Appearance Modelling, Acquisition, Relighting

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Light and Colors

Visible spectrum

Increasing Wavelength ($\lambda$) →
Light and Colors

• Light can be a mixture of many wavelengths
• Spectral power distribution (SPD)
  - $P(\lambda) =$ intensity at wavelength $\lambda$
  - intensity as a function of wavelength
• We perceive these distributions as colors
Measuring Light

• How do we measure light

Measuring = Counting photons
Basic Definitions

• Assume light consists of photons with
  – \( x \): Position
  – \( \vec{\omega} \): Direction of motion
  – \( \lambda \): Wavelength

• Each photon has an energy of:
  \[
  \frac{hc}{\lambda}
  \]
  – \( h \approx 6.63 \cdot 10^{-34} m^2 \cdot kg/s \): Planck's constant
  – \( c = 299,792,458 m/s \): speed of light in vacuum
  – Unit of energy, Joule: \([J = kg \cdot m^2/s^2]\)
Radiometry

• Flux (radiant flux, power)
  – total amount of energy passing through surface or space per unit time
    \[
    \Phi(A) \left[ \frac{J}{s} = W \right]
    \]
  – examples:
    • number of photons hitting a wall per second
    • number of photons leaving a lightbulb per second
Radiometry

- Radiant intensity
  - Power (flux) per solid angle = directional density of flux
  
  \[ I(\hat{\omega}) = \frac{d\Phi}{d\omega} \quad \left[ \frac{W}{sr} \right] \]

  \[ \Phi = \int_{S^2} I(\hat{\omega}) d\omega \]

- example:
  - power per unit solid angle emanating from a point source
Radiometry

• Radiance
  – Radiant intensity per perpendicular unit area

\[ L(x, \omega) = \frac{d^2 \Phi(A)}{d\omega dA^\perp(x, \omega)} \]

\[ = \frac{d^2 \Phi(A)}{d\omega dA(x) \cos \theta} \left[ \frac{W}{m^2 \text{sr}} \right] \]

  – remains constant along a ray
Reflection Models

- **Bidirectional Reflectance Distribution Function (BRDF)**
BRDF

- Bidirectional Reflectance Distribution Function

\[ f_r(x, \bar{\omega}_i, \bar{\omega}_r) = \frac{dL_r(x, \bar{\omega}_r)}{L_i(x, \bar{\omega}_i \cos \theta_i d\bar{\omega}_i} \quad [1/sr] \]

BRDF infinitesimal reflected radiance infinitesimal solid angle
Reflection Equation

• The BRDF provides a relation between incident radiance and differential reflected radiance
• From this we can derive the Reflection Equation

\[
f_r(\mathbf{x}, \mathbf{\omega}_i, \mathbf{\omega}_r) = \frac{dL_r(\mathbf{x}, \mathbf{\omega}_r)}{L_i(\mathbf{x}, \mathbf{\omega}_i) \cos \theta_i \, d\mathbf{\omega}_i}
\]

\[
f_r(\mathbf{x}, \mathbf{\omega}_i, \mathbf{\omega}_r) L_i(\mathbf{x}, \mathbf{\omega}_i) \cos \theta_i = \frac{dL_r(\mathbf{x}, \mathbf{\omega}_r)}{d\mathbf{\omega}_i}
\]

\[
\int_{H^2} f_r(\mathbf{x}, \mathbf{\omega}_i, \mathbf{\omega}_r) L_i(\mathbf{x}, \mathbf{\omega}_i) \cos \theta_i \, d\mathbf{\omega}_i = L_r(\mathbf{x}, \mathbf{\omega}_r)
\]
Reflection Equation

- The reflected radiance due to incident illumination from all directions

\[ L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos \theta_i \, d\omega_i \]
The Rendering Equation

- The outgoing light is the sum of emitted and incoming reflected light.

\[
L_o(x, \omega_o) = L_e(x, \omega_o) + L_r(x, \omega_o)
\]

\[
L_o(x, \omega_o) = L_e(x, \omega_o) + \int_{H^2} f_r(x, \omega_i, \omega_o) L_i(x, \omega_i) \cos \theta_i \, d\omega_i
\]

Outgoing light  Emitted light  Reflected light

Energy is conserved!
Direct Illumination

- All light comes directly from emitters, i.e. light sources

\[ L_o(x, \omega_o) = L_e(x, \omega_o) + \int_{H^2} f_r(x, \omega_i, \omega_o) L_i(x, \omega_i) \cos \theta_i d\omega_i \]

\[ L_i(x, \omega) = L_e(r(x, \omega), -\omega) \]
Global Illumination

• Consider all light – including bounces
Global Illumination

Direct illumination

Indirect illumination

Direct + Indirect
Measuring BRDFs

Matusik et al.: Efficient Isotropic BRDF Measurement, Eurographics Symposium on Rendering 2003
Simpler Reflections

