Compiler Construction

Lecture 9: Deriving interpreter 2

Jeremy Yallop
jeremy.yallop@cl.cam.ac.uk

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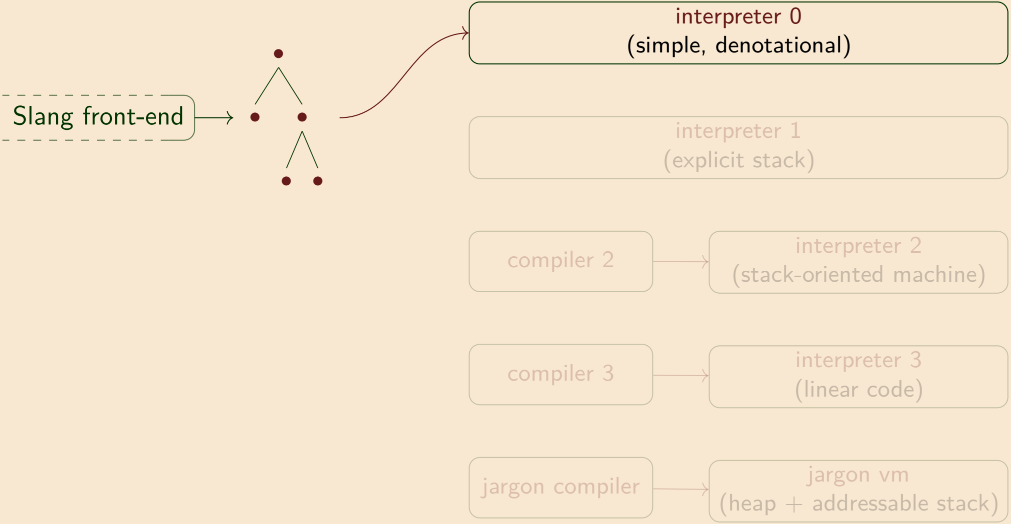
Reminder: the derivation



Slang front-end

interpreter 0
(simple, denotational)

interpreter 1
(explicit stack)

compiler 2

interpreter 2
(stack-oriented machine)

compiler 3

interpreter 3
(linear code)

jargon compiler

jargon vm
(heap + addressable stack)

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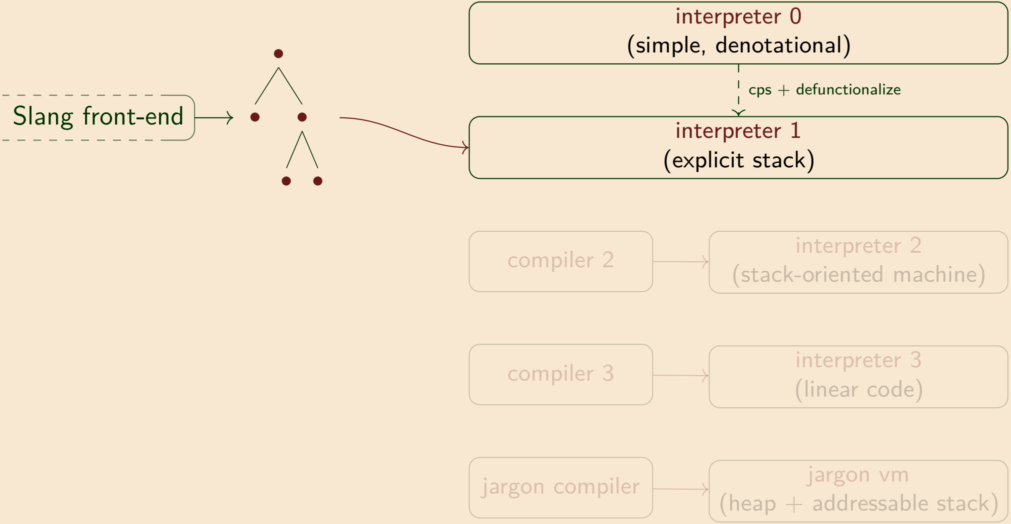

