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RAINBOW GRAPHICS GROUP

Advanced Graphics & Image Processing

Ray tracing (refresher)

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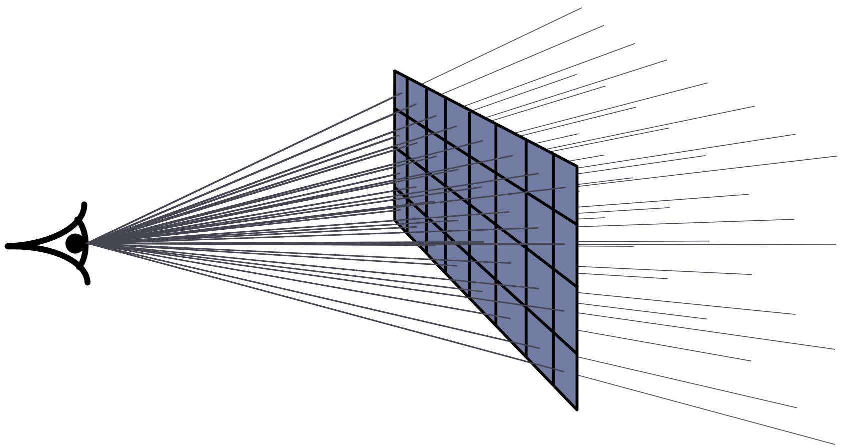
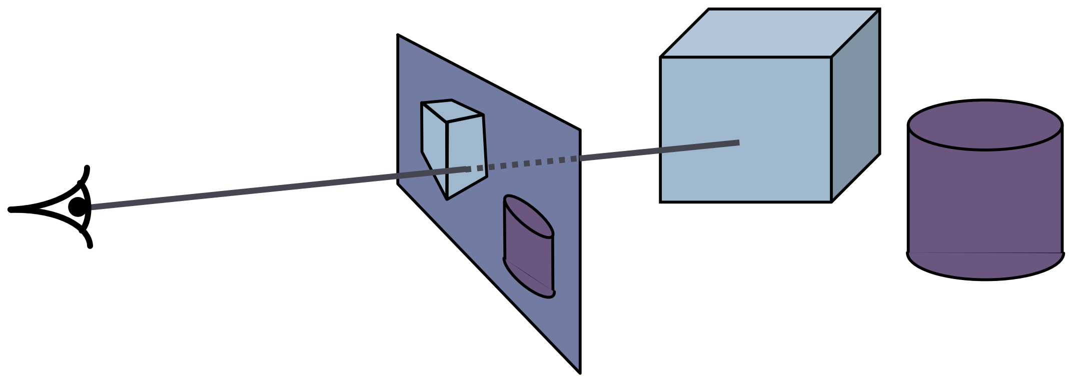
Ray tracing

Given a set of 3D objects, shoot a ray from the eye through the centre of every pixel and see what surfaces it hits

- Identify point on surface and calculate illumination



shoot a ray through each pixel



whatever the ray hits determines the colour of that pixel

Ray tracing algorithm

```
select an eye point and a screen plane

FOR every pixel in the screen plane
    determine the ray from the eye through the pixel's centre
    FOR each object in the scene
        IF the object is intersected by the ray
            IF the intersection is the closest (so far) to the eye
                record intersection point and object
            END IF ;
        END IF ;
    END FOR ;
    calculate colour for the closest intersection point (if any)
END FOR ;
```

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Reflection models and radiometry

Advanced Graphics

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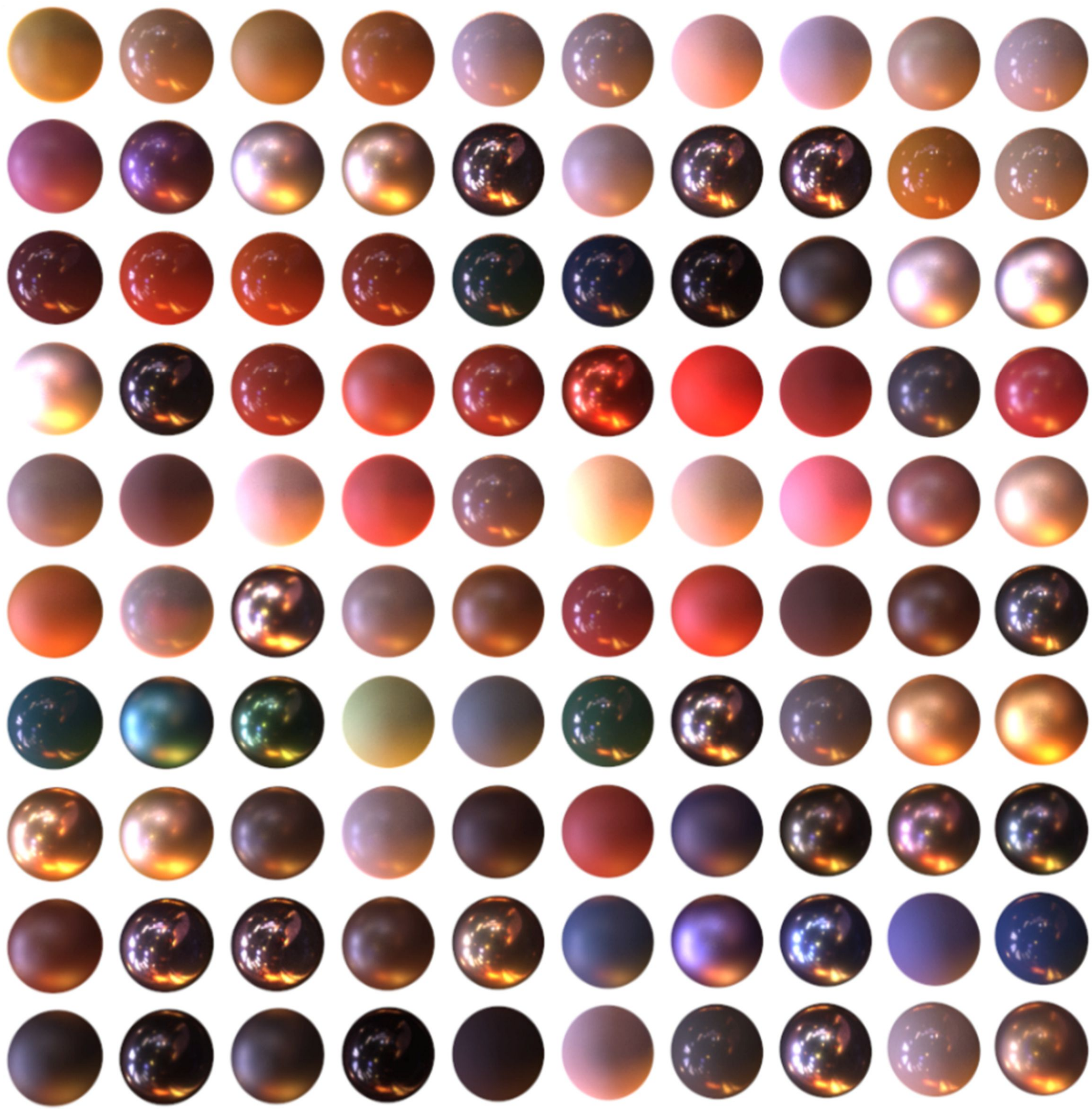
Applications

- To render realistic looking materials
- Applications also in computer vision, optical engineering, remote sensing, etc.
  - To understand how surfaces reflect light



Applications

- Many applications require faithful reproduction of material appearance

Source: http://ikea.com/

Source: http://www.mercedes-benz.co.uk/

Rendering equation

- Most rendering methods require solving an (approximation) of the rendering equation:

Incident light

Integral over the hemisphere of incident light

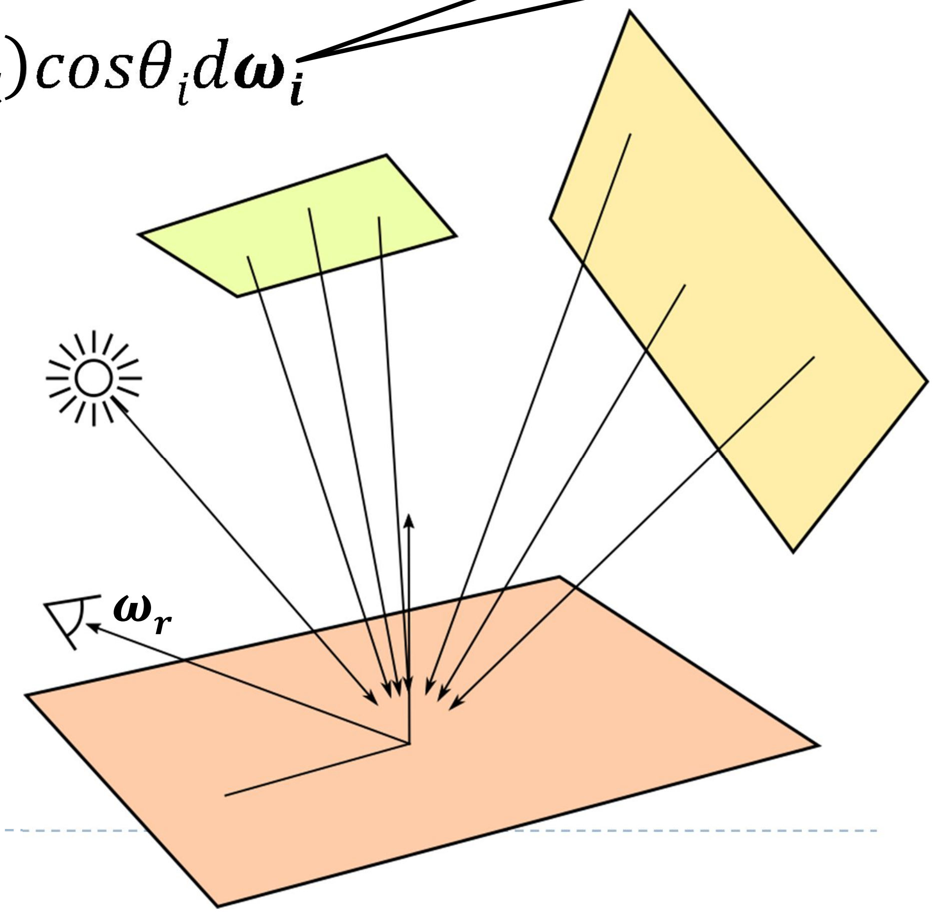
$$L_r(\boldsymbol{\omega_r}) = \int_{\Omega} \rho(\boldsymbol{\omega_i}, \boldsymbol{\omega_r}) L_i(\boldsymbol{\omega_i}) cos\theta_i d\boldsymbol{\omega_i}$$

Reflected light

BRDF

- The solution is trivial for point light sources
- Much harder to estimate the contribution of other surfaces

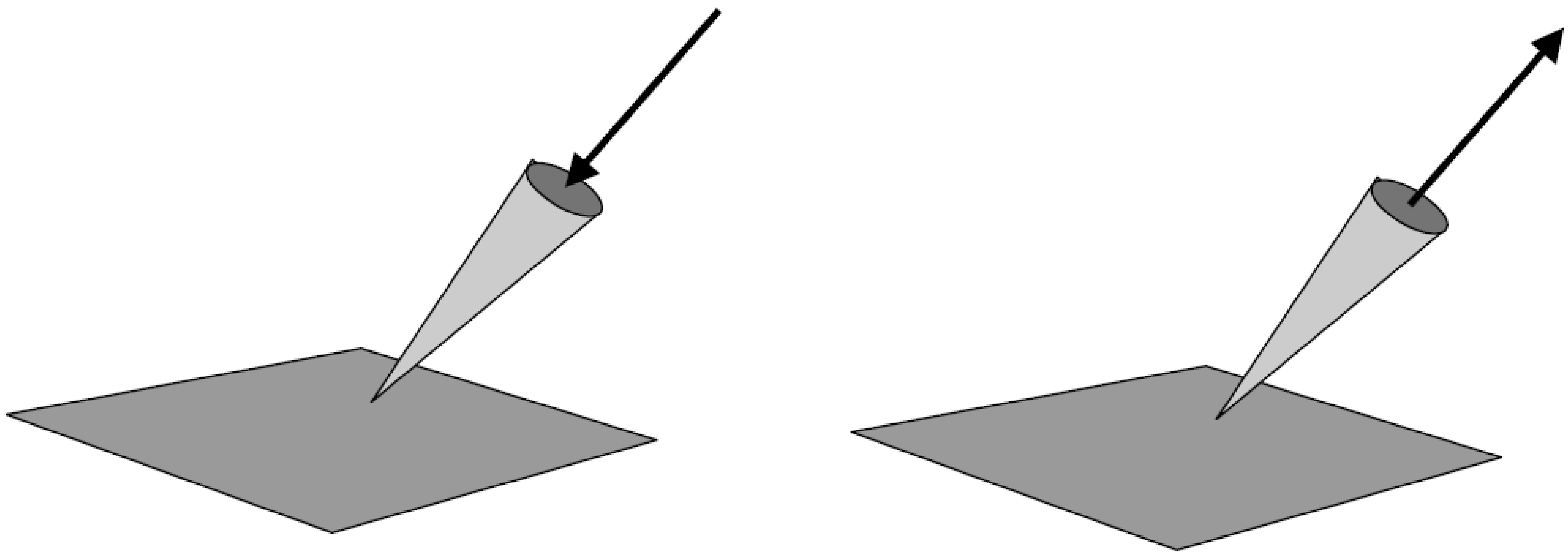
$$\boldsymbol{\omega_i} = [\phi_i, \theta_i]$$



