Formal Models of Language

Paula Buttery

Dept of Computer Science & Technology, University of Cambridge

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We learn a lot about the rabbit from the words in the local context.

- So far, we have been discussing grammars with discrete alphabets and algorithms that have discrete symbols as input.
- Many Natural Language Processing tasks require some notion of similarity between the symbols.
 - e.g. The queen looked angry. Her majesty enjoyed beheading.

To understand the implication of these sentences we need to know that *the queen* and *her majesty* are similar ways of expressing the same thing.

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• But which key words do we include in the collection?

- We could look at a $\pm n$ -word context **window** around the **target** word.
- We could select (and weight) keywords based on their frequency in the window:

rabbit = {the 56, white 22, a 17, was 11, in 10, it 9, said 8, and 8, to 7...}

• This would become a little more informative if we removed the function words:

rabbit ={white 22, said 8, alice 7, king 4, hole 4, hush 3, say 3, anxiously 2...}

queen ={said 21, king 6, shouted 5, croquet 4, alice 4, play 4, hearts 4, head 3... }

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• Two words can be expected to be semantically similar if they have similar word co-occurrence behaviour in texts.

e.g. in large amounts of general text we would expect *queen* and *monarch* to have similar word co-occurrences.

- Simple collections of context words don't help us easily calculate any notion of similarity.
- A trend in modern Natural Language Processing technology is to replace symbolic representation with a **vector representation**
- Every word is encoded into some vector that represents a point in a multi-dimensional word space.

		croquet	grin	hurried	king		shouted	vanished
rabbit	7			2	4	3		1
	4	4		1	6	1	5	
cat	5	1	2					2

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queen	4	4	0	1	6	1	5	0
cat	5	1	2	0	0	0	0	2

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 I wish I could show you our cat Dinah
 I didn't know that Cheshire cats always grinned in fact I didn't know that cats could grin

Similarity

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- Assumption: proximity in word space correlates with similarity in meaning
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- e.g. cosine similarity $cosine(\mathbf{v}_1, \mathbf{v}_2) = \frac{\mathbf{v}_1 \cdot \mathbf{v}_2}{\|\mathbf{v}_1\| \|\mathbf{v}_2\|}$
- Equivalent to dot product of normalised vectors (not affected by magnitude)
- cosine is 0 between orthogonal vectors
- cosine is 1 if $v_1 = \alpha v_2$, where $\alpha > 0$

- In certain circumstances we might select dimensions expertly
- For general purpose vectors we want to simply count in a large collection of texts, the number of times each word appears inside a window of a particular size around the target word.
- This leads to very large sparse vectors (remember Zipf's law)
- There are an estimated 13 *million* tokens for the English language—we can reduce this a bit by removing (or discounting) function words, grouping morphological variants (e.g., *grin, grins, grinning*)
- Is there some *k*-dimensional space (such that *k* << 13*million*) that is sufficient to encode the word meanings of natural language?
- Dimensions might hypothetically encode tense (past vs. present vs. future), count (singular vs. plural), and gender (masculine vs. feminine)...

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To find reduced dimensionality vectors (usually called word embeddings)

- Loop over a massive dataset and accumulate word co-occurrence counts in some form of a large sparse matrix X (dimensions n × n where n is vocabulary size)
- Perform Singular Value Decomposition on X to get a USV^T decomposition of X.

$$\begin{bmatrix} x_{11} & \dots & x_{1n} \\ \vdots & \chi & \vdots \\ x_{n1} & \dots & x_{nn} \end{bmatrix} = \begin{bmatrix} u_{11} & \vdots & \vdots & \vdots \\ \vdots & u_{2} & \dots & u_{n} \\ u_{1n} & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} s_{1} & 0 & 0 & \dots \\ 0 & s_{2} & 0 & \dots \\ 0 & 0 & \ddots & \dots \\ \vdots & \vdots & \vdots & s_{n} \end{bmatrix} \begin{bmatrix} v_{11} & \dots & v_{1n} \\ \dots & v_{2} & \dots \\ \dots & \vdots & \dots \\ \dots & v_{n} & \dots \end{bmatrix}$$

- Note *S* matrix has diagonal entries only.
- Cut diagonal matrix at index k based on desired dimensionality (can be decided by desired percentage variance): (∑_{i=1}^k s_i)/(∑_{i=1}ⁿ s_i)

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- Use rows of *U* for the word embeddings.
- This gives us a *k*-dimensional representation of every word in the vocabulary.