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cps + defunctionalize

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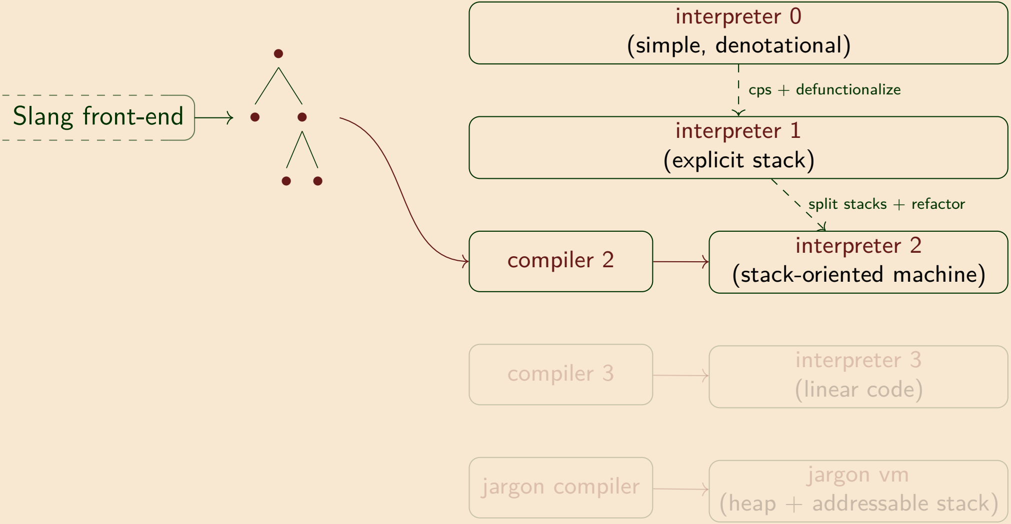


Slang front-end

interpreter 0
(simple, denotational)

cps + defunctionalize

interpreter 1
(explicit stack)

split stacks + refactor

compiler 2

interpreter 2
(stack-oriented machine)

compiler 3

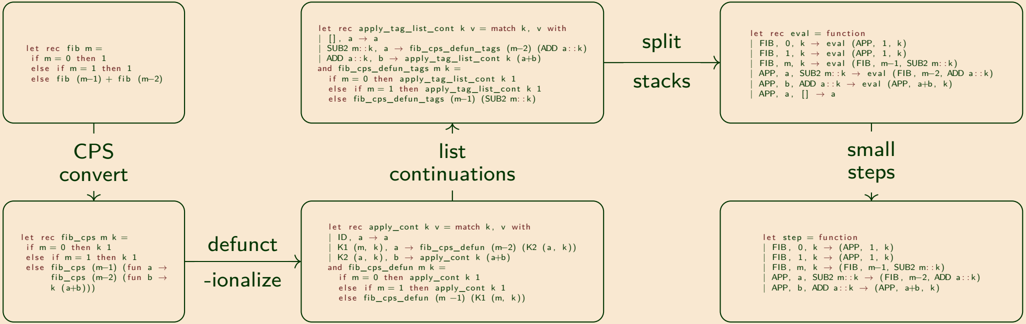
interpreter 3
(linear code)

jargon compiler

jargon vm
(heap + addressable stack)

Reprise

fib transformation recap



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let rec fib m =
 if m = 0 then 1
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 else fib (m−1) + fib (m−2)
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CPS convert

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let rec fib_cps m k =
 if m = 0 then k 1
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 else fib_cps (m−1) (fun a →
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defunct -ionalize

