(a) Explain in detail how a binary search tree represents a dictionary. If reference types are not used, what is the average-case cost of an update operation?

(b) A mutable binary tree is either a leaf or a branch node containing a label and references to two other mutable binary trees. Present the OCaml variant type declaration for mutable binary trees.

(c) Describe how to use this type to implement (mutable) binary search trees. Give OCaml code for the lookup and update operations. The update operation must never copy existing tree nodes.