Radiance

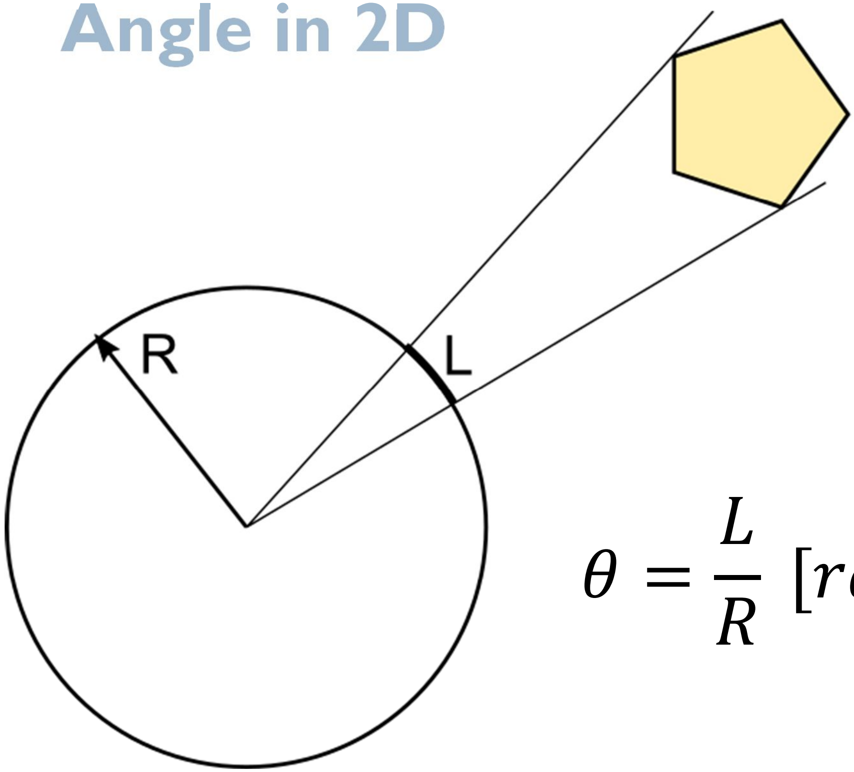
- Power of light per unit projected area per unit solid angle
- Symbol: $L(\boldsymbol{x}, \boldsymbol{\omega}_i)$
- Units: $\frac{W}{m^2 sr}$

Steradian



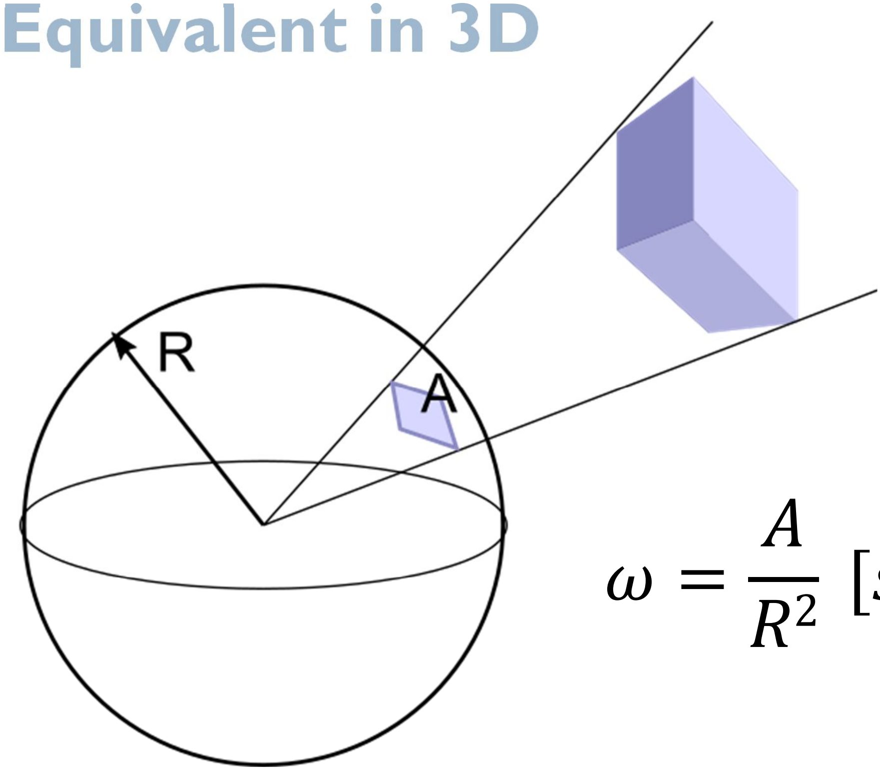
Solid angle

Angle in 2D



$$\theta = \frac{L}{R}\ [rad]$$

Equivalent in 3D



$$\omega = \frac{A}{R^2}\ [sr]$$

Full circle = 2π radians

- Full sphere = 4π steradians

Radiance

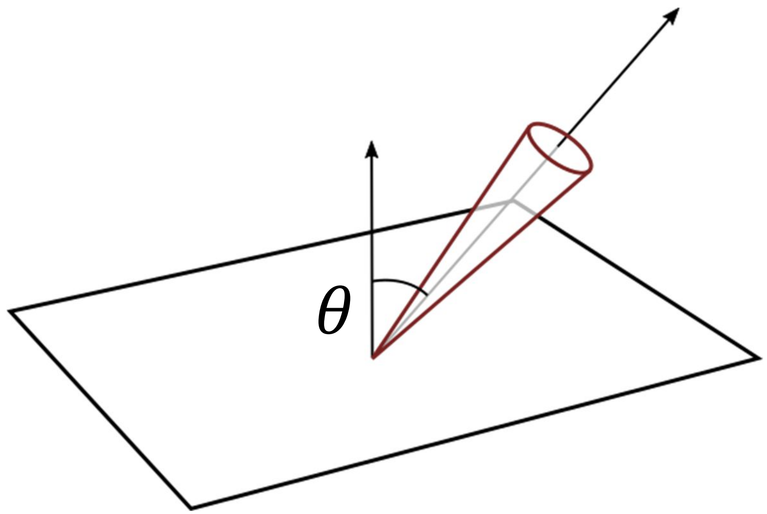
Position

Power of light

$$L(\boldsymbol{x}, \boldsymbol{\omega}_i) = \frac{d\phi}{d\omega\, dA\, cos\theta}$$

Incoming direction

Projected area

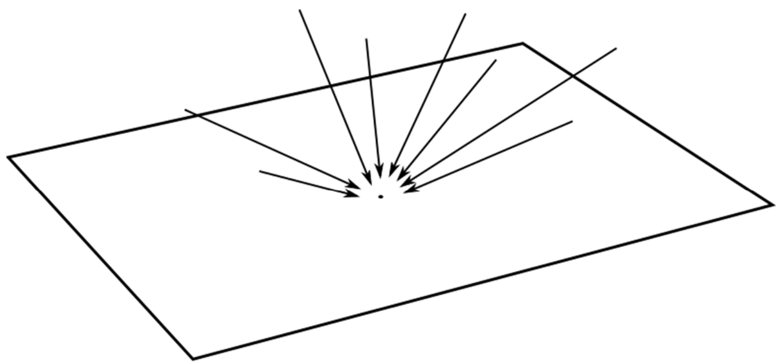


- Power per solid angle per projected surface area
- Invariant along the direction of propagation (in vacuum)
- Response of a camera sensor or a human eye is related to radiance
- Pixel values in image are related to radiance (projected along the view direction)

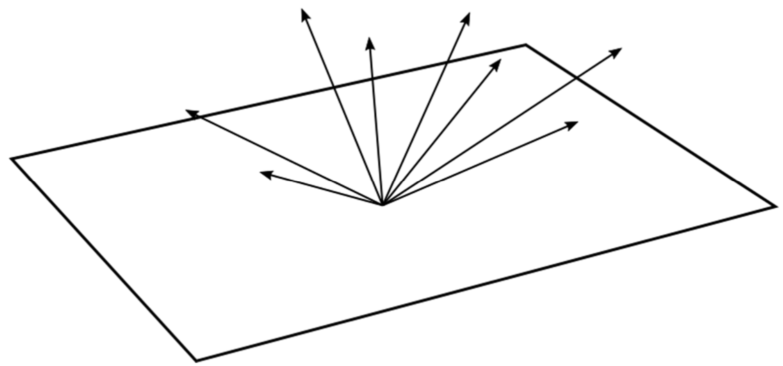
Irradiance and Exitance

- Power per unit area
- Irradiance: H(x) – incident power per unit area
- Exitance / radiosity: E(x) – exitant power per unit area
- Units: $\frac{W}{m^2}$

Irradiance



Exitance / Radiosity



Relation between Irradiance and Radiance

- Irradiance is an integral over all incoming rays
  - Integration over a hemisphere Ω:

$$H = \int_{\Omega} L(\boldsymbol{x}, \boldsymbol{\omega_i}) \, cos\theta \, d\boldsymbol{\omega}$$

  - In the spherical coordinate system, the differential solid angle is:

$$d\boldsymbol{\omega} = \sin\theta d\theta \, d\phi$$

  - Therefore:

$$H = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\frac{\pi}{2}} L(\boldsymbol{x}, \boldsymbol{\omega_i}) \, cos\theta \, sin\theta \, d\theta \, d\phi$$

  - For constant radiance:

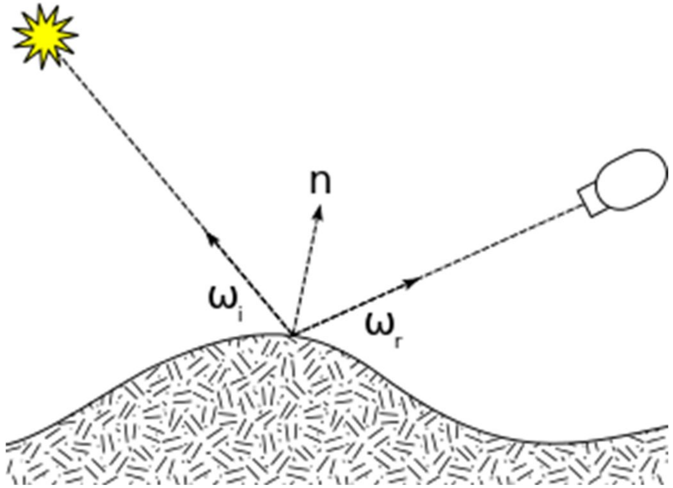
$$H = \pi L$$

BRDF: Bidirectional Reflectance Distribution Function

Differential radiance of reflected light

$$\rho(\boldsymbol{\omega_i}, \boldsymbol{\omega_r}) = \frac{dL_r(\boldsymbol{\omega_r})}{dH_i(\boldsymbol{\omega_i})} = \frac{dL_r(\boldsymbol{\omega_r})}{L_i(\boldsymbol{\omega_i}) cos\theta_i d\boldsymbol{\omega_i}}$$

