COMPUTER SCIENCE TRIPOS

Friday 3 June 1977. 1.30 to 4.30

PAPER 3

Candidates should answer five questions

1. The maximum error in solving $Ax=b$, where A is an $n \times n$ matrix, can be about $2^n \varepsilon$, where ε is a typical rounding error. Explain why this has not prevented systems of equations up to 1000×1000 from being solved.

2. A Minsky machine has a finite number of registers, each of which can hold a non-negative integer value. Describe in detail how you would represent a list of such integers within a single register of the machine.

Adopting this representation, design a Minsky machine that, given lists of integers $(b_1, b_2, .., b_n)$, $(c_1, c_2, .., c_n)$ in registers B, C, will HALT after computing the list of integers $(a_1, a_2, .., a_n)$ to register A, where $a_i = \min(b_i, c_i)$, $1 \le i \le n$. Use as many auxiliary registers as you please, but ensure that your program will work for any value of n.

3. Explain the following concepts in relation to the solution of the initial value problem for ordinary differential equations using step-by-step methods:

(i) One-point and multi-point methods
(ii) Convergence
(iii) Absolute stability
(iv) Stability of the method
(v) Stiff equations.

4. Show that interrupts are minimised by allocating the buffer store in proportion to $\sqrt{r_i}$, where r_i is the rate of transfer to the ith device and where one interrupt is generated each time the buffer store allocation is full.

5. Write a LISP function, together with any necessary subsidiary functions, that will create a list of all the atoms that are in a piece of LISP data structure. Modify the program so that it can cope with re-entrant structures, and, with the aid of a suitable small example, show how it works.

6. Describe the way in which eigenfunctions and eigenvalues may be used to discuss the convergence of an iterative method for solving elliptic partial differential equations. Illustrate these ideas by discussing the optimum value of α in the method based on the use of $1+\alpha L$ when L is the Laplacian operator. Explain in what way the nature of residual error after a large number of iterations depends on whether α is greater than or less than the optimum. For some iterative methods the eigenfunctions and eigenvalues are complex. How does this affect the argument?

7. Describe the operation of a raster scan VDU and explain in detail how character formation is achieved. How could such a display be enhanced to provide simple graphics facilities, such as the display of graphs and bar charts?

8. There are many computational problems that are referred to as being NP-complete. What does this mean, and why would it be important if an efficient method for solving one of these problems was found? A travelling salesman plans a round trip journey in which he will move from one city to the next by choosing always as his next stop the closest city not previously visited. Show by giving sample routemaps that this does not necessarily produce an optimal solution.

9. What are the properties of the data transmission code whose codewords are 1011, 0110, 1101? Describe the use of Slepian's Standard Array and show what may be done to decode long codes more efficiently.

10. Explain the difference between *editing* and *selection*. Produce an algorithm to give the approximate solution to the equation

$$y^5 - y^2 + e = 0$$

for small e, and write an algebra program for this algorithm. What part does editing or selection take in your solution?

11. Define a *relation* and explain how a relation may be described by a simple data description language.

There are a number of climbing areas (e.g. Glencoe); in each climbing area there are a number of crags of a particular rock at a known grid reference (e.g. Craig Bwlch y Moch, dolerite, SH575407); on each crag there are climbs which have a name, a length, a grade, date of first ascent and originator (e.g. Christmas Curry, 250 feet, just severe, 25/12/1953, J.J. Moulam). You may assume that crags and climbs have universally unique names.

Each area is covered by a number of climbing guides, each of which describes the climbs on a number of crags, up to a certain date (e.g. Climbers' Club Guides Tryfan and Glyder Fach, 1964).

A climber has a history of climbs he has led and followed, some of which may be first ascents.

Describe, using a suitable DDL, a set of relations well-formed to represent this information. Do not attempt to describe storage structures, submodels or constraints.

Show in any corresponding data manipulation language how the following queries may be expressed:

1) which climbs have been led by J. Brown?

2) what is the name and length of climbs in Snowdonia, on slate, which are of grade hard severe?

3) which new routes have been put up since the last publication on the crags covered by the guide book "Climbers' Guide Sheffield and Stanage End"?

TURN OVER

12. Discuss the detailed implementation of binary semaphores in a multiprocessor computer system. What higher level synchronisation facilities would you build using semaphores? Are there differences between the facilities needed by system and user programs?

13. Construct the characteristic finite state machine for the following grammar:

$$
\begin{aligned}
S &\rightarrow A \; ; \\
A &\rightarrow B \; \% \; A \\
A &\rightarrow B \\
B &\rightarrow i \\
B &\rightarrow (\; A \;)
\end{aligned}
$$

Show how this machine can be used to form the basis of a parser for the grammar, paying particular attention to any inadequate states that it may contain.