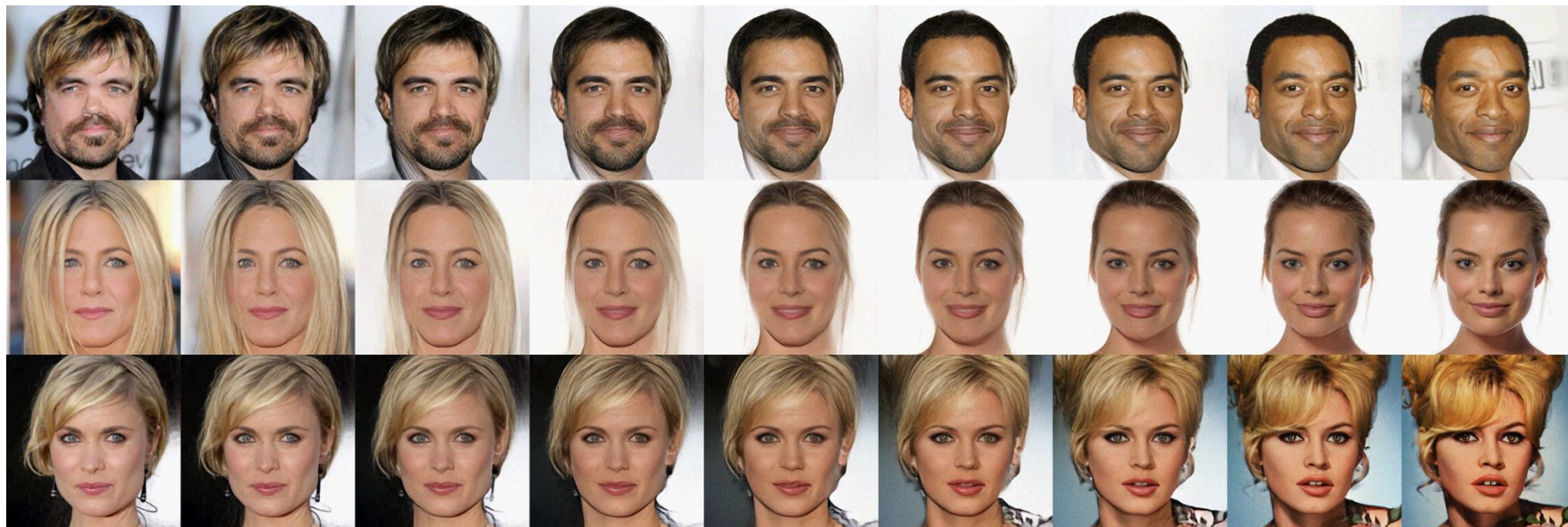


(a) Forward

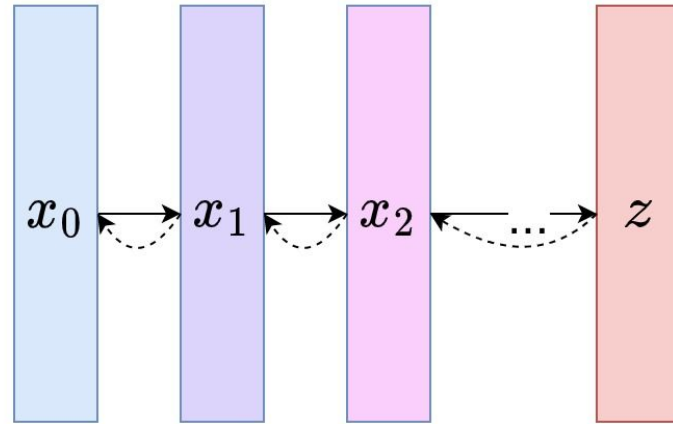


(b) Inverse

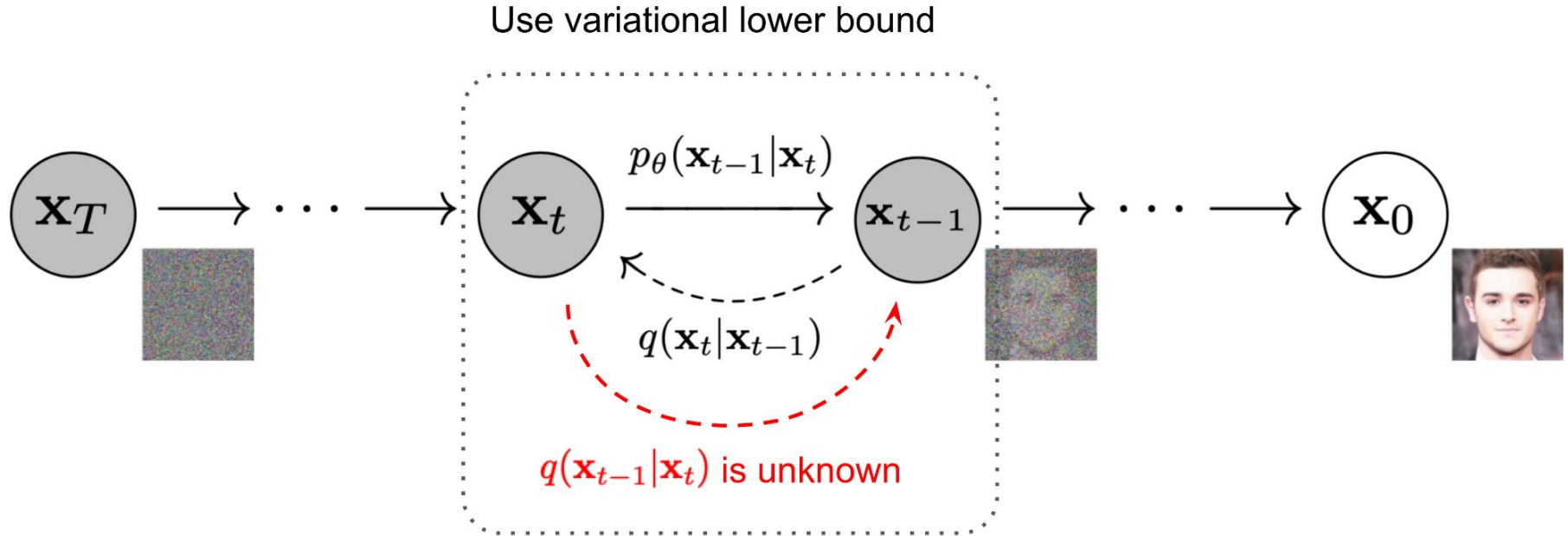
