Foundations of Computer Science
Lecture #11: Procedural Programming

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let rec addLen n = function
| [] -> n
| x :: xs -> addLen (n+1) xs

Example:
addLen 0 [1,2,3]

Calling addLen with same arguments will always produce the same result. We can infer result through function expansion and reduction of expressions. This allows us to:

→ Prove algorithm correctness
→ Understand and predict algorithm outcome
Procedural programs can change the machine state.
They can interact with its environment.
They use control structures like branching, iteration and procedures.
They use data abstractions of the computer’s memory:
- references to memory cells
- arrays: blocks of memory cells
- linked structures, especially linked lists

concept: memory cells that are mutable
What are References?

In functional programming:
The store is an *invisible* device inside the computer

In procedural / imperative programming:
The store is *visible*
What are References?

In functional programming:
The store is an *invisible* device inside the computer

In procedural / imperative programming:
The store is *visible*

- References are *storage locations*
- They can be:
  (a) created
  (b) inspected
  (c) updated
ML Primitives for References

\[ \tau \text{ ref} \] type of references to type \( \tau \)

\[ \text{ref } E \] create a reference

\[ initial \text{ contents} = \text{the value of } E \]

\[ ! P \] return the \textit{current contents} of reference \( P \) \footnote{dereferencing}

\[ P := E \] update the contents of \( P \) to the value of \( E \)
ML Primitives for References

\( \tau \text{ ref} \)  \textit{type} of references to type \( \tau \)

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\( ! P \)  return the \textit{current contents} of reference \( P \)  \textit{dereferencing}

\( P := E \)  \textit{update} the contents of \( P \) to the value of \( E \)

\( P \) for 'pointer'

pointer to a 'box'

contents of that 'box'
### ML Primitives for References

- **τ ref**  
  *type* of references to type τ

- **ref E**  
  *create* a reference  
  *initial contents* = the value of E

- **! P**  
  return the *current contents* of reference P  
  *(dereferencing)*

- **P := E**  
  *update* the contents of P to the value of E

Three new ML functions / operators:

- **ref** : `'a -> 'a ref**  
  *(a) create box*

- **!** : `'a ref -> 'a**  
  *(b) inspect box content*

- **:=** : `'a ref -> 'a -> unit**  
  *(c) update box content*
# let p = ref 5 (* create a reference *)
val p : int ref = {contents = 5}

# p := !p + 1 (* p now holds value 6 *)
- : unit = ()

# let ps = [ ref 77; p ]
val ps : int ref list = [{contents = 77}; {contents = 6}]

# List.hd ps := 3
- : unit = ()

# ps
- : int ref list = [{contents = 3}; {contents = 6}]
Trying Out References

# let p = ref 5 (* create a reference *)
val p : int ref = {contents = 5}

# let z = p
val z : int ref = {contents = 5}

# p := !p + 1 (* p now holds value 6 *)
- : unit = ()

# p
- : int ref = {contents = 6}

# z
- : int ref = {contents = 6}

Aliasing: two values refer to the same mutable cell
Commands: Expressions with Effects

- Basic commands update references, write to files, etc.
- $C_1; \ldots; C_n$ causes a series of expressions to be evaluated and returns the value of $C_n$.
- A typical command returns the empty tuple: ()
- \textbf{if} $B$ \textbf{then} $C_1$ \textbf{else} $C_2$ behaves like the traditional control structure if $C_1$ and $C_2$ have effects.
- Other ML constructs behave naturally with commands, including \texttt{match} expressions and recursive functions.
Commands: Expressions with Effects

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- $C_1; \ldots; C_n$ causes a series of expressions to be evaluated and returns the value of $C_n$.
- A typical command returns the empty tuple: ()
- `if B then C_1 else C_2` behaves like the traditional control structure if $C_1$ and $C_2$ have effects.
- Other ML constructs behave naturally with commands, including `match` expressions and recursive functions.

Example:

```plaintext
> 1 + (print_endline "abc"; 3; 101);

abc
- : int = 102
```
let rec addLen n = function
  | [] -> n
  | x :: xs -> addLen (n+1) xs

addLen 0 [1,2,3]
addLen 1 [2,3]
addLen 2 [3]
addLen 3 []
=> returns 3
Iteration: the **while** Command

```ocaml
# let tlopt = function
  | [] -> None
  | _::xs -> Some xs
val tlopt : 'a list -> 'a list option = <fun>

# let length xs =
  let lp  = ref xs in (* list of uncounted elements *)
  let np  = ref 0  in (* accumulated count *)
  let fin = ref false in
  let fin = ref false in
  while not !fin do
    match tlopt !lp with
    | None -> fin := true
    | Some xs ->
      lp := xs;
      np := 1 + !np
  done;
  !np (* the final count is returned *)
val length : 'a list -> int = <fun>
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Iteration: the while Command

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Iteration: the *while* Command

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val length : 'a list -> int = <fun>
```
Example: length with Mutability

evaluation steps:

length([1;2;3])
==> lp = ref [1,2,3]

tlopt [1;2;3] != None ==> true
lp := [2,3]; np := 1+0;

tlopt [2;3] != None ==> true
lp := [3]; np := 1+1

tlopt [3] != None ==> true
lp := []; np := 1+2

tlopt [] != None ==> false
fin := true
==> return !np
==> returns 3
let tlopt = function
  | [] -> None
  | _::xs -> Some xs

