3D Geometry Capture
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Sources of Geometry

Acquisition from the real world

Modeling applications
The West Cambridge Digital Twin project
Shape Acquisition

- Digitizing real world objects

3D Scanning → Registration → Pre-processing → Reconstruction
3D Scanning

Touch Probes
+ Precise
- Small objects

Optical Scanning
+ Fast
- Glossy objects

Active

Passive
Active Systems

• Triangulation Laser
  – Laser beam and camera
  – Laser dot is photographed
  – The location of the dot in the image allows triangulation: we get the distance to the object
Active Systems
Structured light
Active Systems
Structured light
Active Systems

• Structured light
  – Pattern of visible or **infrared** light is projected onto the object
  – The distortion of the pattern (recorded by the camera) provides geometric information
  – Very fast – 2D pattern at once
  – Complex distance calculation → prone to noise
Active Systems

• LIDAR
  – Measures the time it takes the laser beam to hit the object and come back
Passive Systems

Multi-view Stereo

\[(x, y, z)\]

Right camera focal point

Left camera focal point

Epipolar line

Left camera projection plane

Right camera projection plane

\[(x_L, y_L)\]

\[(x_R, y_R)\]
Passive Systems
3D Imaging

- Ultrasound, CT, MRI
- Discrete volume of density data
- First need to segment the desired object (contouring)
3D Scanning

- Challenges

  Noise, outliers, irregularity

  Incompleteness

  Inconsistency
Shape Acquisition

• Digitizing real world objects

3D Scanning → Registration → Pre-processing → Reconstruction
Registration

• Bringing scans into a common coordinate frame
Registration

$M_1 \approx T(M_2), T$: translation + rotation
Registration

\[ M_1 \approx T_2(M_2) \approx \cdots \approx T_n(M_n) \]
Registration

- How many points are needed to define a unique rigid transformation?
- The first problem is finding pairs

\[ \mathbf{p}_1 \rightarrow \mathbf{q}_1 \]
\[ \mathbf{p}_2 \rightarrow \mathbf{q}_2 \]
\[ \mathbf{p}_3 \rightarrow \mathbf{q}_3 \]
\[ R\mathbf{p}_i + t \approx \mathbf{q}_i \]
Registration

• ICP: Iterative Closest Point
• Idea: Iterate
  – (1) Find correspondences
  – (2) Use them to find a transformation
Registration

• ICP: Iterative Closest Point
• Intuition:
  – With the right correspondences, problem solved
  – If you don’t have the right ones, can still make progress
Registration

• ICP: Iterative Closest Point
Registration

• ICP: Iterative Closest Point -- algorithm
  – Select (e.g., 1000) random points
  – Match each to closest point on other scan
  – Reject pairs with distance too big
  – Construct error function:
    \[ E := \sum_i (R p_i + t - q_i)^2 \]
  – Minimize
    • closed form solution in: [http://dl.acm.org/citation.cfm?id=250160](http://dl.acm.org/citation.cfm?id=250160)
Shape Acquisition

• Digitizing real world objects
Pre-processing

• Cleaning, repairing, resampling
Pre-processing

• Sampling for accurate reconstructions
Reconstruction

• Mathematical representation for a shape
Reconstruction

Connect-the-points Methods

+ Theoretical error bounds
– Expensive
– Not robust to noise

Approximation-based Methods

+ Efficient to compute
+ Robust to noise
– No theoretical error bounds
Reconstruction

• Approximating an implicit function

\[ f : \mathbb{R}^3 \rightarrow \mathbb{R} \]
with value > 0 outside the shape and < 0 inside
Reconstruction

• Approximating an implicit function

\[ f : \mathbb{R}^3 \to \mathbb{R} \]

with value > 0 outside the shape and < 0 inside

\[ \{ \mathbf{x} : f(\mathbf{x}) = 0 \} \]

extract zero set
Least Squares

• Problem

\[ f(x) = ax + b \]
Least Squares

- Problem

\[ f(x) = ax + b \]

\[ \min_{a,b} \sum_{i=1}^{n} (f(x_i) - y_i)^2 \]

\[ \min_{a,b} \sum_{i=1}^{n} (ax_i + b - y_i)^2 \]
Least Squares

• Multi-dimensional problem

\[ f(x) : \mathbb{R}^d \rightarrow \mathbb{R} \]

\[
\min_{f \in \Pi_m^d} \sum_{i} (f(x_i) - f_i)^2
\]

\( \Pi_m^d \) : polynomials of degree \( m \) in \( d \) dimensions

\[
f(x) = b(x)^T c
\]

\( m = 2, d = 2 \)

\[
b(x) = [1 \ x \ y \ x^2 \ y^2 \ xy]^T
\]

\[
f(x) = c_0 + c_1 x + c_2 y + c_3 x^2 + c_4 y^2 + c_5 xy
\]
Least Squares

- Multi-dimensional problem

\[ f(x) : \mathbb{R}^d \rightarrow \mathbb{R} \quad \min_{f \in \Pi_m^d} \sum_i (f(x_i) - f_i)^2 \]

\[ f(x) = \mathbf{b}(x)^T \mathbf{c} \]

\[ \min_{\mathbf{c}} E(\mathbf{c}) \quad E(\mathbf{c}) = \sum_i \left( \mathbf{b}(x_i)^T \mathbf{c} - f_i \right)^2 \]
Least Squares