Differential irradiance of incoming light

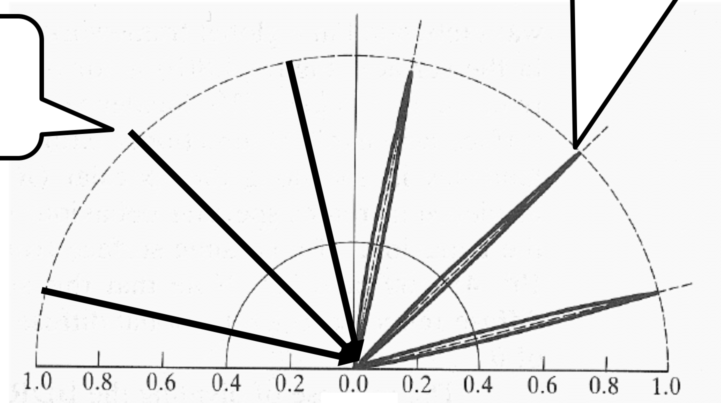


Source: Wikipedia

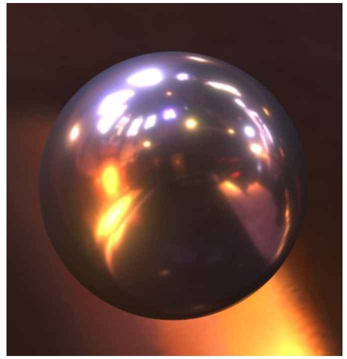
- BRDF is measured as a ratio of reflected radiance to irradiance
  - Because it is difficult to measure $L_i(\boldsymbol{\omega_i})$, BRDF is not defined as the ratio $L_r(\boldsymbol{\omega_r})/L_i(\boldsymbol{\omega_i})$

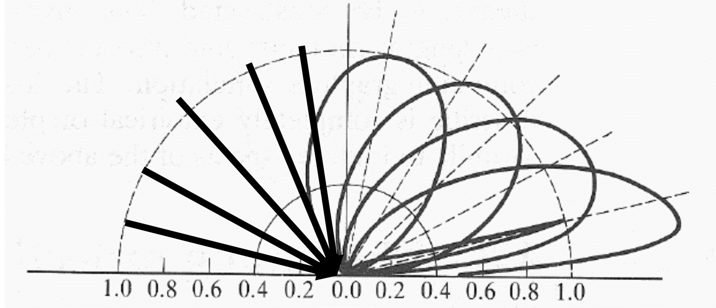
BRDF of various materials

- The diagrams show the distribution of reflected light for the given incoming direction
- The material samples are close but not accurate matches for the diagrams

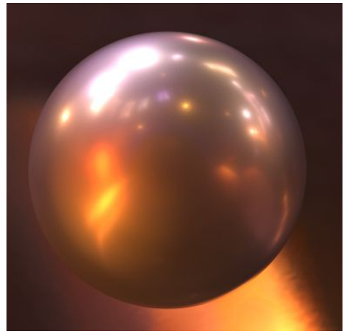


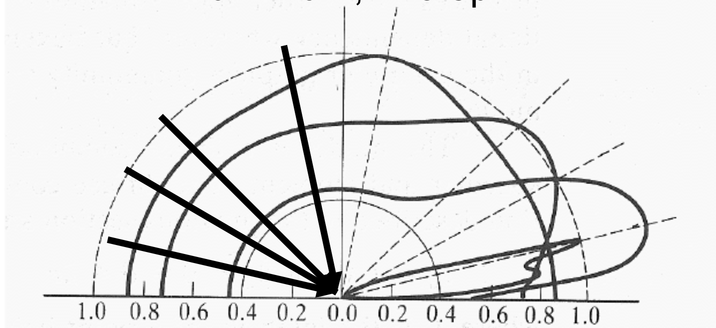
Aluminium; λ=2.0μm





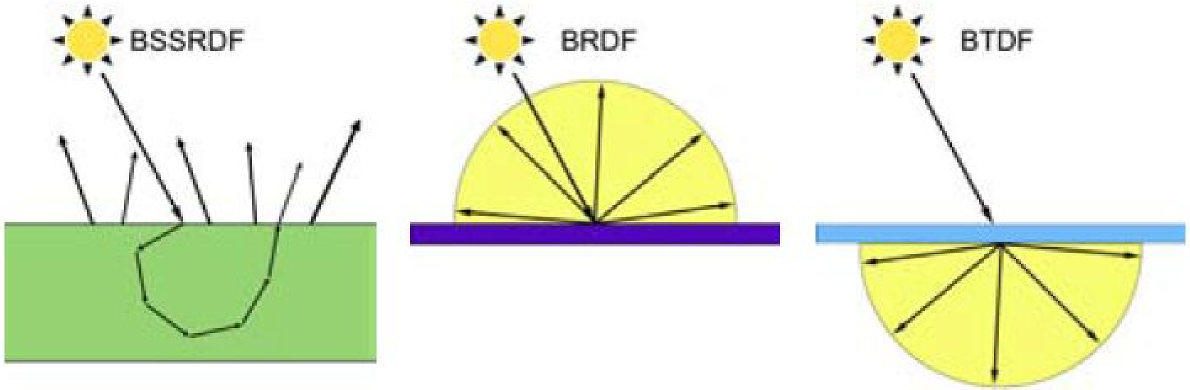
Aluminium; λ=0.5μm





Magnesium alloy; λ=0.5μm

Other material models

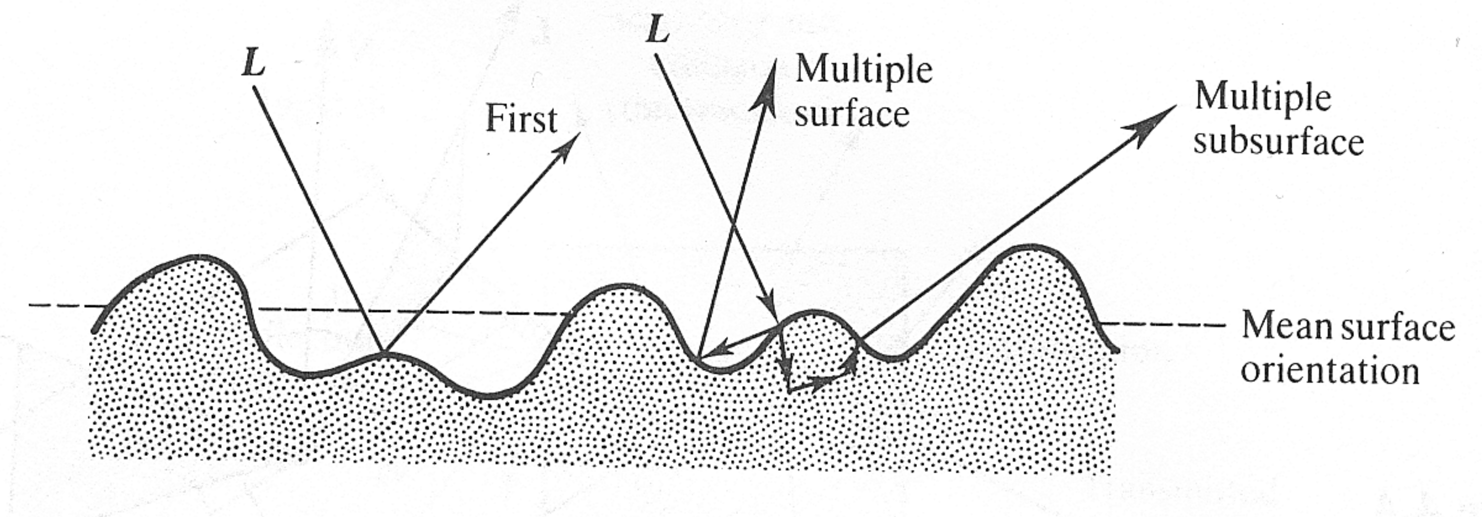


Source:
Guarnera et al. 2016

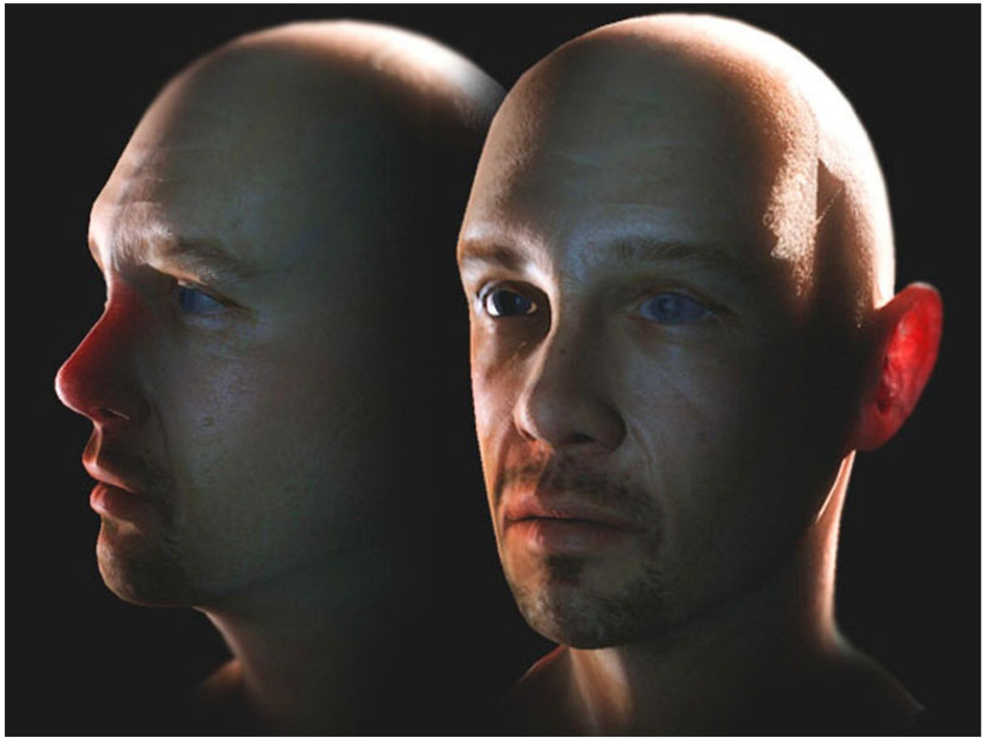
- Bidirectional Scattering Surface Reflectance Distribution F.
- **Bidirectional Reflectance Distribution Function**
- Bidirectional Transfer Distribution Function
- But also: BTF, SVBRDF, BSDF
- In this lecture we will focus mostly on BRDF

Sub-surface scattering

- Light enters material and is scattered several times before it exits
  - Examples - human skin: hold a flashlight next to your hand and see the color of the light
- The effect is expensive to compute
  - But approximate methods exist



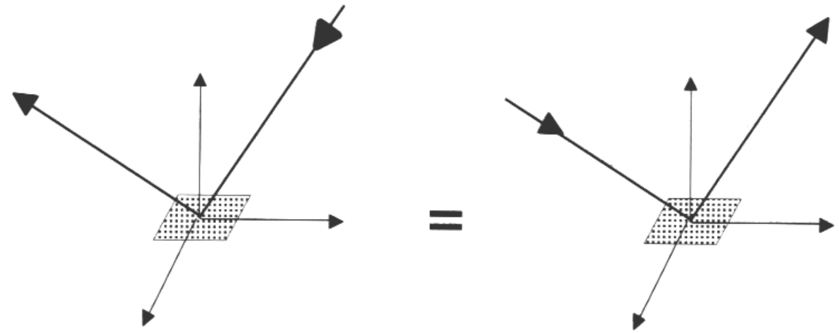
Subsurface scattering - examples



BRDF Properties

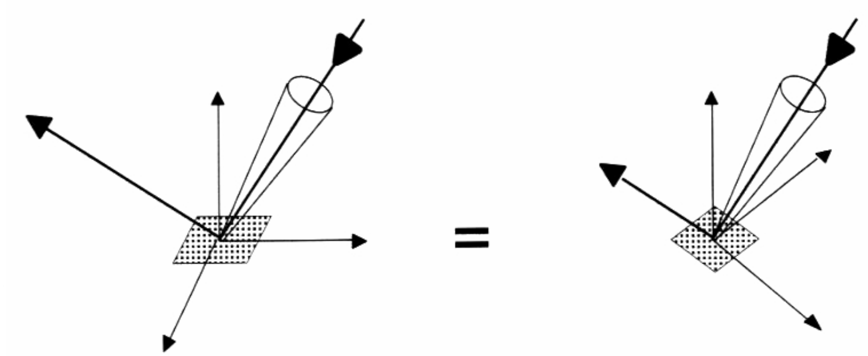
- Helmholtz reciprocity principle
  - BRDF remains unchanged if incident and reflected directions are interchanged

$$\rho(\boldsymbol{\omega_r}, \boldsymbol{\omega_i}) = \rho(\boldsymbol{\omega_i}, \boldsymbol{\omega_r})$$



