Economics, Law and Ethics

Part IB CST

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Lecture 4: Auction theory and game theory

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with many thanks to Ross Anderson

Overview

- Auctions:
  - Types of auctions
  - Equivalence
  - What goes wrong
  - Advertising auctions
- Game theory:
  - Cooperation or conflict
  - Strategies
  - Types of games
  - Broader implications

Auctions

- Around for millennia; standard way of selling livestock, fine art, mineral rights, bonds…
- Many other sales from corporate takeovers to house sales are also really auctions
- Auctions are a big success of the Internet, from eBay to Google
- Some unpleasant side-effects
- Rapidly growing interest in theoretical computer science: auction resources in distributed systems
- Many issues of asymmetric info, signaling, strategic play… – plus some solid theory!

Types of auction

- English, or ascending-bid: start at reserve price and raise till a winner is left (art, antiques)
- Dutch, or descending-bid: start high and cut till somebody bids (flowers)
- First-price sealed-bid auction: one bid per bidder (government contracts)
- Second-price sealed-bid auction, or Vickrey auction: highest bidder wins and pays second-highest bid (postage stamps)
- All-pay auction: everyone pays at every round until one remaining bidder gets the goods (war, litigation, winner-takes-all market race)

The Aalsmeer flower auction

Strategic equivalence

- A Dutch auction and a first-price sealed-bid auction give the same result: the highest bidder gets the goods at his reservation price
- They are ‘strategically equivalent’
- Ditto the English auction and the second-price sealed-bid auction (modulo the bid increment)
- But the two pairs are not strategically equivalent!
  - in a second-price auction it’s best to bid truthfully
  - in a Dutch / first-price auction, you should bid low if you think your valuation is much higher than everybody else’s

Revenue equivalence

- This is weaker – not ‘who will win’ but ‘how much money on average’
- According to the revenue equivalence theorem, you get the same revenue from any well-behaved auction under ideal conditions
- These include risk-neutral bidders, no collusion, Pareto efficiency (highest value bidder gets goods), reserve price, independent valuations, …
- Then bidders adjust their strategies and the English, Dutch and all-pay auction yield the same
- So when you design an auction, you must focus on any ways the conditions aren’t ideal

What goes wrong (1)

- In a ‘private-value auction’, each bidder’s value v_i is exogenous (think: sculpture). In a second-price auction, everything you buy is a bargain
- In a ‘public-value auction’, each item has a true price which bidders estimate at $v + \varepsilon_I$ (think mineral leases; spectrum auctions). The buyer is the sucker who overestimated the most!
- This is called ‘the winner’s curse’
- Many real auctions lie somewhere between these two extremes

What goes wrong (2)

- Bidding rings – bidders collude to buy low, have a private auction later, split the proceeds
- First-price auctions are harder to rig; with second-price, New Zealand bids of $7m and $5000
- Entry detection / deterrence: an early (1991) ITV franchise auction required bidders to draw up a detailed programming plan. In Midlands & Central Scotland, industry knew there was no competition; bids under 1p per head (vs £9–16 elsewhere)
- Predation: ‘we’ll top any other bid’ in takeovers
- Sniping and other boundary effects

What goes wrong (3)