- Multi-dimensional problem

\[
\min \limits_{c} E(c) \quad E(c) = \sum_{i} (b(x_i)^T c - f_i)^2
\]

\[m = 1, d = 1\]

\[
E(c) = \sum_{i} (c_0 + c_1 x_i - f_i)^2
\]

\[m = 2, d = 1\]

\[
E(c) = \sum_{i} (c_0 + c_1 x_i + c_2 x_i^2 - f_i)^2
\]
Least Squares

- Multi-dimensional problem

\[
\min_{\mathbf{c}} E(\mathbf{c}) \quad E(\mathbf{c}) = \sum_{i} (b(x_i)^T \mathbf{c} - f_i)^2
\]

\[
E(\mathbf{c}) = \sum_{i} (c_0 + c_1 x + c_2 y + c_3 x^2 + c_4 y^2 + c_5 xy - f_i)^2
\]

\[m = 2, \, d = 2\]
Least Squares

• Solution of the multi-dimensional problem

$$\min_{c} E(c) \quad E(c) = \sum_{i} (b(x_i)^T c - f_i)^2 \quad b(x_i) = [b_1(x_i) \cdots b_m(x_i)]^T$$

$$\frac{\partial E(c)}{\partial c_k} = \sum_{i} 2b_k(x_i) \left[ b(x_i)^T c - f_i \right] = 0$$

$$\frac{\partial E(c)}{\partial c} = 2 \sum_{i} b(x_i) \left[ b(x_i)^T c - f_i \right] = 0$$

$$\sum_{i} b(x_i) b(x_i)^T c = \sum_{i} b(x_i) f_i$$

$$c = \left[ \sum_{i} b(x_i) b(x_i)^T \right]^{-1} \sum_{i} b(x_i) f_i$$
Least Squares

• Solution of the multi-dimensional problem

Example

\[ m = 2, d = 1 \quad E(c) = \sum_i (c_0 + c_1 x + c_2 x^2 - f_i)^2 \]

\[
\sum_i \begin{bmatrix} 1 & x_i & x_i^2 \\ x_i & x_i^2 & x_i^3 \\ x_i^2 & x_i^3 & x_i^4 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \sum_i \begin{bmatrix} 1 \\ x_i \\ x_i^2 \end{bmatrix} f_i
\]
Weighted Least Squares

- Multiply the terms with given weights

**LS**  \( \min_c E(c) \quad E(c) = \sum_i (b(x_i)^T c - f_i)^2 \)

**WLS**  \( \min_c E(c) \quad E(c) = \sum_i (b(x_i)^T c - f_i)^2 w_i \)
Moving Least Squares

• Idea: make the weights local
Moving Least Squares

- Idea: make the weights local

\[ f(x) = \min_{\mathbf{f}_x \in \Pi_m^d} \sum_{i} \phi(||\mathbf{x} - \mathbf{x}_i||) \left( \mathbf{f}_x(\mathbf{x}_i) - f_i \right)^2 \]

Local approximation
Weights depend on \( x \)
Moving Least Squares

- Idea: make the weights local

\[
c(x) = \arg\min_c \ E_x(c) = \sum_i \phi(||x - x_i||) \ (b(x_i)^T c - f_i)^2
\]

\[
f(x) = b(x)^T c(x)
\]

In comparison, LS:

\[
c = \arg\min_c \ E(c) = \sum_i \ (b(x_i)^T c - f_i)^2
\]

\[
f(x) = b(x)^T c
\]
Moving Least Squares

- Local solution

\[
c(x) = \left[ \sum_i \phi_i(x)b(x_i)b(x_i)^T \right]^{-1} \sum_i \phi_i(x)b(x_i)f_i
\]

\[
\phi_i(x) = \phi(||x - x_i||)
\]

\[
f(x) = b(x)^T c(x)
\]
Moving Least Squares

- Local solution

Example $m = 1, d = 1$

$$\min_{c_0, c_1} \sum_i \phi_i(x) \left( c_0 + c_1 x_i - f_i \right)^2$$

$$f_x(x) = c_0 + c_1 x$$

$$f(x) = f_x(x)$$
Implicit MLS Surfaces

• Basic problem
  – Given sample points & attributes
  – Compute a function
    \[ f(x) : \mathbb{R}^2 \text{ or } \mathbb{R}^3 \rightarrow \mathbb{R} \]
  – such that the curve/surface is given by
    \[ S = \{x | f(x) = 0, \nabla f(x) \neq 0\} \]
Implicit MLS Surfaces

Query Point $x$

$\mathbf{f}_x(x)$

Neighborhood defined by $\phi(||x - \cdot||)$

$f(x) = \mathbf{f}_x(x)$
Implicit MLS Surfaces

Example $m = 1, d = 2$

$$f_x(x) = c_0(x) + c_1(x)x + c_2(x)y$$
Implicit MLS Surfaces

How can we avoid the trivial solution

\[ f(x) = 0 \ \forall x \]

Gradient constraints

\[ \| \nabla f_x(x) \| = 1 \quad \nabla f(x_i) = n_i \]

Reproduce local functions

\[ f_i(x) = n_i^T (x - x_i) \]
Implicit MLS Surfaces

• Example

\( m = 1, d = 2 \)

\[ f_x(x) = n_x^T x + o_x \quad ||n_x|| = 1 \]

\[
(n_x, o_x) = \arg\min_{n,o} \sum_i \phi_i(x) \left( n^T x_i + o \right)^2 \quad ||n|| = 1
\]
Implicit MLS Surfaces

• Examples in 3D

Feature Preserving Point Set Surfaces based on Non-Linear Kernel Regression, Eurographics 2009
Implicit MLS Surfaces

• Examples in 3D

Spatio-Temporal Geometry Fusion for Multiple Hybrid Cameras using Moving Least Squares Surfaces, Eurographics 2014
Shape Acquisition

• Digitizing real world objects

3D Scanning  Registration  Pre-processing  Reconstruction
Neural Radiance Fields
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