- Smooth surface: isotropic BRDF
  - reflectivity independent of rotation around surface normal
  - BRDF has only 3 instead of 4 directional degrees of freedom

$$\rho(\theta_i, \theta_r, \phi_r - \phi_i)$$



BRDF Properties

- Characteristics
  - BRDF units [1/sr]
    - Not intuitive
  - Range of values:
    - From 0 (absorption) to ∞ (reflection, δ -function)
  - Energy conservation law

$$\int_{\Omega} \rho(\omega_r, \omega_i) cos\theta_i d\omega_i \leq 1$$

    - No self-emission
    - Possible absorption
  - Reflection only at the point of entry ($x_i = x_r$)
    - No subsurface scattering

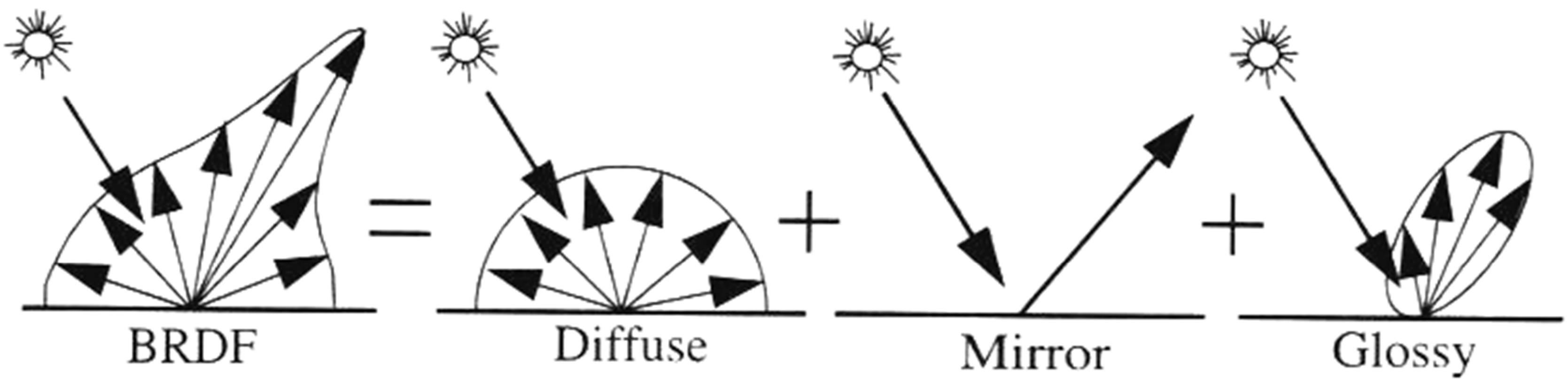
BRDF Measurement

- Gonio-Reflectometer
- BRDF measurement
  - point light source position (θ, φ)
  - light detector position (θ_o, φ_o)
- 4 directional degrees of freedom
- BRDF representation
  - m incident direction samples (θ, φ)
  - n outgoing direction samples (θ_o, φ_o)
  - mn reflectance values (large!!!)

Stanford light gantry

BRDF Modeling

- It is common to split BRDF into diffuse, mirror and glossy components
- Ideal diffuse reflection
  - Lambert's law
  - Matte surfaces
- Ideal specular reflection
  - Reflection law
  - Mirror
- Glossy reflection
  - Directional diffuse
  - Shiny surfaces

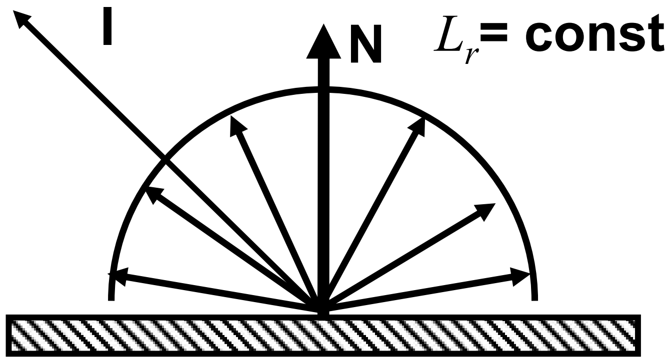


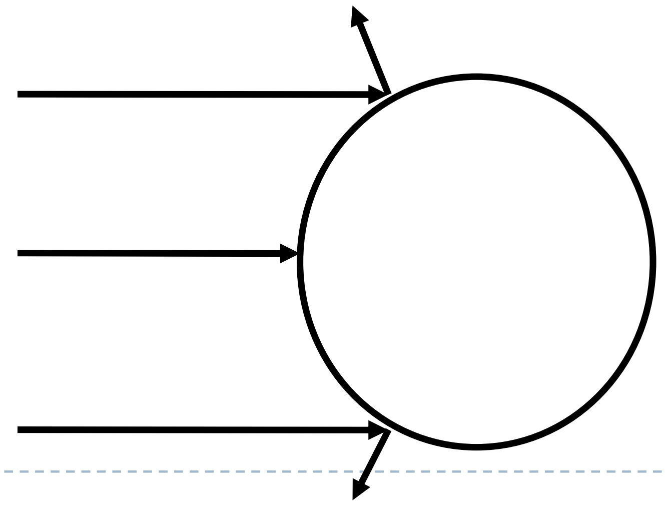
Diffuse Reflection

- Light equally likely to be reflected in any outgoing direction (independent of incident direction)
- Constant BRDF $\rho(\omega_r, \omega_i) = k_d = const$

$$L_r(\omega_r) = \int_{\Omega} k_d L_i(\omega_i) cos\theta_i d\omega_i = k_d \int_{\Omega} L_i(\omega_i)\cos\theta_i d\omega_i = k_d H$$

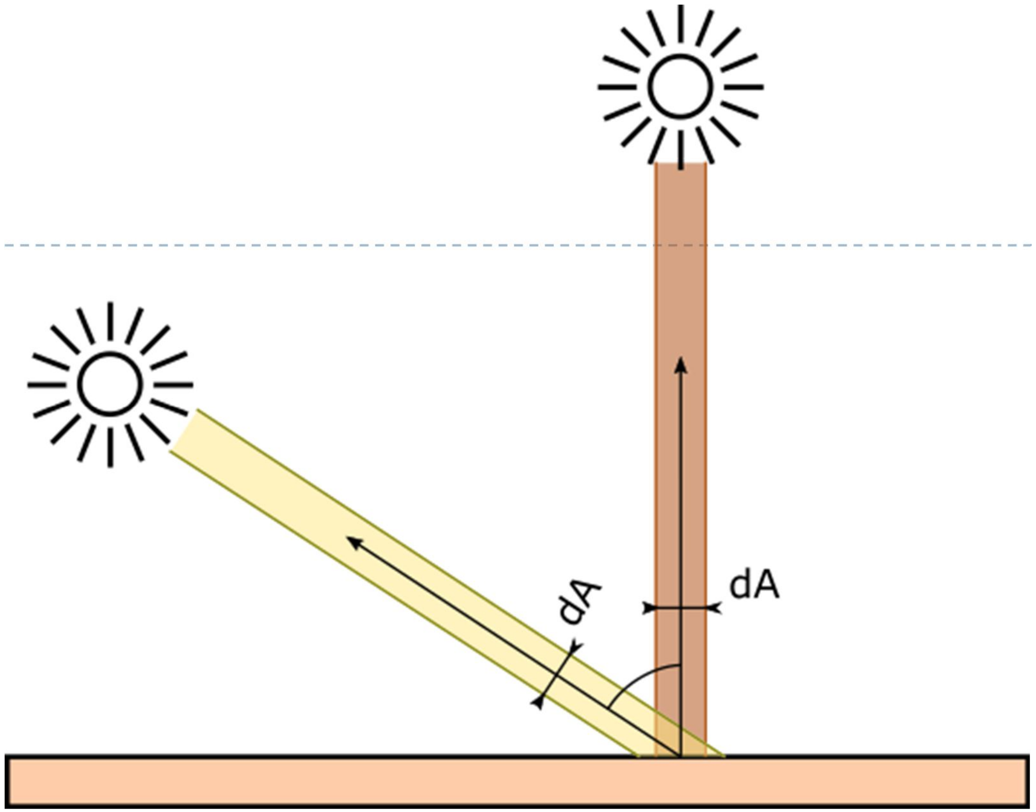
  - k_d: diffuse coefficient, material property [1/sr]



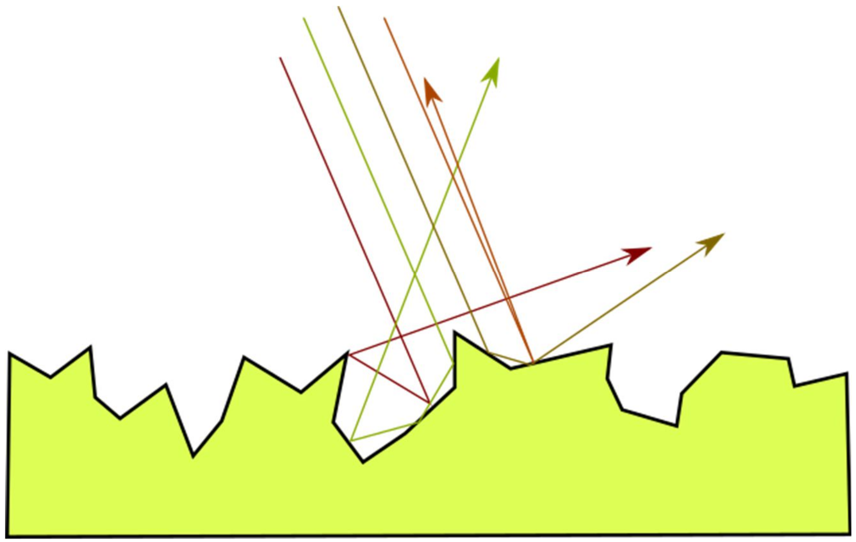


Diffuse reflection

- Cosine term
  - The surface receives less light per unit area at steep angles of incidence



- Rough, irregular surface results in diffuse reflection
  - Light is reflected in random direction
  - (this is just a model, light interaction is more complicated)