let length xs =
  let lp  = ref xs in
  let np  = ref 0  in
  let fin = ref false in
  while not !fin do
    match tlopt !lp with
    | None -> fin := true
    | Some xs ->
      lp := xs;
      np := 1 + !np
    done;
  !np
val length : 'a list -> int = <fun>

let rec addLen n = function
  | [] -> n
  | x :: xs ->
    addLen (n+1) xs
# exception TooMuch of int

exception TooMuch of int

# let makeAccount initBalance =
let balance = ref initBalance in
let withdraw amt =
    if amt > !balance then
        raise (TooMuch (amt - !balance))
    else begin
        balance := !balance - amt;
        !balance
    end
in
withdraw

val makeAccount : int -> int -> int = <fun>
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val makeAccount : int -> int -> int = <fun>

returns a function that returns contents of 'balance', not the cell itself
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      balance := !balance - amt;
      !balance
    end
  in
  withdraw

val makeAccount : int -> int -> int = <fun>

returns a function that returns contents of ‘balance’, not the cell itself.
let my_account = makeAccount 30;
my_account : int -> int = <fun>

let my_new_balance = my_account 10;
my_new_balance : int = 20

let my_new_balance = my_account ~10;
my_new_balance : int = 30
# let student = makeAccount 500
val student : int -> int = <fun>

# let director = makeAccount 4000000
val director : int -> int = <fun>

# student 5 (* coach fare *)
- : int = 495

# director 150000 (* Tesla *)
- : int = 3850000

# student 500 (* oh oh *)
Exception: TooMuch 5.
ML Primitives for Arrays

# ["a"; "b"; "c"]
(* allocate a fresh string array *)
- : string array = ["a"; "b"; "c"]

# Array.make 3 'a'
(* array of size 3 with cell containing 'a' *)
- : char array = [ 'a'; 'a'; 'a' ]

# let aa = Array.init 5 (fun i -> i * 10)
(* array of size 5 initialised to (fun i) *)
val aa : int array = [ 0; 10; 20; 30; 40 ]

# Array.get aa 3
(* retrieve the 4th cell in the array *)
- : int = 30

# Array.set aa 3 42
(* set the 4th cell's value to 42 *)
- : unit = ()
# Array.make
- : int -> 'a -> 'a array = <fun>

# Array.init
- : int -> (int -> 'a) -> 'a array = <fun>

# Array.get
- : 'a array -> int -> 'a = <fun>

# Array.set
- : 'a array -> int -> 'a -> unit = <fun>
• We must write \( !p \) to get the contents of \( p \)
• We write just \( p \) for the address of \( p \)

• We can store *private* reference cells in functions; simulating object oriented programming

• OCaml's assignment syntax is \( V := E \) instead of \( V = E \)

• OCaml has similar control structures: *while/done*, *for/done* and *match/with*

• OCaml has short syntax for updating arrays \( x. (1) \) and the access is safe against buffer overflows
What More Is There to ML?

With references, we can now make mutable linked lists

```
# type 'a mlist =
  | Nil
  | Cons of 'a * 'a mlist ref

type 'a mlist = Nil | Cons of 'a * 'a mlist ref
```
Two ways to visualize references to references:

(1) Using pointers:

(2) Using nested boxes:
Linked (Mutable) Lists

# type 'a mlist =
| Nil
| Cons of 'a * 'a mlist ref

type 'a mlist = Nil | Cons of 'a * 'a mlist ref

→ The tail can be redirected!

# let rec mlistOf = function
| [] -> Nil
| x :: l -> Cons (x, ref (mlistOf l))

mlist : 'a list -> 'a mlist = <fun>
Extending a List to the Rear

pointing to a ‘box’

# let extend mlp x =
    let last = ref Nil in
    mlp := Cons (x, last);
    last
> val extend = fn : ‘a mlist ref * ‘a -> ‘a mlist ref
Example of Extending a List

```ocaml
# let mlp = ref (Nil: string mlist);;
val mlp : string mlist ref = {contents = Nil}

# extend mlp "a";;
- : string mlist ref = {contents = Nil}

# let mlp = ref (Nil : string mlist);;
val mlp : string mlist ref = {contents = Nil}

# let it = extend mlp "a" ;;
val it : string mlist ref = {contents = Nil}

# extend it "b" ;;
- : string mlist ref = {contents = Nil}

# mlp ;;
- : string mlist ref =
{contents = Cons ("a",
    {contents = Cons ("b", {contents = Nil})})}}
```
\[ \text{mlp} \rightarrow x \rightarrow \text{Nil} \]
\[ \text{it} \rightarrow y \rightarrow \text{Nil} \]
\[ \text{last} \rightarrow \text{Nil} \]
ref (Cons (x, ref (Cons (y, ref Nil))))
Destructive Concatenation

# let rec joining mlp ml2 =
  match !mlp with
  | Nil -> mlp := ml2
  | Cons (_, mlp1) -> joining mlp1 ml2
val joining : 'a mlist ref * 'a mlist -> unit = <fun>

# let join ml1 ml2 =
  let mlp = ref ml1 in
  joining mlp ml2;
!mlp
val join : 'a mlist -> 'a mlist -> 'a mlist = <fun>
# let ml1 = mlistOf ["a"];;
val ml1 : string mlist = Cons ("a", {contents = Nil})
# let ml2 = mlistOf ["b";"c"];;
val ml2 : string mlist = 
  Cons ("b", {contents = Cons ("c", {contents = Nil})})
# join ml1 ml2 ;;

What does this return?

- : string mlist =
Cons ("a",
  {contents = Cons ("b",
    {contents = Cons ("c", {contents = Nil})})})