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let rec apply_cont k v = match k, v with
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list continuations

```
let rec apply_tag_list_cont k v = match k, v with
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split stacks

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small steps

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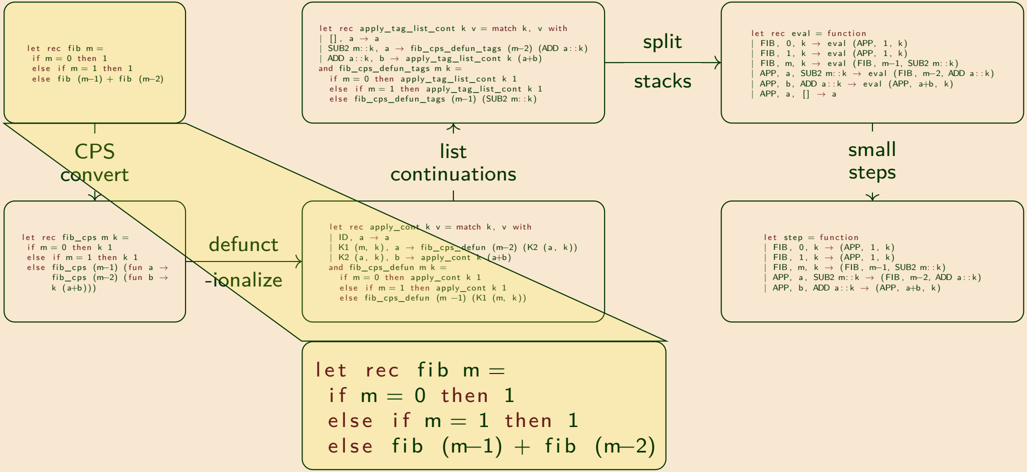
fib transformation recap



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list
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```

small
steps

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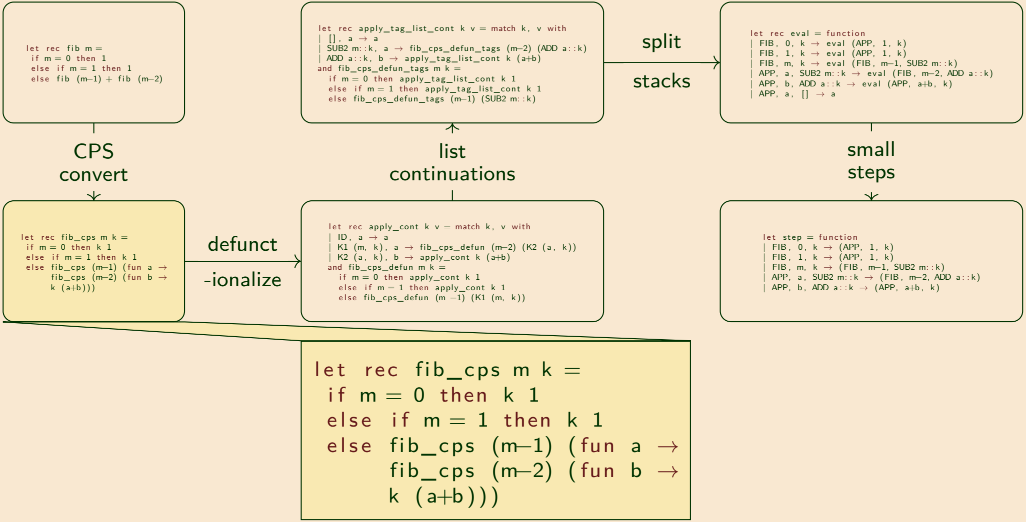
fib transformation recap