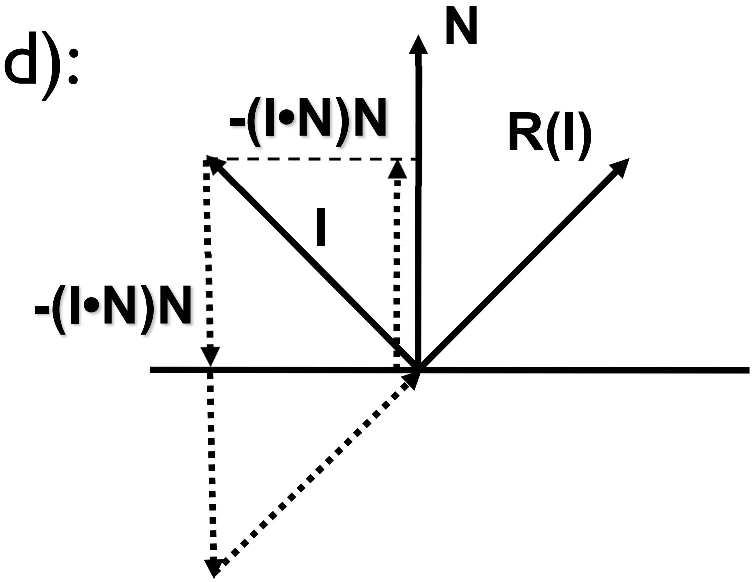


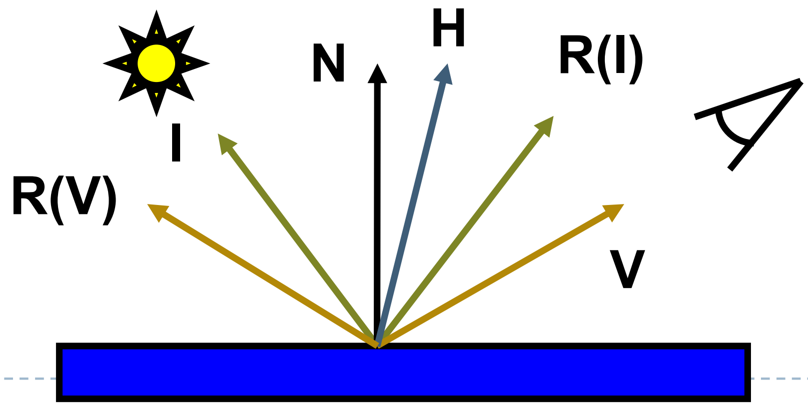
Reflection Geometry

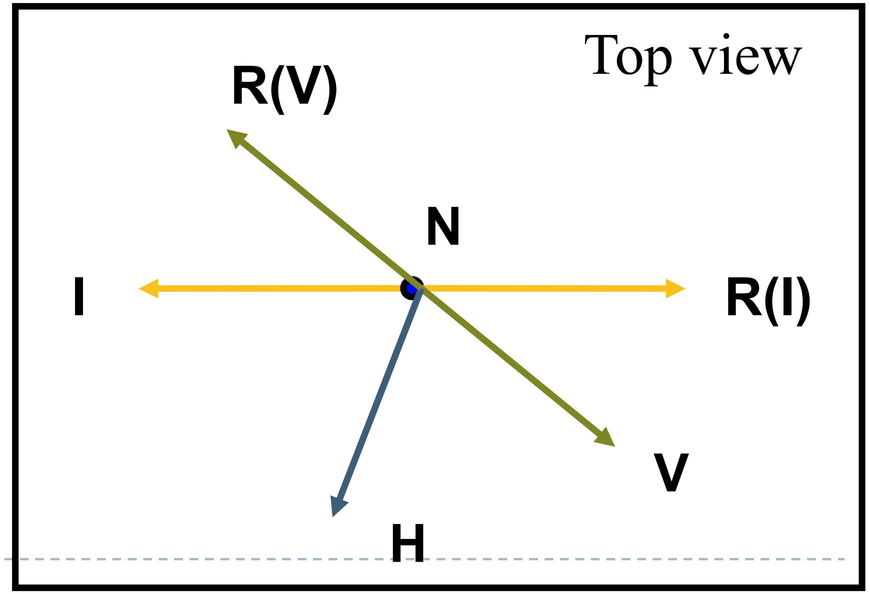
- Direction vectors (all normalized):
  - N: surface normal
  - I: vector to the light source
  - V: viewpoint direction vector
  - H: halfway vector

    H= (I + V) / |I + V|
  - R(I): reflection vector

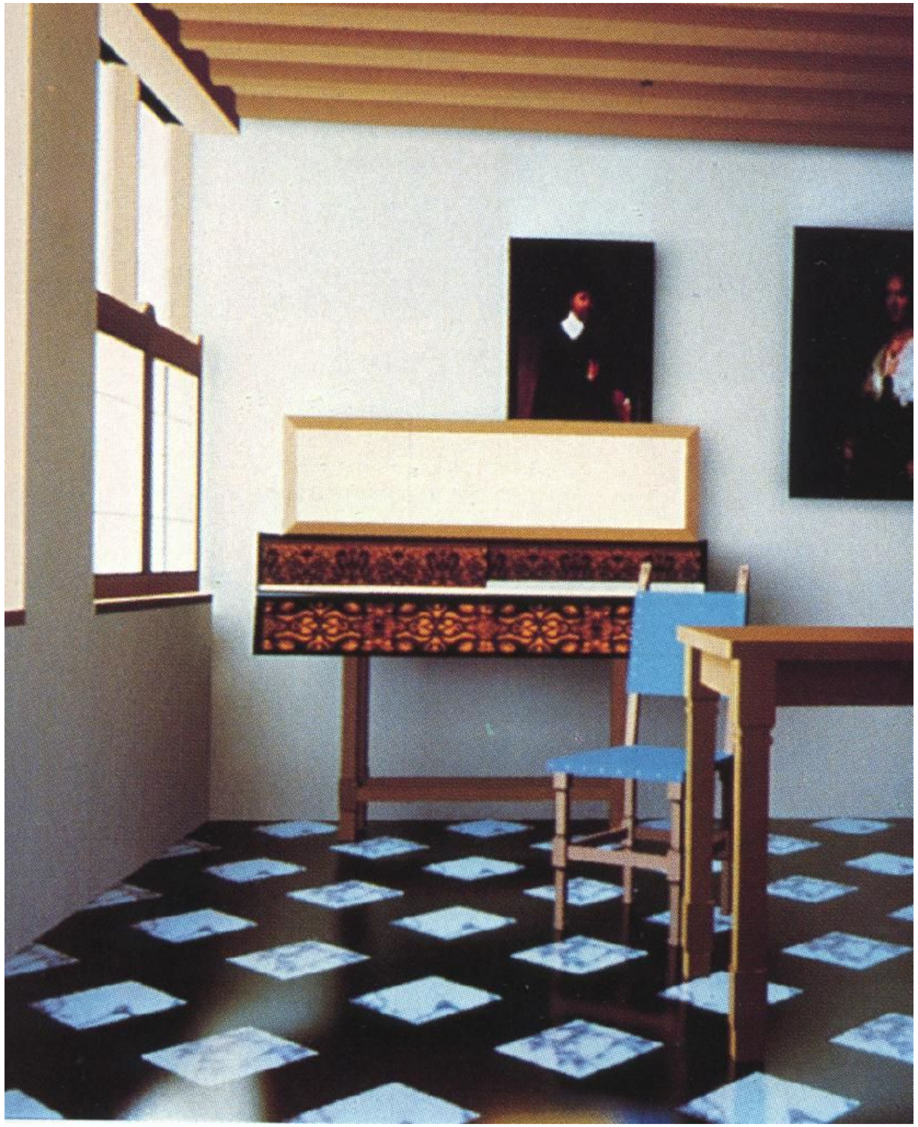
    R(I)= I - 2(I•N)N
  - Tangential surface: local plane





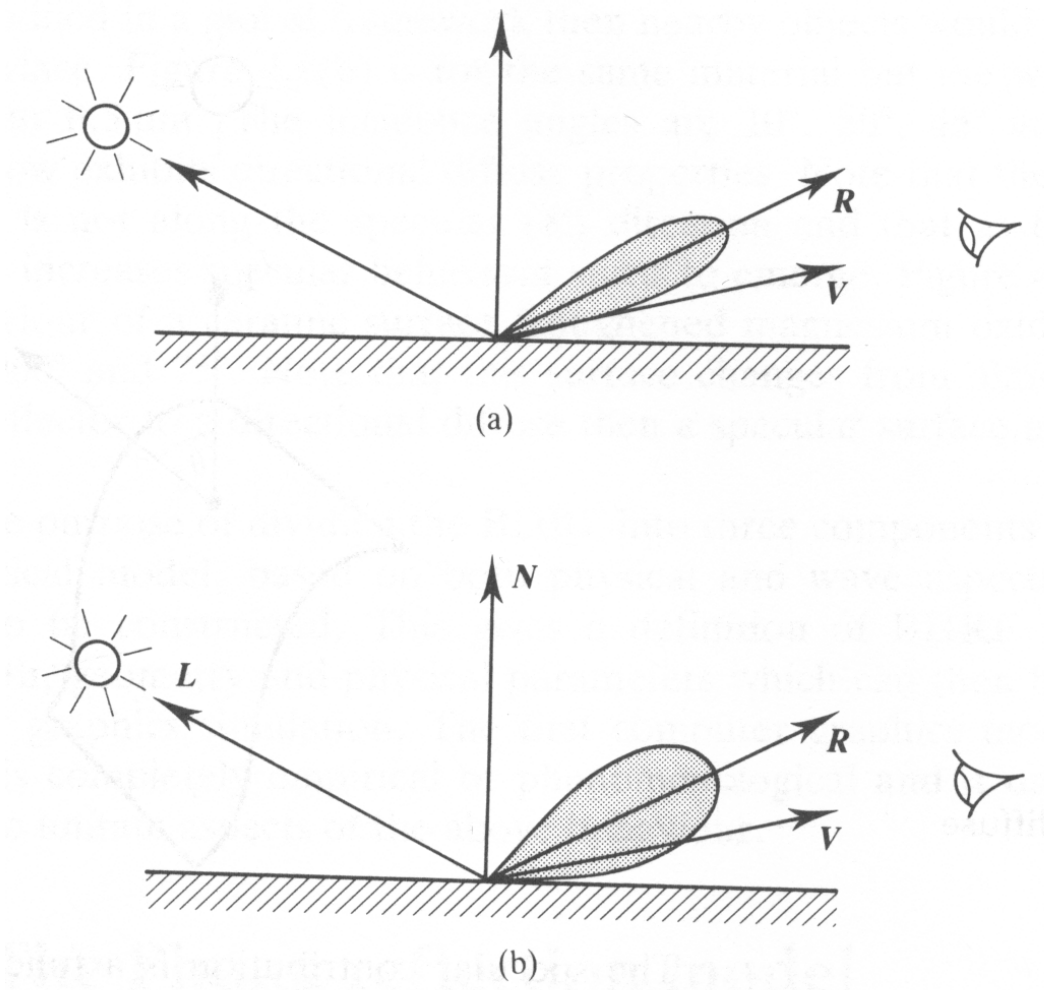


Glossy Reflection



Glossy Reflection

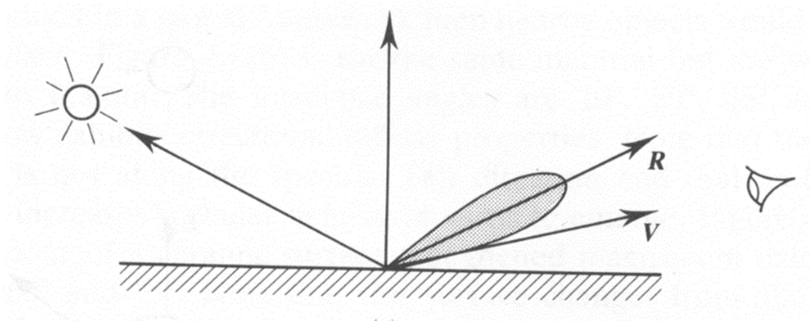
- Due to surface roughness
- Empirical models
  - Phong
  - Blinn-Phong
- Physical models
  - Blinn
  - Cook & Torrance