# Flows - latent manipulation



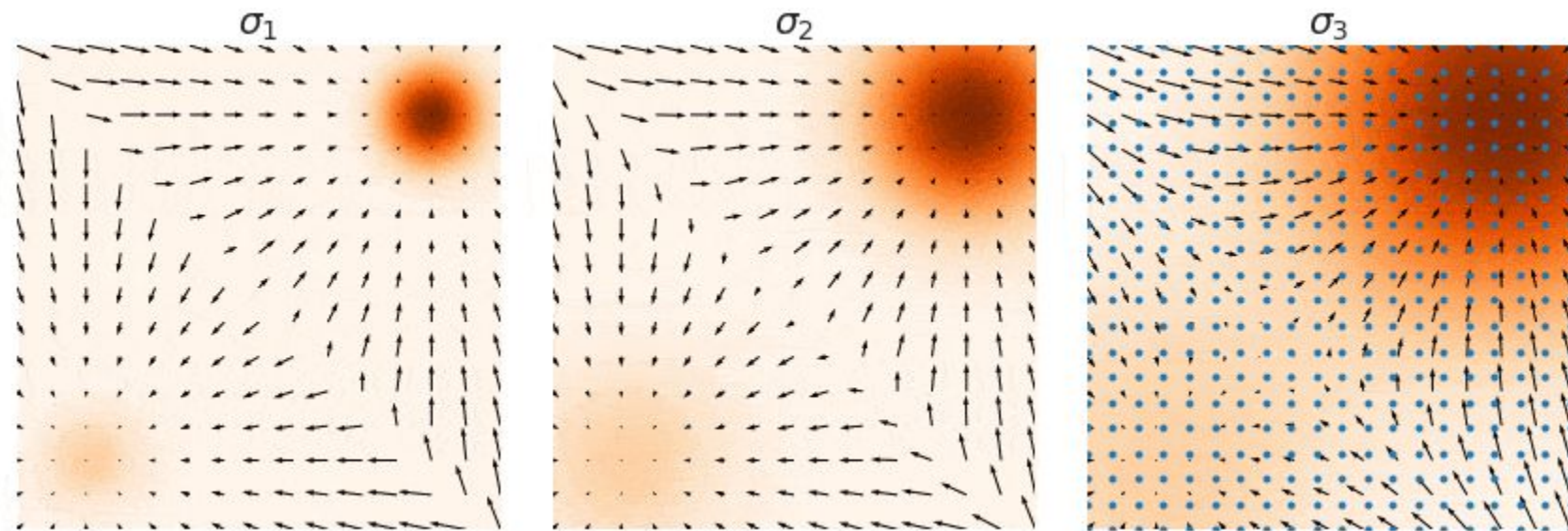
# Diffusion models



# Diffusion models

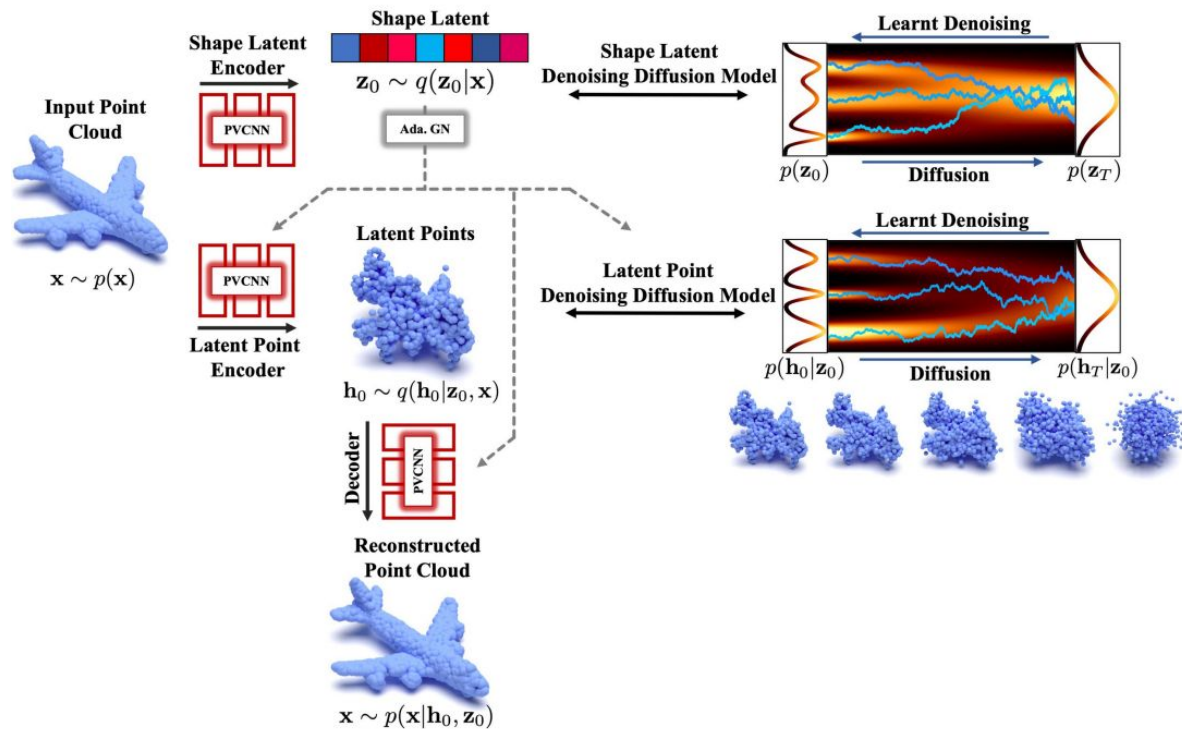


# Score matching



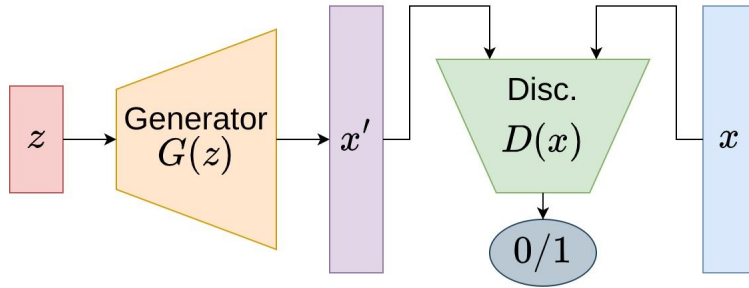
$$\mathbf{x}_{i+1} \leftarrow \mathbf{x}_i + \epsilon \nabla_{\mathbf{x}} \log p(\mathbf{x}) + \sqrt{2\epsilon} \mathbf{z}_i, \quad i = 0, 1, \dots, K$$