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list continuations

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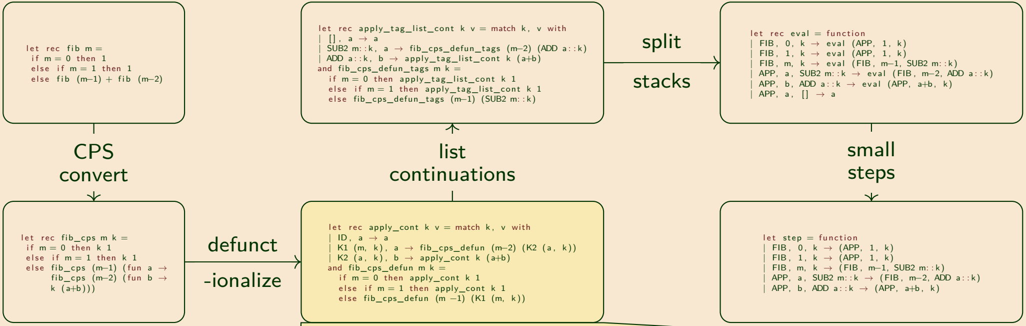

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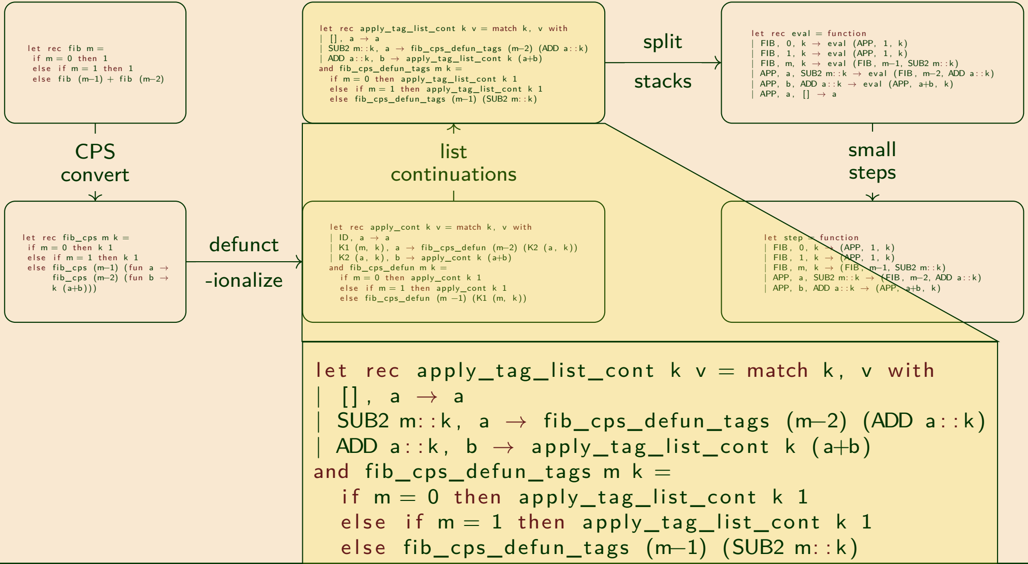

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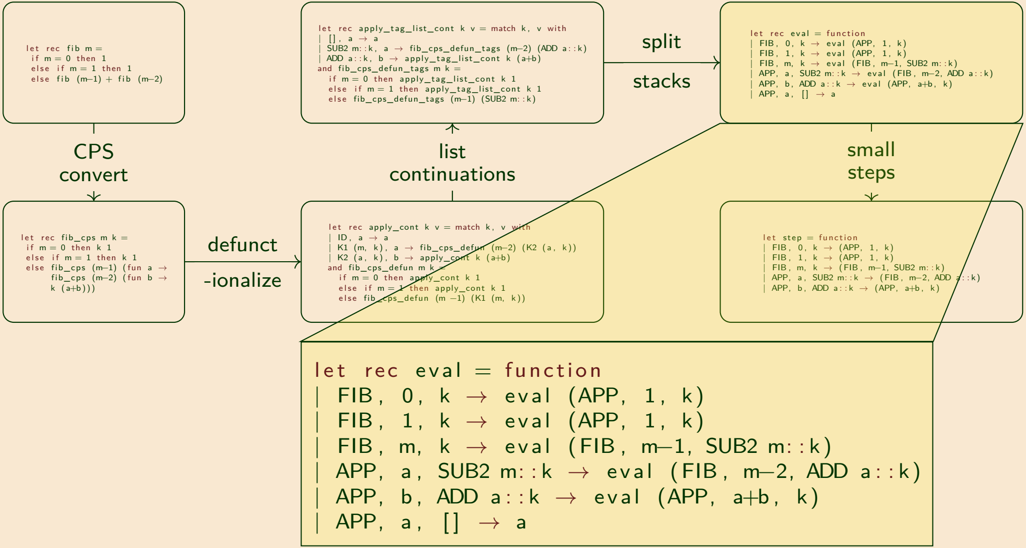

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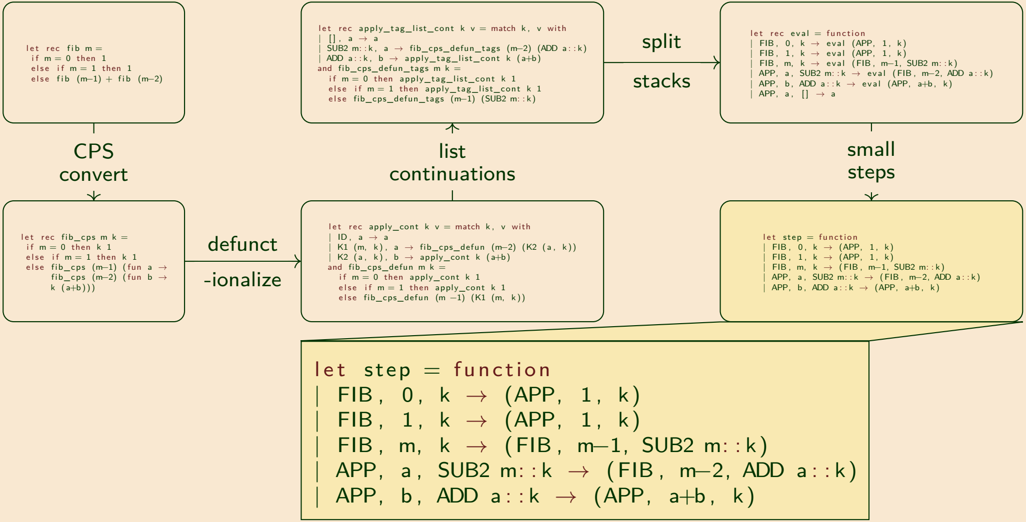
fib transformation recap



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list continuations

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The derivation again, with an evaluator

Plan:

1. start with a simple interpreter
2. derive stack machine & “compiler” that translates expressions to instructions

The simple interpreter:

```
type expr =
| INT of int
| ADD of expr * expr
| MUL of expr * expr
```

```
(* simple recursive expression evaluator *)
let rec eval = function
| INT a → a
| ADD (e1, e2) → eval e1 + eval e2
| MUL (e1, e2) → eval e1 * eval e2
```

Example: eval (ADD (MUL (INT 3, INT 4), INT 5)) ⇝ 17

CPS-convert

CPS-converting `eval` produces `eval_cps` (and `eval_2`):

```
type cont_2 = int → int

let rec eval_cps e k = match e with
| INT a → k a
| ADD (e1, e2) → eval_cps e1 (fun v1 →
                 eval_cps e2 (fun v2 →
                 k (v1 + v2)))
| MUL (e1, e2) → eval_cps e1 (fun v1 →
                 eval_cps e2 (fun v2 →
                 k (v1 * v2)))

let eval_2 e = eval_cps e (fun x → x)
```

Defunctionalize

Defunctionalizing eval_cps produces eval_cps_defun (and eval_3):

```
type cont_3 =
| ID
| OUTER_ADD of expr * cont_3
| OUTER_MUL of expr * cont_3
| INNER_ADD of int * cont_3
| INNER_MUL of int * cont_3

let rec apply f x = match f, x with
| ID, v → v
| OUTER_ADD (e2, k), v1 → eval_cps_defun e2 (INNER_ADD (v1, k))
| OUTER_MUL (e2, k), v1 → eval_cps_defun e2 (INNER_MUL(v1, k))
| INNER_ADD (v1, k), v2 → apply k (v1 + v2)
| INNER_MUL (v1, k), v2 → apply k (v1 * v2)
and eval_cps_defun (e, k) = match e with
| INT a → apply k a
| ADD (e1, e2) → eval_cps_defun e1 (OUTER_ADD(e2, k))
| MUL (e1, e2) → eval_cps_defun e1 (OUTER_MUL(e2, k))

let eval_3 e = eval_cps_defun e ID
```

Convert continuations to lists

Converting continuations to lists gives `eval_cps_defun_tags` (& `eval_4`):