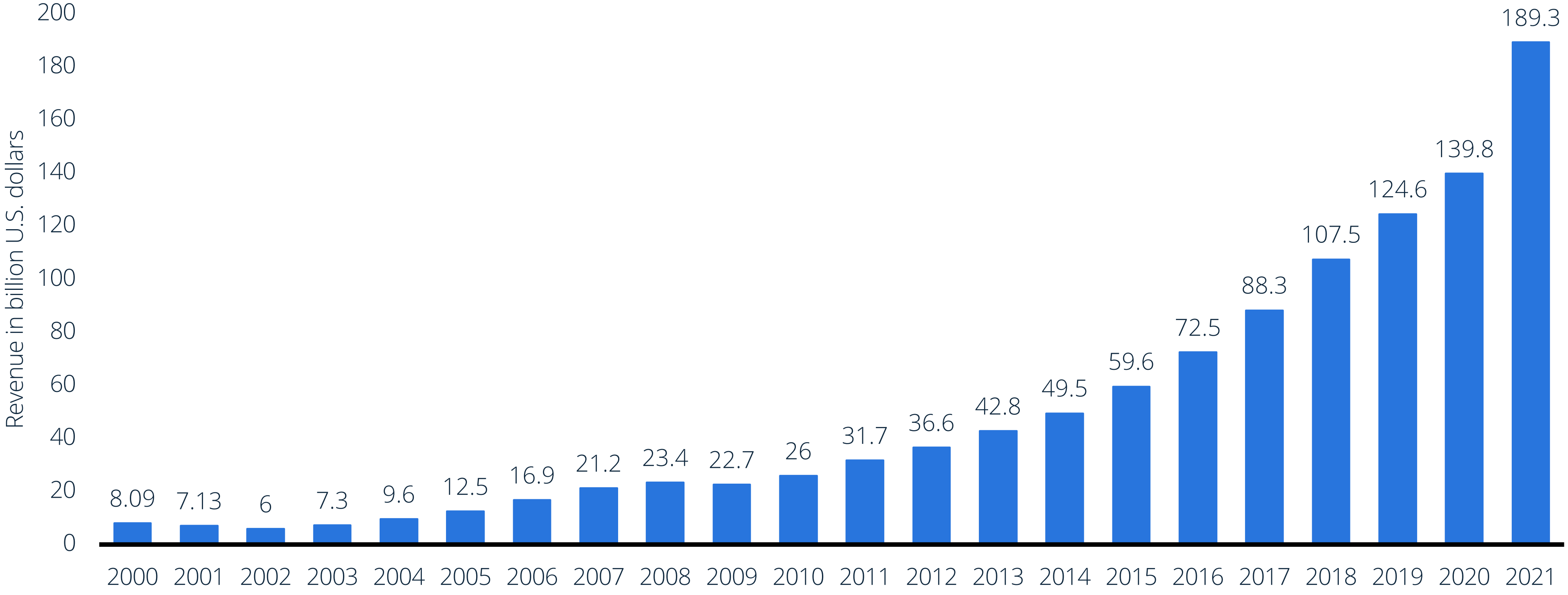
- Risk aversion: if you prefer a certain profit of £1 to a 50% chance of £2, you'll bid higher at a first-price auction
- Signaling games: show aggression by a price hike
- E.g. in simultaneous auctions, as in the USA, signal "we want SF, LA, SD and if you compete with us there we'll push prices up in your patch")
- Budget constraints: if bidders are cash-limited, all-pay auctions are more profitable
- Externalities between bidders – e.g. arms sales

Combinatorial auctions

- Externalities lead to preferences for particular bundles of goods: landing slots at airports, spectrum, mineral rights…
- Bid (\$x for A+B+C) or (\$y for A+D+E) or…
- Critical app for CS: routing in presence of congestion (bid for AB and BC, or AD and DC…)
- The allocation problem is NP-complete; practical algorithms work up to a few thousand objects
- Also: how can we make the auction strategy-proof (i.e. truth-telling is the best strategy)?
- New field of ‘algorithmic mechanism design’

Ad auctions

Online advertising revenue in the United States 2000 to 2021 (in US$bn)

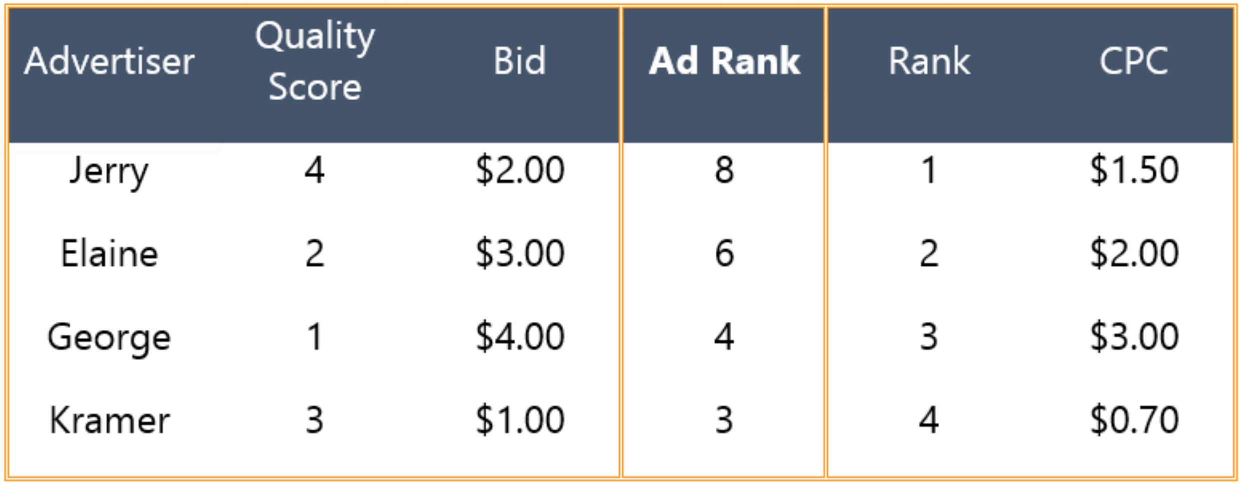


Note(s): United States; 2000 to 2021
Source(s): PwC; IAB (U.S.); ID 183816

Ad auctions (2)