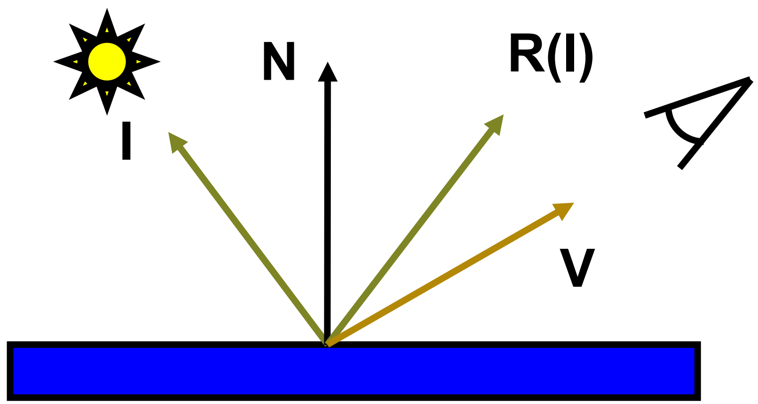
Phong Reflection Model

- Cosine power lobe

$$\rho(\omega_r, \omega_i) = k_s (R \cdot V)^{k_e}$$



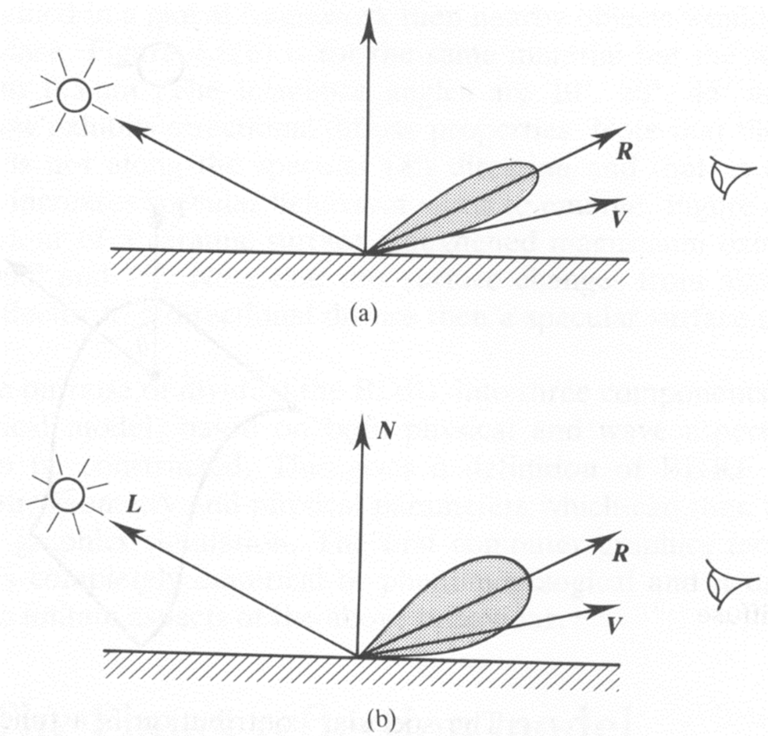
- Dot product & power
- Not energy conserving/reciprocal
- Plastic-like appearance

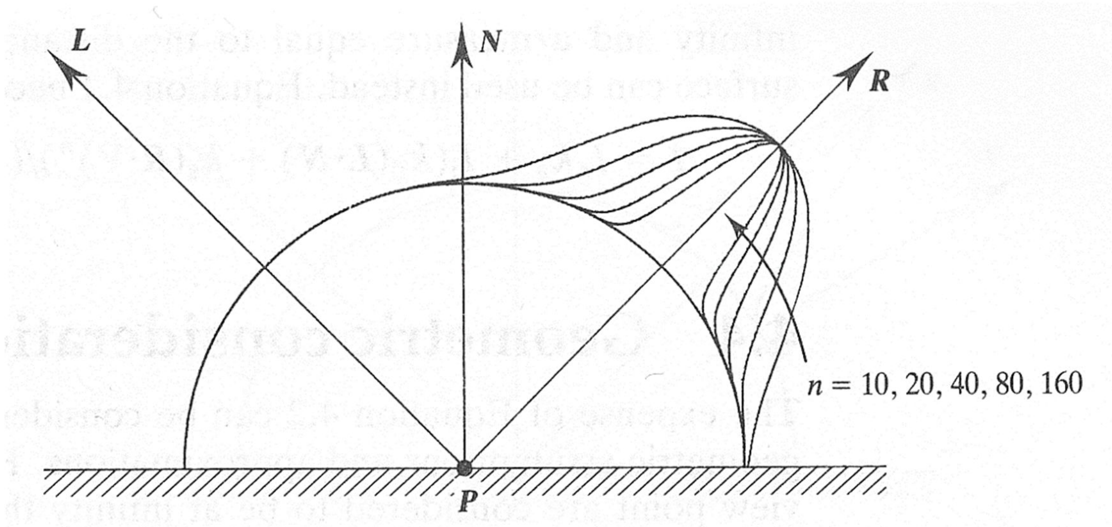


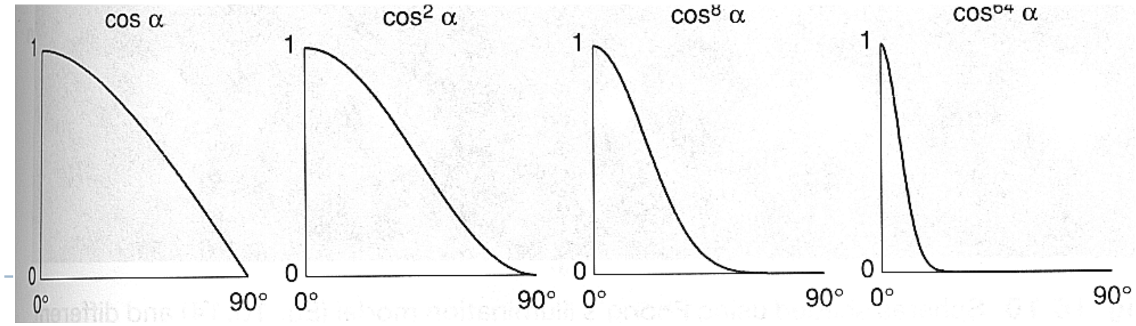
Phong Exponent k_e

$$\rho(\omega_r, \omega_i) = k_s (R \cdot V)^{k_e}$$

- Determines the size of highlight







Phong Illumination Model

- Extended light sources: *l* point light sources

$$L_r = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (R(I_l) \cdot V)^{k_e} \quad \text{(Phong)}$$

$$L_r = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (H_l \cdot N)^{k_e} \quad \text{(Blinn)}$$

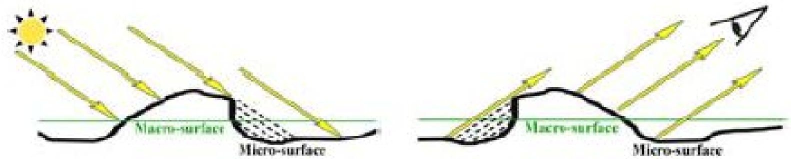
- Colour of specular reflection equal to light source
- Heuristic model
  - Contradicts physics
  - Purely local illumination
    - Only direct light from the light sources
    - No further reflection on other surfaces
    - Constant ambient term
- Often: light sources & viewer assumed to be far away

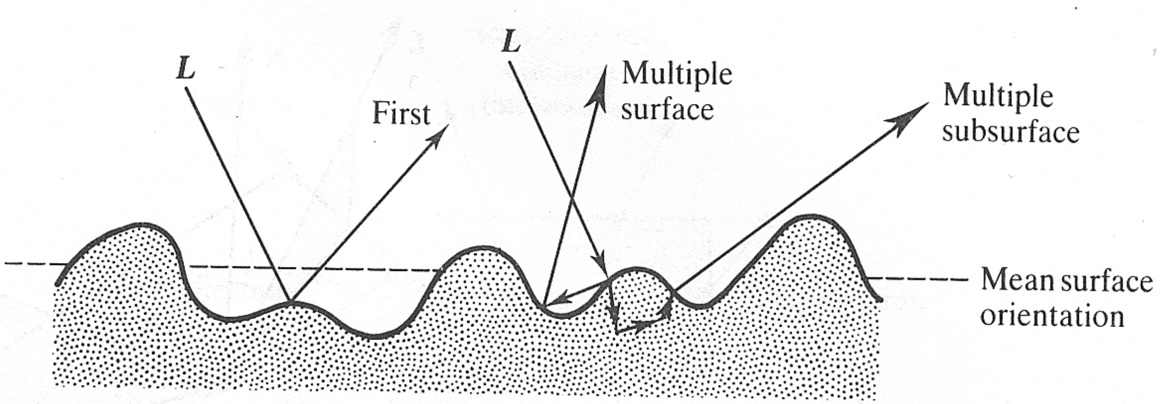
Micro-facet model

- We can assume that at small scale materials are made up of small facets
  - Facets can be described by the distribution of their sizes and directions D
  - Some facets are occluded by other, hence there is also a geometrical attenuation term G
  - And we need to account for Fresnel reflection (see next slides)

$$\rho(\omega_i, \omega_r) = \frac{D \cdot G \cdot F}{4 \cos \theta_i \theta_r}$$

Rendering of glittery materials [Jakob et al. 2014]





Cook-Torrance model

- Can model metals and dielectrics
- Sum of diffuse and specular components

$$\rho(\omega_i, \omega_r) = \rho_d(\omega_i) + \rho_s(\omega_i, \omega_r)$$

- Specular component:

$$\rho_s(\omega_i, \omega_r) = \frac{D(h)\ G(I,V)\ F(\omega_i)}{\pi \cos\theta_i \cos\theta_r}$$

- Distribution of microfacet orientations: $D(h) = \cos\theta_r\, e^{-\left(\frac{\alpha}{m}\right)^2}$

Roughness parameter

- Geometrical attenuation factor
  - To account to self-masking and shadowing

$$G(I,V) = min\left\{1, \frac{2(N \cdot H)(N \cdot V)}{V \cdot H}, \frac{2(N \cdot H)(N \cdot I)}{V \cdot H}\right\}$$

GGX model

- Multiple PBR models have been defined by modifying the definitions of the D and G functions.

$$\rho_s(\omega_i, \omega_r) = \frac{D(h)\; G(I, V)\; F(\omega_i)}{\pi \cos\theta_i \cos\theta_r}$$

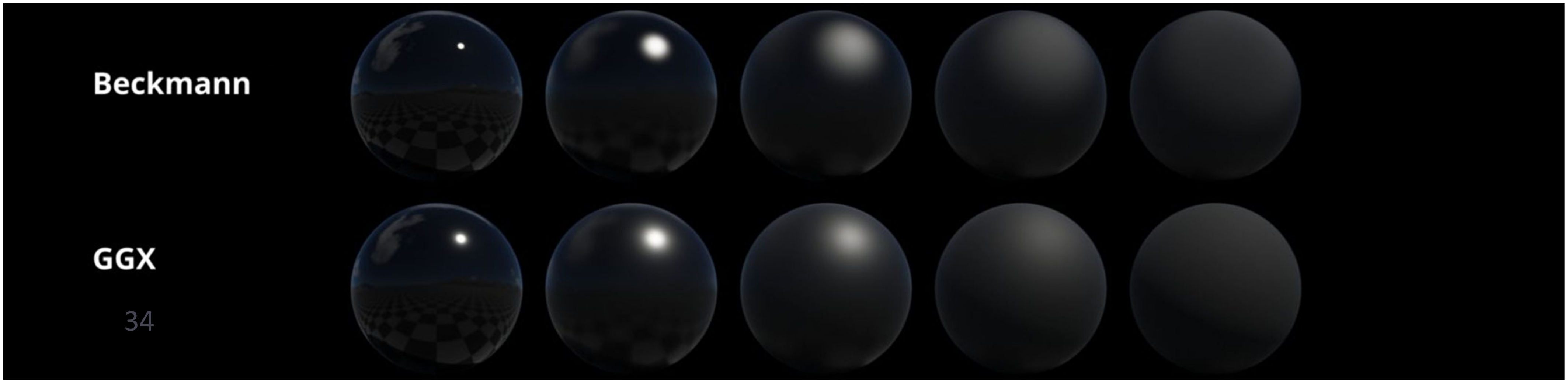
- Distribution of microfacet orientations:

$$D_{\mathrm{GGX}}(\vec{h}) = \frac{\alpha^2}{\pi\left[(\vec{h}\cdot\hat{n})^2(\alpha^2-1)+1\right]^2}$$