```
type tag = O_ADD of expr
         | I_ADD of int
         | O_MUL of expr
         | I_MUL of int

type cont_4 = tag list

let rec apply f x = match f, x with
| [], v → v
| O_ADD e2 :: k, v1 → eval_cps_defun_tags e2 (I_ADD v1 :: k)
| O_MUL e2 :: k, v1 → eval_cps_defun_tags e2 (I_MUL v1 :: k)
| I_ADD v1 :: k, v2 → apply k (v1 + v2)
| I_MUL v1 :: k, v2 → apply k (v1 ∗ v2)

and eval_cps_defun_tags e k = match e with
| INT a → apply k a
| ADD (e1, e2) → eval_cps_defun_tags e1 (O_ADD e2 :: k)
| MUL (e1, e2) → eval_cps_defun_tags e1 (O_MUL e2 :: k)

let eval_4 e = eval_cps_defun_tags e []
```

Eliminate mutual recursion & split: step + driver

Eliminating mutual recursion & splitting eval into step + driver gives:

```
type acc =        (* "Accumulator" containing either
| A_INT of int     (* an int, or *)
| A_EXP of expr    (* an expression *)

let step : cont * acc → cont * acc = function
| k            , A_EXP (INT a)        → (k            , A_INT a)
| k            , A_EXP (ADD (e1, e2)) → (O_ADD e2 :: k, A_EXP e1)
| k            , A_EXP (MUL (e1, e2)) → (O_MUL e2 :: k, A_EXP e1)
| O_ADD e2 :: k, A_INT v1             → (I_ADD v1 :: k, A_EXP e2)
| O_MUL e2 :: k, A_INT v1             → (I_MUL v1 :: k, A_EXP e2)
| I_ADD v1 :: k, A_INT v2             → (k            , A_INT (v1+v2))
| I_MUL v1 :: k, A_INT v2             → (k            , A_INT (v1*v2))
| []           , A_INT v              → ([]           , A_INT v)

let rec driver : cont * acc → int = function
| [], A_INT v → v
| state       → driver (step state) (* tail recursive *)

let eval_5 e = driver ([], A_EXP e)
```

Two stacks, for values and expressions

An expression stack and a value stack

There are really **two independent stacks** here: one **for “expressions”** / one **for values** .

```
type directive = E of expr  (* ‘‘expressions’’ *)
               | DO_ADD
               | DO_MUL
type directive_stack = directive list

type value_stack = int list

(* The state is two stacks *)
type state_6 = directive_stack * value_stack

val step_6 : state_6 → state_6
val driver_6 : state_6 → int

val eval_6 : expr → int
```

Refactor step to use two stacks

The refactored implementation of `step` manipulates the two stacks separately:

```
let step_6 : state_6 → state_6 = function
 | E (INT v)      ::ds,         vs → (                    ds,        v::vs)
 | E (ADD (e1,e2))::ds,         vs → (E e1::E e2::DO_ADD::ds,          vs)
 | E (MUL (e1,e2))::ds,         vs → (E e1::E e2::DO_MUL::ds,          vs)
 | DO_ADD         ::ds, v2::v1::vs → (                    ds, v1 + v2::vs)
 | DO_MUL         ::ds, v2::v1::vs → (                    ds, v1 * v2::vs)
 | _ → failwith ”eval : runtime error!”

let rec driver_6 : state_6 → int = function
  | ([], [v]) → v
  | state → driver_6 (step_6 state)

let eval_6 (e : expr) : int = driver_6 ([E e], [])
```

An eval_6 trace

	ds	vs
inspect	`[E(ADD(MUL(INT 89,INT 2),MUL(INT 10,INT 4)))]`	`[]`
	`[DO_ADD;E(MUL(INT 10, INT 4));E(MUL(INT 89, INT 2))]`	`[]`
compute	`[DO_ADD;E(MUL(INT 10, INT 4));DO_MUL;E(INT 2);E(INT 89)]`	`[]`
	`[DO_ADD;E(MUL(INT 10, INT 4));DO_MUL;E(INT 2)]`	`[89]`
	`[DO_ADD;E(MUL(INT 10, INT 4));DO_MUL]`	`[89;2]`
inspect	`[DO_ADD;E(MUL(INT 10, INT 4))]`	`[178]`
compute	`[DO_ADD;DO_MUL;E(INT 4);E(INT 10)]`	`[178]`
	`[DO_ADD;DO_MUL;E(INT 4)]`	`[178;10]`
	`[DO_ADD;DO_MUL]`	`[178;10;4]`
	`[DO_ADD]`	`[178;40]`
	`[]`	`[218]`

(The top of each stack is on the right)

Interleaving *inspect* + *compute*

The evaluator is **interleaving** two quite distinct computations:

1. **inspect**: decomposition of an expression *e* into sub-expressions $e_1, e_2, \ldots$
2. **compute**: the computation of + and ×

Idea Since *e* is known from the start, complete inspect before starting compute

In general: refactor interpreter as translator + lower-level interpreter.