# Hybrid models

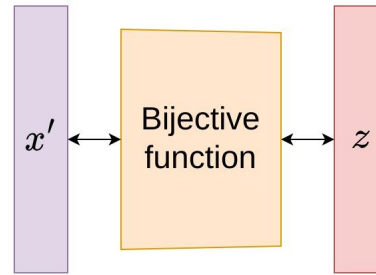




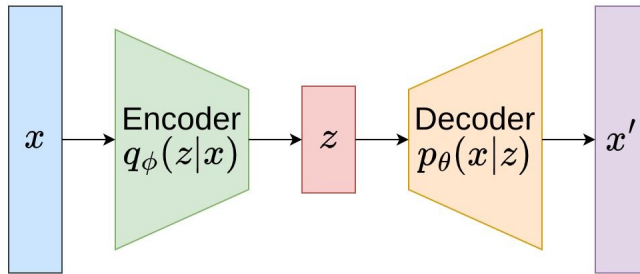
# Evaluation with likelihoods



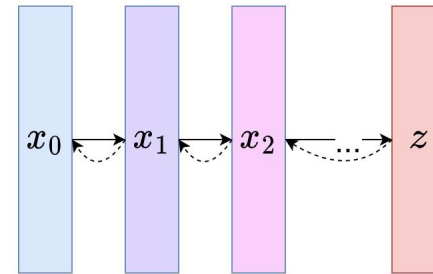
Generative adversarial network



Normalizing flow



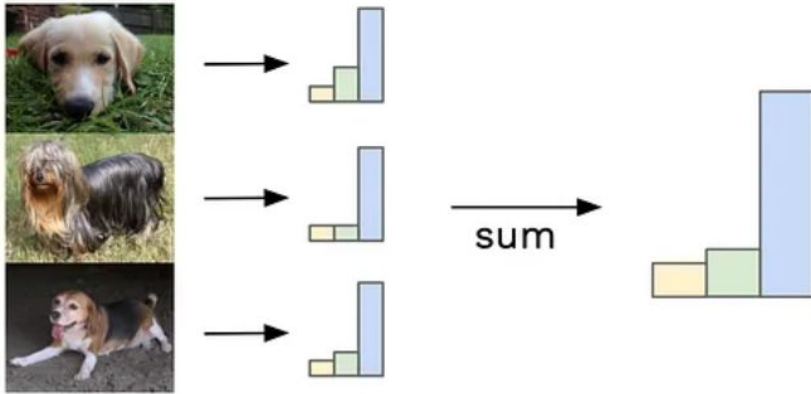
Variational autoencoder



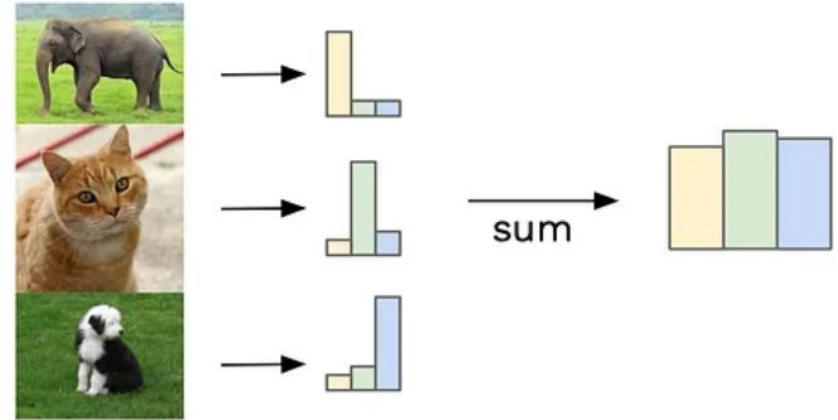
Diffusion method

# Inception score

Similar labels sum to give focussed distribution



Different labels sum to give uniform distribution



# Frechet Inception distance

- Squared Wasserstein distance between generated and reference distributions
- Use intermediate activations of pre-trained classifier
- Assume distributions are multivariate normals

$$\text{FID} = \|\mu - \mu_w\|_2^2 + \text{tr}(\Sigma + \Sigma_w - 2(\Sigma^{1/2}\Sigma_w\Sigma^{1/2})^{1/2}).$$

# Requirements

- Handle high dimensional data
- Fast, efficient sampling
- High sample quality
- Diverse samples
- [Optional] Density evaluation
- [Optional] Low dimensional latent

# Summary

The Generative Learning Trilemma