Things to note:

- Need all the counts before we do the SVD reduction.
- The matrix is extremely sparse (most words do not co-occur)
- The matrix is very large ($\approx 10^6 \times 10^6$)
- SVD is quadratic

Points of methodological variation:

- Due to Zipf distribution of words there is **large variance** in co-occurrence frequencies (need to do something about this e.g. discount/remove stop words)
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Predict models can be more efficient than **count** models

- **word2vec** is a **predict model**, in contrast to the distributional models already mentioned which are **count models**.
- Instead of computing and storing a large matrix from a very large dataset, use a model that learns **iteratively**, eventually encoding the probability of a word given its context.
- The parameters of the model are the word embeddings.
- The model is trained on a certain objective.
- At every iteration we run our model, evaluate the errors, and then adjust the model parameters that caused the error.

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- Two simple word2vec architectures:
- **Continuous Bag of Words** CBOW: given some context word embeddings, predict the target word embedding.
- **Skip-gram**: given a target word embedding, predict the context word embeddings (below).



- skip-gram model predicts relationship between a centre word w_t and its context words: p(context|w_t) = ...
- Predict context word embeddings based on the target word embedding.
- A loss function is used to score the prediction (usually **cross-entropy** loss function).

- Adjust the word embeddings to minimise the loss function.
- Repeat over many positions of *t* in a very big language corpus.



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- There has been a change in state-of-the-art for some applications: (e.g. Google Translate)
- Multi-modal experiments have become more straightforward (by combining vector representations)
- But these models are statistical (need very large amounts of data and have to find a way to handle unseen words)
- There has been a lot of hype and not much work on the problems the distributional models can't solve.

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Methods for predict word models are fast moving research

- There are many different methods for training word embeddings.
- A method can be considered better than a previous method if it gives us an improvement for a task.
- e.g. using contextual embeddings for grammatical error detection



- Part III project by Sam Bell 2019

Word embeddings can correlate with human intuitions

Researchers test their word embeddings against datasets of **human similarity judgements**:

- For a test set of words, participants rate word pairs for **relatedness** (e.g. Miller & Charles, Rubenstein & Goodenough)
- A rank of relatedness can be drawn up between items in the test set.
- A **rank correlation** between embeddings and human judgements can be calculated.
- Good embeddings have a correlation of 0.8 or better with the human judgements.

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• Can we do vector-oriented reasoning based on the offsets between words?

- Derive the vector between the pair of words *man* and *woman* and then add it to *king*.
- The nearest word to the region of vector space that results will be the answer to the analogy.



- *Mikolov* found that word2vec embeddings are good at capturing syntactic and semantic regularities in language, and that each relationship is characterised by a relation-specific vector offset.
- Note that the space is very sparse and that there are word pairs for which this does not work...

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- Humans have the capacity to translate thoughts into words, and to infer others' thoughts from their words.
- There must be some mental representations of meaning that are mapped to language, but we have no direct access to these representations.

$$\begin{array}{c|c} - & W \rightarrow \boxed{encoder} - & X \rightarrow \boxed{channel}_{p(y|x)} - & Y \rightarrow \boxed{decoder} - & W' \rightarrow \\ \hline mental & words & words' & mental \\ \hline representation & representation' & \end{array}$$

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- The brain encodes information through continuous signals of activation.
- Language symbols are transmitted via continuous signals of sound/vision.
- *Pereira et al.* trained a system using brain imaging data and word embeddings.
- Demonstrated the ability to generalise to new meanings from limited imaging data.

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