- Pioneered by Google
- Basic idea: second-price auction mechanism but tweaked to optimise platform revenue
- Bidders bid prices p_i, platform estimates ad quality e_i, and then ad rank $a_i = p_i.e_i$
- Ad quality e_i = relevance . clickthrough rate
- So how do we work out who wins the auction and how much they pay?

Ad auctions (3)



Advertiser	Quality Score	Bid	**Ad Rank**	Rank	CPC
Jerry	4	$2.00	8	1	$1.50
Elaine	2	$3.00	6	2	$2.00
George	1	$4.00	4	3	$3.00
Kramer	3	$1.00	3	4	$0.70

- Here George bids $4 and Jerry $2 but Jerry wins the auction because of higher ad quality – the platform expects he’ll get four times the clicks
- Jerry pays a cost per click of only $1.50 (bid times competitor ad rank / own ad rank)

Ethical aspects of ad auctions

- Translated to social media, ad ‘quality’ can easily segue into ‘virality’
- Then if your ads are good clickbait, and your followers follow them, you pay less
- See Martinez ‘How Trump conquered Facebook – without Russian ads’ (web page)
- Many sites tend to serve ever more provocative and extreme content...

Cooperation or conflict

- One way of getting what you want is to make it, or make something else of value and trade for it – ‘Economics’
- Another way is to just take it, whether by force or via the ballot box – ‘Politics’
- Choices between cooperation and conflict are made at all sorts of levels all the time
- They can evolve in complex combinations
- The main tool we use to analyse them is game theory

Game theory

- The study of problems of cooperation and conflict among independent decision-makers
- We focus on games of strategy, rather than chance
- We abstract to players, choices, payoffs, strategies
- There are
  - games of perfect information (such as chess and go)
  - games of imperfect information (which are often more interesting to analyse)

Strategic form

- Example: matching pennies. Alice and Bob throw H or T. If they're different, Alice gets Bob's penny; else he gets hers. The strategic form is

Bob

Alice

	H	T
H	-1, 1	1, -1
T	1, -1	-1, 1

- This is an example of a zero-sum game: Alice's gain = Bob's loss

Dominant strategy equlibrium

- In the following game, Bob's better off playing left; similarly Alice is always better off playing bottom

Bob

Alice

	Left	Right
Top	1, 2	0, 1
Bottom	2, 1	1, 0

- A strategy is an algorithm: input state, output play
- Here, each player's optimal play is a constant
- The is called a 'dominant strategy equilibrium'

Nash equlibrium

- Consider this game:

Bob

Alice

	Left	Right
Top	2, 1	0, 0
Bottom	0, 0	1, 2

- Each player's optimal strategy depends on what they think the other will do
- Two strategies are in Nash equilibrium when A's choice is optimal given B's, and vice versa
- Here there are two: top left and bottom right
- This game is sometimes called 'Battle of the sexes'

Pure v mixed strategies

- With deterministic algorithms, some games have no Nash equilibrium

Bob

Alice

	scissors	paper	stone
scissors	0,0	1, -1	-1, 1
paper	-1, 1	0,0	1, -1
stone	1, -1	-1, 1	0,0

- Alice plays scissors → Bob will play stone → Alice will play paper …
- Fix: randomised algorithm. Called a ‘mixed’ strategy; deterministic algorithms are called ‘pure’

Prisoners' dilemma

- Two prisoners are arrested on suspicion of planning a robbery. The police tell them separately: if neither confesses, one year each for gun possession; if one confesses he goes free and the other gets 6 years; if both confess then each will get 3 years

Benjy

Alfie

	confess	deny
confess	-3, -3	0, -6
deny	-6, 0	-1, -1

- (confess, confess) is the dominant strategy equilibrium
- It's obviously not optimal for the villains!
- Is this a problem? If so, what's the solution?

Prisoners' dilemma (2)

- You might answer 'serves them right'!
- But this can't apply to all instances of the dilemma
  - Defence spending
  - Fishing quotas
  - Free riders in file-sharing systems
  - Reducing carbon emissions
  - …
- Tough but inescapable conclusion: if the game is truly as described, there is no escape. Both will cheat rather than cooperate, with bad outcome
- To fix it, you need to change the game somehow!