Hendrik Lensch, Efficient Image-Based Appearance Acquisition of Real-World Objects, Ph.D. thesis, 2004
Diffuse Reflection

• For diffuse reflection, the BRDF is a constant:

\[ L_r(x, \vec{\omega}_r) = \int_{H^2} f_r(x, \vec{\omega}_i, \vec{\omega}_r) L_i(x, \vec{\omega}_i) \cos \theta_i \, d\vec{\omega}_i \]

\[ L_r(x) = f_r \int_{H^2} L_i(x, \vec{\omega}_i) \cos \theta_i \, d\vec{\omega}_i \]

\[ L_r(x) = f_r \, E_i(x) \]
Photometric Stereo

Goal: estimate surface normal from observed light, e.g. camera

\[ L_o(x, \omega_o, \lambda_{RGB}) = \int_{H^2} f_r(x, \omega_i, \omega_o, \lambda_{RGB}) L_i(x, \omega_i, \lambda_{RGB}) (\omega_i \cdot \hat{n}) d\omega_i \]
Photometric Stereo

Goal: estimate surface normal from observed light, e.g. camera

\[ L_o(x, \omega_o, \lambda_{RGB}) = \int_{H^2} f_r(x, \omega_i, \omega_o, \lambda_{RGB}) L_i(x, \omega_i, \lambda_{RGB})(\omega_i \cdot \bar{n}) d\omega_i \]

Assumptions:
- Delta directional lighting
  \[ L_i(x, \omega_i, \lambda_{RGB}) = L_i(\omega_i) = 1 \]
- Lambertian BRDF
  \[ f_r(x, \omega_i, \omega_o, \lambda_{RGB}) = f_r(\lambda_{RGB}) = \frac{\rho_{d,RGB}}{\pi} \]
- Orthographic projection
Photometric Stereo

Goal: estimate surface normal from observed light, e.g. camera

\[ L_o(x, \omega_o, \lambda_{RGB}) = \frac{\rho_{d,RGB}}{\pi} (\vec{n} \cdot \vec{\omega}_i) \]
Photometric Stereo

Goal: estimate surface normal from observed light, e.g. camera

\[
L_o(x, \omega_o, \lambda_{RGB}) = \frac{\rho_{d,RGB}}{\pi}(\vec{n} \cdot \vec{\omega}_i)
\]

\[
A_{n \times 3} \ast x_{3 \times 1} = b_{n \times 1}
\]

\[
A = \begin{bmatrix}
\omega_{i,1}^T \\
\vdots \\
\omega_{i,n}^T
\end{bmatrix}

b = \begin{bmatrix}
I_{\lambda,1} \\
\vdots \\
I_{\lambda,n}
\end{bmatrix}

x = \frac{\rho_{d,\lambda}}{\pi} \ast \vec{n}^T
\]
Bidirectional scattering-surface reflectance distribution function
Measuring the Human Face

Light Stage

Acquiring the Reflectance Field of a Human Face
Measuring the Human Face

Incident illumination $R_i(u_i, v_i, \theta_i, \phi_i)$

Radiant field of illumination $R_r(u_r, v_r, \theta_r, \phi_r)$

Surface enclosing the scene
Assume directional lighting for incident illumination.
Measuring the Human Face

Idea:

1. Turn one light on at a time and capture an image from the same view

2. Construct a new lighting setup as a sum of the images
Measuring the Human Face

At each pixel, we have radiance that correspond to different lighting directions.
Relighting the Human Face

$$R_{xy}(\theta, \phi)$$

$$\hat{L}(x, y) = \sum_{\theta, \phi} R_{xy}(\theta, \phi) L_i(\theta, \phi) \delta A(\theta, \phi)$$

Output pixel value

Map of incident illumination at this pixel

Reflectance function for this pixel

$$\delta A(\theta, \phi) = \sin \phi$$
\[ \hat{L}(x, y) = \sum_{\theta, \phi} R_{xy}(\theta, \phi) L_i(\theta, \phi) \delta A(\theta, \phi) \]

\[ \delta A(\theta, \phi) = \sin \phi \]
Relighting the Human Face

Real image

Environment

Relit face
Neural Materials and Lighting

Volume rendering

Input: position and direction

Output: colour and density
Neural Materials and Lighting

Volume rendering

Output: colour and density

\[(RGB\sigma)\]

Integrate to get the final pixel colour

\[C(r) = \int_{t_n}^{t_f} T(t)\sigma(r(t))c(r(t), d)dt\]

\[T(t) = \exp\left(-\int_{t_n}^{t} \sigma(r(s))ds\right)\]

\[RGB\]
Given a set of views, the network parameters are optimized and new views can be synthesized.
Neural Materials and Lighting

**Problem** radiance = combination of geometry, materials, and lighting

**Solution** disentangle them

Neural Radiance Fields for Outdoor Scene Relighting