Foundations of Computer Science

*This question has been translated from Standard ML to OCaml*

(a) Explain how $O$-notation is used to express efficiency of algorithms. [5 marks]

(b) Arrange the following list of complexity classes in order of decreasing efficiency in $n$. Briefly justify each relationship.

\[
O(5n^2) \quad O(e^n) \quad O(n^{1/3}) \quad O(n^3 - 3n^2) \quad O(\log n) \quad O(n2^n)
\]

[4 marks]

(c) Suppose that $f$ is a function from integers to integers such that $i \leq j$ implies $f(i) \leq f(j)$. Then there is an efficient algorithm to solve the equation $f(k) = y$, given the desired $y$ and a range of values in which to search for $k$: the idea is repeatedly to halve this range. Code this algorithm as the OCaml function `search` whose arguments are $f$, $y$, and the range $(a, b)$. Its result should be the greatest $k$ such that $f(k) \leq y$ and $a \leq k \leq b$, provided such a $k$ exists. [11 marks]