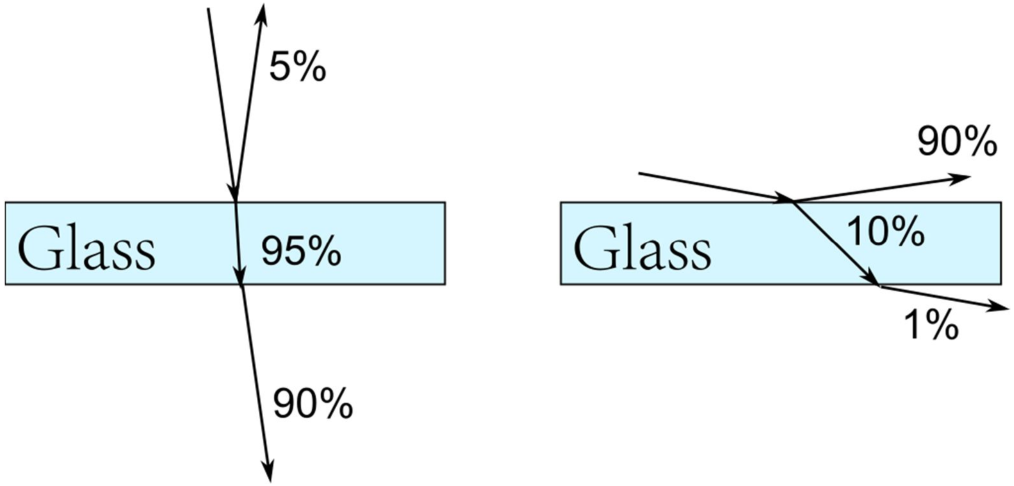
Roughness parameter

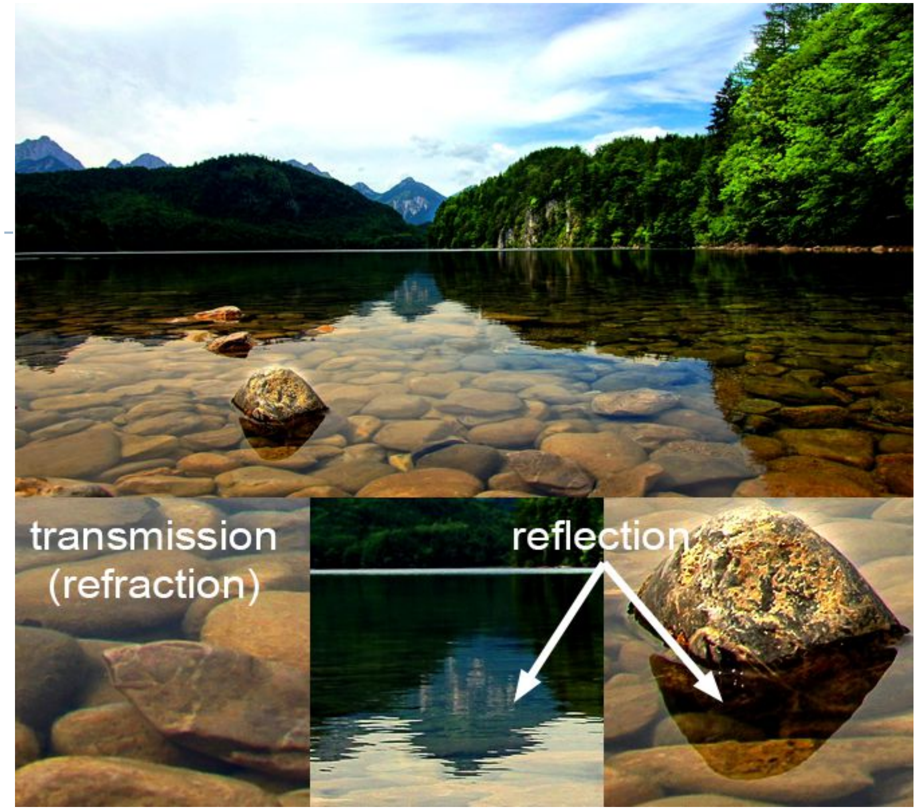
- More computationally efficient than Beckmann
- More realistic, especially high roughness materials
- Longer tails (higher intensity reflections at grazing angles)
- Currently used by most real-time renderers

Source: https://planetside.co.uk/news/terragen-4-5-releas



Fresnel term





Example from: https://www.scratchapixel.com/lessons/3d-basic-rendering/introduction-to-shading/reflection-refraction-fresnel

- The light is more likely to be reflected rather than transmitted near grazing angles
- The effect is modelled by Fresnel equation: it gives the probability that a photon is reflected rather than transmitted (or absorbed)

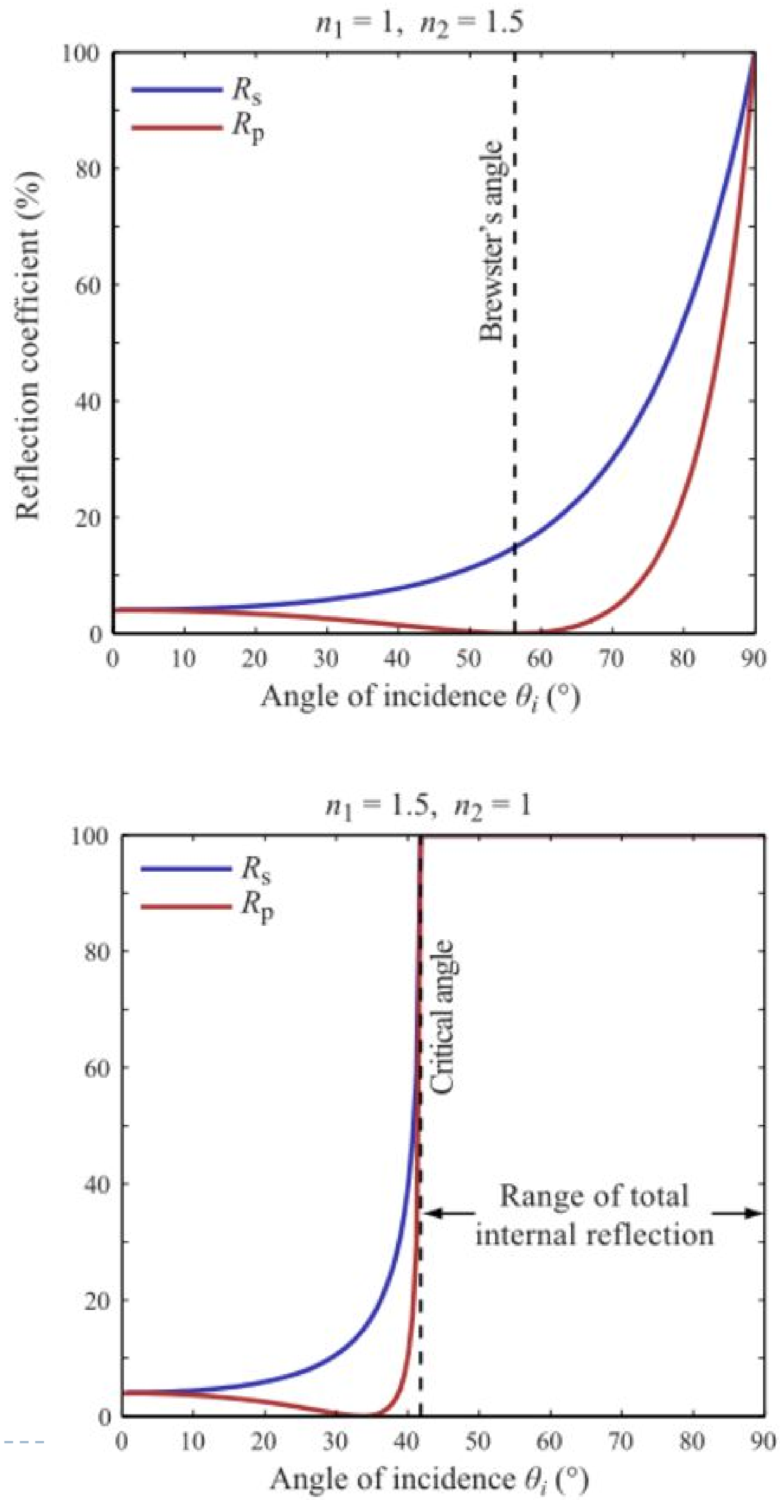
Fresnel equations

- Reflectance for s-polarized light:

$$R_s = \left|\frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t}\right|^2 = \left|\frac{n_1\cos\theta_i - n_2\sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_i\right)^2}}{n_1\cos\theta_i + n_2\sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_i\right)^2}}\right|^2,$$

- Reflectance for p-polarized light:

$$R_p = \left|\frac{n_1\cos\theta_t - n_2\cos\theta_i}{n_1\cos\theta_t + n_2\cos\theta_i}\right|^2 = \left|\frac{n_1\sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_i\right)^2} - n_2\cos\theta_i}{n_1\sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_i\right)^2} + n_2\cos\theta_i}\right|^2.$$



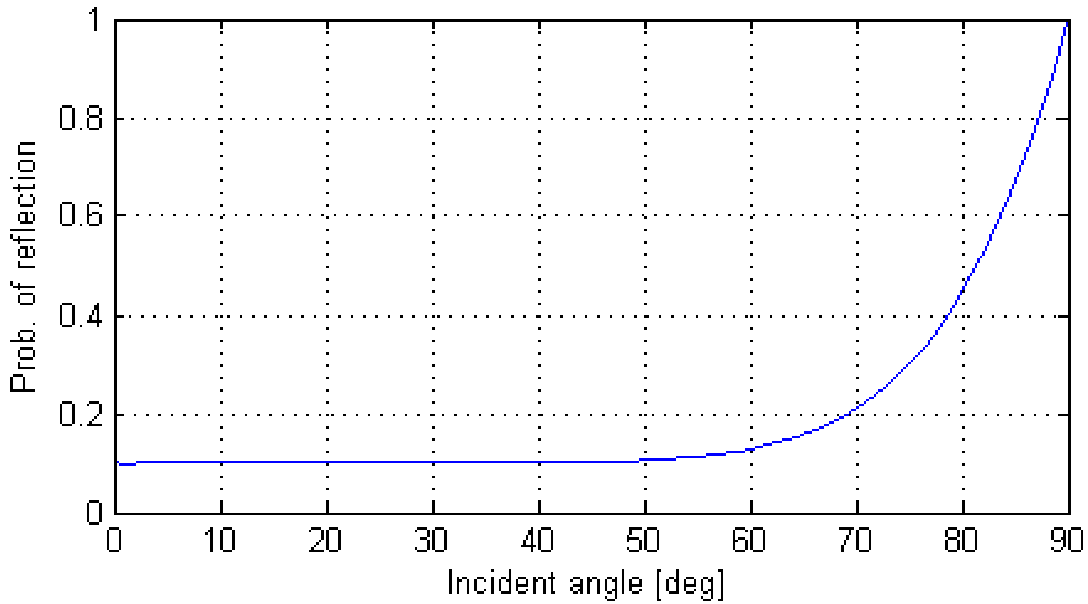
Fresnel term

- In Computer Graphics the Fresnel equation is approximated by Schlick's formula [Schlick, 94]:

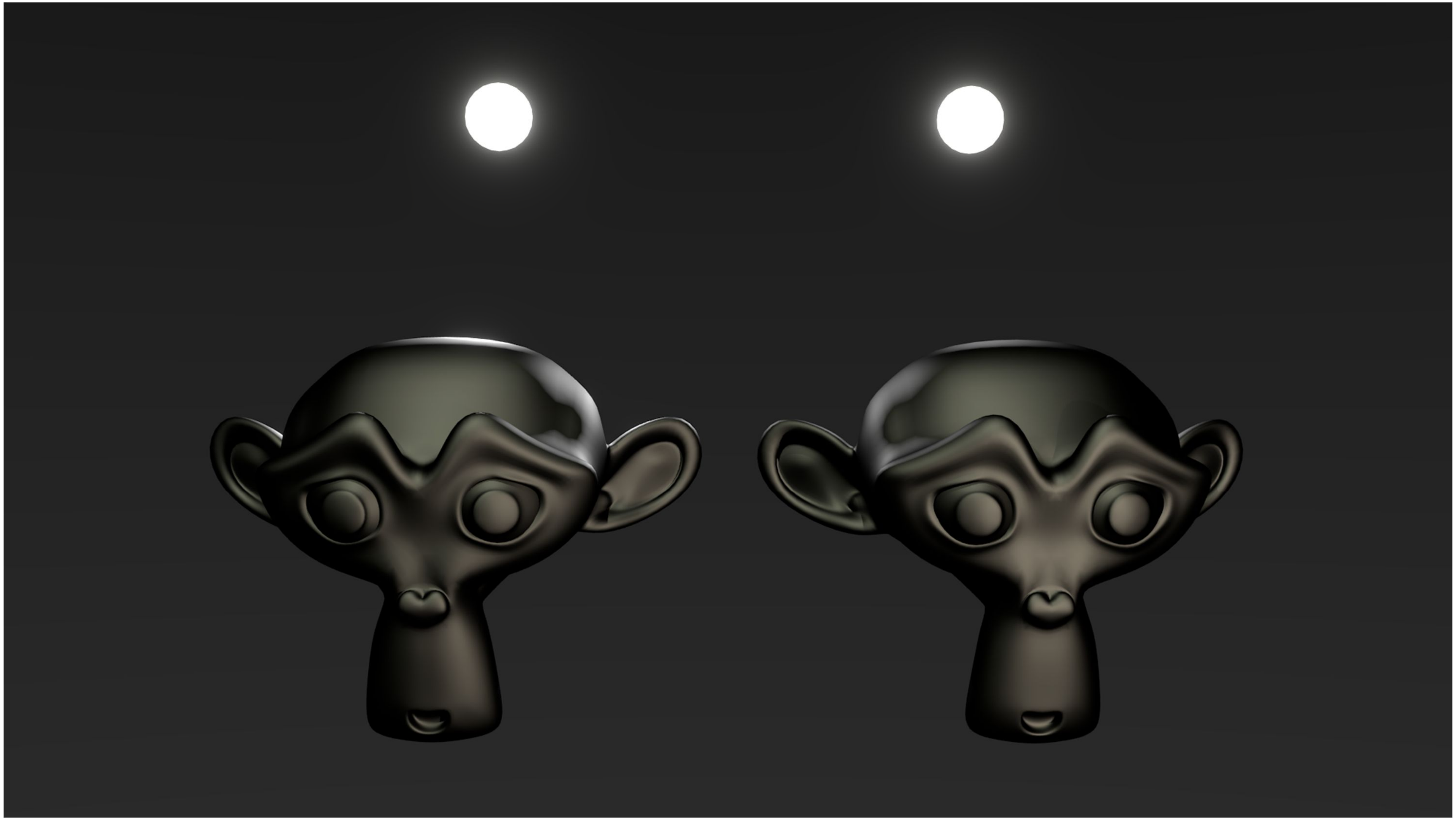
$$R(\theta,\lambda) = R_0(\lambda) + \big(1 - R_0(\lambda)\big)(1 - cos\theta)^5$$

- where $R_0(\lambda)$ is reflectance at normal incidence and λ is the wavelength of light
- For dielectrics (such as glass):

$$R_0(\lambda) = \left(\frac{n(\lambda) - 1}{n(\lambda) + 1}\right)^2$$

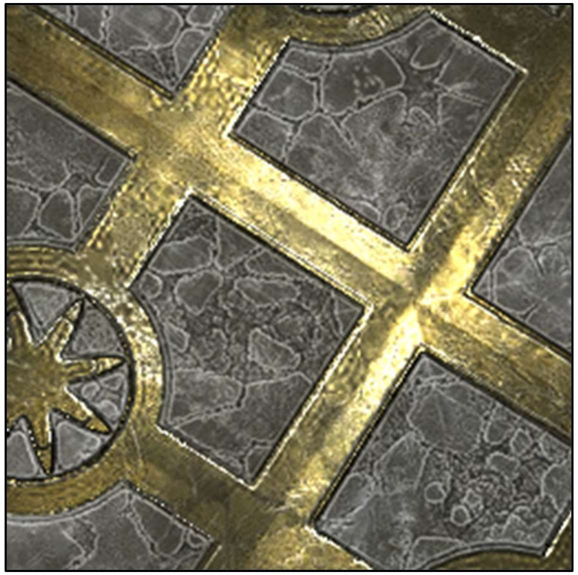


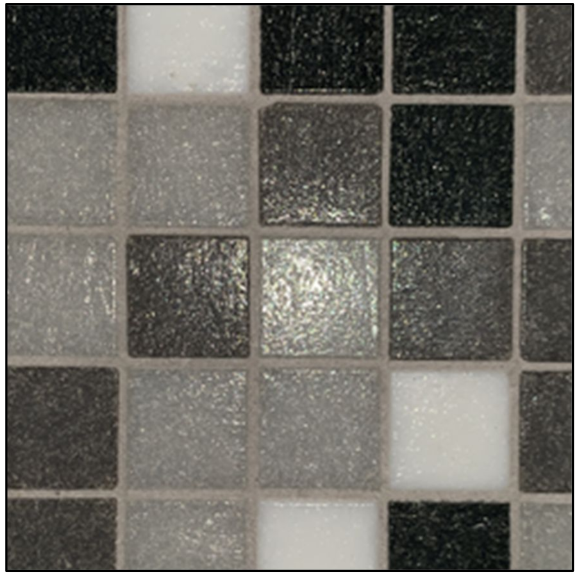
Which one is Phong / Cook-Torrance ?



Spatially-Varying Materials

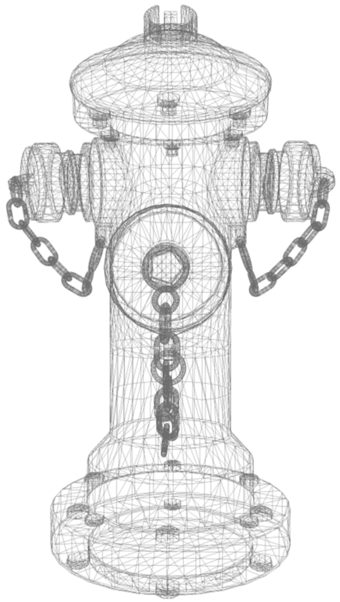
- In spatially-varying materials the reflectance has an additional dependence on the position over the material surface: $f_r(\vec{\omega}_i, \vec{\omega}_o, \vec{x})$

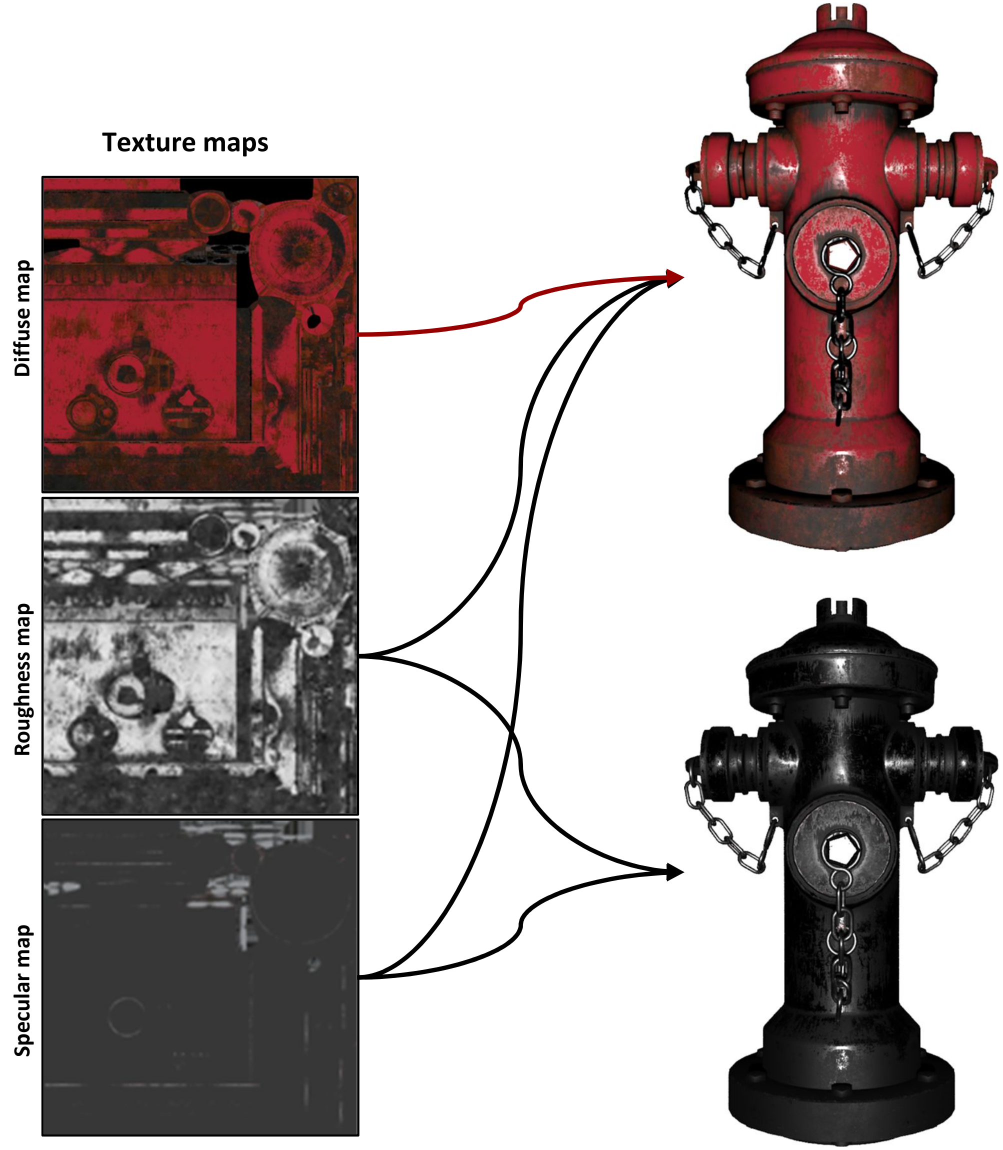




Spatially-Varying Materials

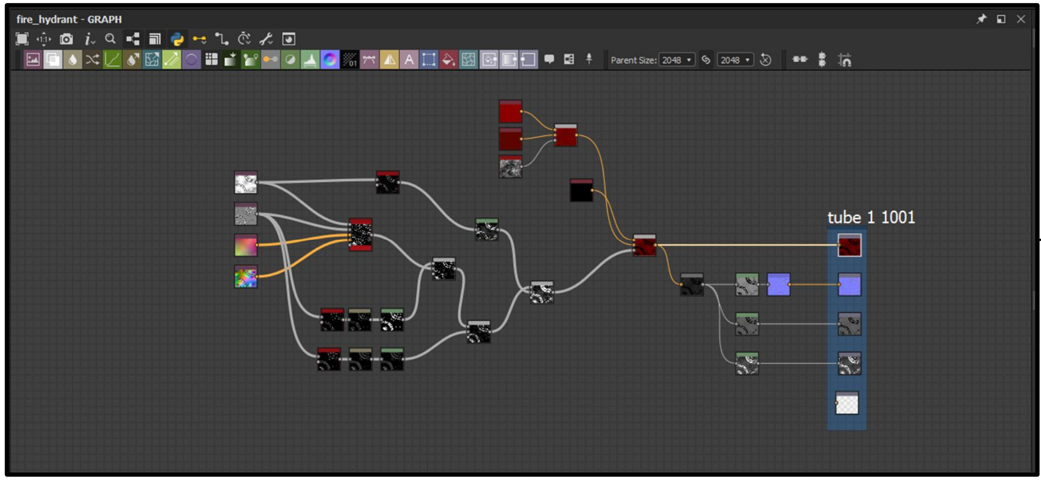
A common representation for SVBRDFs is via textures that encode analytic BRDF model parameters.





Spatially-Varying Materials

- In most commercial renderers, SVBRDFs are designed through procedural graphs. This gives the user great editing flexibility.
- Texture map representations can also be used as input or output of the graph (baking).



Source: Blender.

Image based lighting (IBL)

1. Capture an HDR image of a light probe

2. Create an illumination (cube) map

3. Use the illumination map as a source of light in the scene

The scene is surrounded by a cube map

Source: Image-based lighting, Paul Debevec, HDR Symposium 2009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Blender monkeys + IBL (path tracing)

Further reading

- A. Watt, 3D Computer Graphics
  - Chapter 7: Simulating light-object interaction: local reflection models
- Matt Pharr, Wenzel Jakob, Greg Humphreys, "Physically Based Rendering From Theory to Implementation" (2017)
- Eurographics 2016 tutorial
  - D. Guarnera, G. C. Guarnera, A. Ghosh, C. Denk, and M. Glencross
  - BRDF Representation and Acquisition
  - **DOI:** 10.1111/cgf.12867
- Some slides have been borrowed from Computer Graphics lecture by Hendrik Lensch
  - http://resources.mpi-inf.mpg.de/departments/d4/teaching/ws200708/cg/slides/CG07-Brdf+Texture.pdf