```
interpret_higher (e) = interpret_lower(compile(e))
```

(Examples: interpret Python by compiling to bytecode;
interpret machine code by compiling to micro-code)

Two stages, for compilation and evaluation

Refactoring: *compilation* + evaluation

> Never put off till run-time what you can do at compile-time.
> – David Gries

First stage: **inspect** (before starting compute):

```
type instr = (* low-level instructions *)
| Ipush of int
| Iplus
| Imult

type code = instr list

type state_7 = code * value_stack

let rec compile : expr → code = function
| INT a          →                                    [Ipush a]
| ADD (e1, e2) → compile e1 @ compile e2 @ [Iplus]
| MUL (e1, e2) → compile e1 @ compile e2 @ [Imult]
```

Refactoring: compilation + *evaluation*

Second stage: **compute** (without interleaving inspect):

```
let step_7 : state_7 → state_7 = function
  | Ipush v :: is,            vs → (is,    v    :: vs)
  | Iplus   :: is, v2::v1::vs → (is, v1 + v2 :: vs)
  | Imult   :: is, v2::v1::vs → (is, v1 ∗ v2 :: vs)
  | _                          → failwith ”eval : runtime error!”

let rec driver_7 = function
  | ([], [v]) → v
  | _ → driver_7 (step_7 state)

let eval_7 e = driver_7 (compile e, []) I
```

An eval_7 trace

inspect

```
compile (ADD(MUL(INT 89, INT 2), MUL(INT 10, INT 4)))
  ⇝
[push 89; push 2; mul; push 10; push 4; mul; add]
```

compute

```
[add; mul; push 4; push 10; mul; push 2; push 89]  []
[add; mul; push 4; push 10; mul; push 2]           [89]
[add; mul; push 4; push 10; mul]                   [89; 2]
[add; mul; push 4; push 10]                        [178]
[add; mul; push 4]                                 [178; 10]
[add; mul]                                         [178; 10; 4]
[add]                                              [178; 40]
[]                                                 [218]
```

instruction stack

value stack

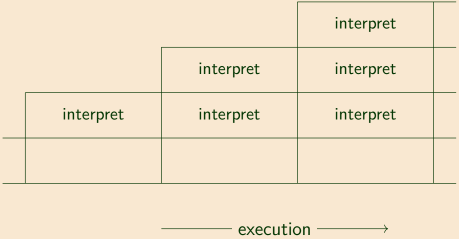
(The top of each stack is on the right)

Application to intepreter 0

interpret is implicitly using OCaml’s runtime stack

interp_0.ml

```
let rec interpret (e, env, store) =
  match e with
   | Integer n → (INT n, store)
   | Op(e1, op, e2) →
     let (v1, store1) = interpret(e1, env, store) in
     let (v2, store2) = interpret(e2, env, store1) in
     (do_oper(op, v1, v2), store2)
     :
```



Every call to interpret builds an activation record on OCaml’s runtime stack.

Interpreter 2 will make this stack explicit

interp_0.ml ⇝ interp_1.ml ⇝ interp_2.ml

Use derivation: `eval` ⇝ `eval_7` as a guide to derive interpreter 0 ⇝ interpreter 2.

Interpreter 0 is analagous to `eval`, a naive recursive evaluator.

1. Convert to **continuation-passing style**
2. **Defunctionalize**

Interpreter 1 is analogous to `eval_6`.
It has one continuation stack for expressions, values and environments

1. **Split the stack** into instruction stack + and a value/environment stack
2. **Stage** as compiler + lower-level interpreter

Interpreter 2 is analogous to `eval_7`

Properties of Interpreter 2

Interpreter 2 is a **high-level stack-oriented machine** with these properties:

- It makes the OCaml runtime stack explicit
- Complex values are pushed onto stacks
- It has one stack for values and environments
- It has one stack for instructions
- The heap is used only for references
- Its instructions have tree-like structure

(we will not look at the details of interpreter 1)

Data types: interpreter 0 vs interpreter 2

interp_0.mli