The evolution of cooperation

- If PD played repeatedly, there's a fix!
- 'Tit-for tat': cooperate at round 1, then at round n do what the other guy did at n-1
- Simulation competitions run by Bob Axelrod played off many iterated-game strategies; tit-for-tat did consistently well
- In the presence of noise, tit-for-tat gets locked into (defect, defect). So: forgive the other guy occasionally
- People have realised in the last 30 years or so that strategy evolution explains a lot of behaviour

Price-fixing

- If it costs $250 to fly someone LHR-JFK, do airlines compete and charge $255 or collude and charge $500?
- Competition laws forbid price-fixing cartels, but the same behaviour can arise implicitly
- Try charging $500 and see what other airlines do. If they ‘defect’ by competing, play tit-for-tat
- If you’re the regulator, how do you cope?

Stag hunt

- People can hunt rabbits on their own, but have to work together to hunt a stag. If your buddy runs off after a rabbit, the stag will escape

Frank

Bernard

	chase hare	hunt stag
chase hare	2, 2	5, 0
hunt stag	0, 5	10, 10

- Difference from PD: (stag, stag) is now a Nash equilibrium
- You'll only chase a rabbit if you believe your buddy will defect
- Thus while PD is payoff-dominant, stag hunt is risk-dominant

Volunteer's dilemma

- Multi-player chicken: if one person volunteers, everyone else benefits, but if no-one volunteers then everyone suffers a big loss

Everyone else

Me

	someone acts	no-one acts
act	benefit - cost	benefit - cost
don't act	benefit	big loss

- The Arab Spring: "If everyone goes on the street and says 'the government is finished', it's finished. If you go on the street and say 'the government is finished', you're finished"
- Evolution of leadership: first move = fitness signal

Chicken

- In ‘Rebel without a cause’, Jim (James Dean) and Buzz (Corey Allan) drive stolen cars at a canyon and try to jump out last to prove their manhood

Jim

Buzz

	jump	drive on
jump	2, 2	1, 3
drive on	3, 1	0, 0

- Here, (1,3) and (3,1) are Nash equilibria
- Bertrand Russell suggested this as a model of nuclear confrontation in the Cold War
- But what about the iterated version?

Game theory and evolution

- John Maynard Smith proposed the ‘Hawk-dove’ game as a simple model of animal behaviour. Consider a mixed population of aggressive and docile individuals:

	Hawk	Dove
Hawk	(v-c)/2, (v-c)/2	v, 0
Dove	0, v	v/2, v/2

- Food v at each round; doves share; hawks take food from doves; hawks fight (with risk of death c)
- If v > c, whole population becomes hawk (dominant strategy)
- What happens if c > v?

Game theory and evolution (2)

- If $c > v$, a small number of hawks will prosper as most interactions will be with doves. Equilibrium reached at hawk probability p setting hawk payoff = dove payoff

	Hawk	Dove
Hawk	(v-c)/2, (v-c)/2	v, 0
Dove	0, v	v/2, v/2

- I.e. $p(v-c)/2 + (1-p)v = (1-p)v/2$

$\Leftrightarrow pv - pc + 2v - 2pv = v - pv$

$\Leftrightarrow -pc = -v$

$\Leftrightarrow p = v/c$

Broader implications

- Nash, Axelrod, Maynard Smith and others opened up many applications
- Politics: models of conflict, and of when religions are dominated by fundamentalists
- Criminologists: model the Mafia as alternative contract enforcement, and tattoos as signalling
- Computer science: how do you get AS operators to tell the truth about Internet routing? How do you get them to secure BGP? Will bitcoin converge, fork or collapse? ...

Broader implications (2)

- In pre-state societies, if you see a man you don't recognise, you'd better kill him first (Diamond, "The World Until Yesterday")
- Now we live in largely peaceful societies (Pinker, "The Better Angels of our Nature")
- Evolutionary basis of morality: fairness from tit-for-tat, hierarchy from hawk-dove, maybe conservative / liberal preferences too (Haidt)
- Cooperation developed by states, religions, literature, markets, rights, TV …

Broader implications (3)

- If institutions that involve social cooperation are replaced by online mechanisms, what happens then?
- TV caused people to become more solitary when it replaced clubs, churches and pubs as the social focus (“Bowling Alone”)
- What if more of our cooperative social mechanisms are replaced by echo chambers?
- The spread of broadband was correlated with a rise in political polarisation...