```
type address

type store = address → value

and value =
| REF of address
| INT of int
| BOOL of bool
| UNIT
| PAIR of value * value
| INL of value
| INR of value
| FUN of ((value * store)

type env = Ast.var → value
```

interp_2.mli

```
type address = int

type value =
| REF of address
| INT of int
| BOOL of bool
| UNIT
| PAIR of value * value
| INL of value
| INR of value
| CLOSURE of bool * closure
and closure = code * env

and instruction =
| PUSH of value           | LOOKUP of var
| POP                     | BIND of var
| FST                     | SND
| DEREF                   | APPLY
| MK_PAIR                 | MK_INL
| MK_CLOSURE of code      | MK_REC of var * code
| ...
```

Interpreter 2

Correctness of the driver

Correctness criterion (informal):

If `e` passes the front-end and `Interp_0.interpret e = v`
then `driver (compile e, []) = <v>` (where `<v>` represents `v`)

We therefore need `compile e` to leave the code for `e` on top of the stack

```
val compile : expr → code
```

From interpreter 0 to interpreter 2: a few cases

interp_0.ml

```
let rec interpret (e, env, store) = match e with
  | Fst e → (match interpret(e, env, store) with
             | PAIR (v1, _), store' → (v1, store')
             | _ → complain "Runtime error: expecting a pair")
  | If(e1, e2, e3) → let (v, store') = interpret(e1, env, store) in
                     (match v with
                     | BOOL true → interpret(e2, env, store')
                     | BOOL false → interpret(e3, env, store')
                     | v → complain "Runtime error: expecting a boolean!")
```

interp_2.ml

```
let rec compile = function
| Fst e → compile e @ [FST]
| If(e1, e2, e3) → compile e1 @ [TEST(compile e2, compile e3)]
:
let step = function
| FST :: ds, V(PAIR (v, _)) :: evs → (ds, V v :: evs)
| (TEST (c1, c2)) :: ds, V (BOOL true)::evs → (c1 @ ds, evs)
| (TEST (c1, c2)) :: ds, V (BOOL false)::evs → (c2 @ ds, evs)
| POP::ds, s::evs → (ds, evs)
.
```

From interpreter 0 to interpreter 2: cases for functions

interp_0.ml

```
let rec interpret (e, env, store) = match e with
  | Lambda(x, e) → FUN (fun (v, s) →
                                interpret(e, update(env, (x, v)), s)), store
  | App(e1, e2) →
    let (v2, store1) = interpret(e2, env, store) in
    let (v1, store2) = interpret(e1, env, store1) in
    (match v1 with
    | FUN f → f (v2, store2)
    | v → complain "Runtime error: expecting a function")
```

interp_2.ml

```
let rec compile = function
| Lambda(x, e) → [MK_CLOSURE(BIND x :: compile e @ [SWAP; POP])]
| App(e1, e2) → compile e2 @ compile e1 @ [APPLY; SWAP; POP]
:

let step = function
| (BIND x::ds, V v::evs) → (ds, EV [(x, v)]::evs)
| (MK_CLOSURE c::ds, evs) → (ds, V (mk_fun(c, evs_to_env evs))::evs)
| (APPLY::ds, V (CLOSURE (_, (c, env)))::V v::evs) →
                                    (c @ ds, V v::EV env::evs)
```

Example: Compiled code for `rev_pair.slang`

rev_pair.slang

```
let rev_pair (p : int * int)
            : int * int =
  (snd p, fst p)
in
  rev_pair (21, 17)
end
```

compile →

bytecode

```
MK_CLOSURE
   ([BIND p; LOOKUP p;
    SND; LOOKUP p; FST;
    MK_PAIR; SWAP; POP]);
BIND rev_pair;
PUSH 21;
PUSH 17;
MK_PAIR;
LOOKUP rev_pair;
APPLY;
SWAP;
POP;
SWAP;
POP
```

Next time: flattening the code