Compiler Construction

Lecture 2: Lexing

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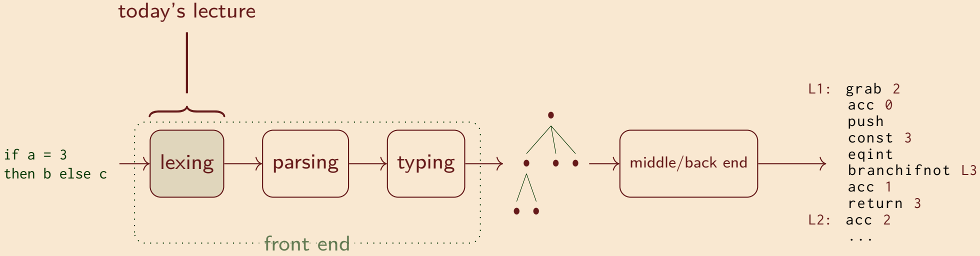
Lent 2023

What is a lexer?



today's lecture

```
if a = 3
then b else c
```

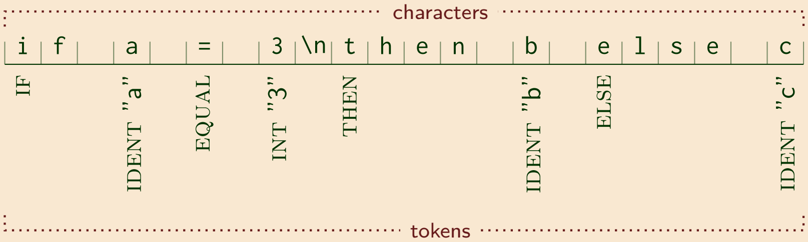
lexing → parsing → typing → middle/back end

front end

```
L1:  grab 2
     acc 0
     push
     const 3
     eqint
     branchifnot L3
     acc 1
     return 3
L2:  acc 2
     ...
```

What is lexing?

Lexing converts a sequence of characters into a sequence of tokens.



What do lexers look like?

A *lexer* is typically specified as a sequence mapping regexes to tokens:

regular expressions

if	⇒	IF
then	⇒	THEN
else	⇒	ELSE
=	⇒	EQUAL
[a-zA-Z]+ **as** s	⇒	IDENT s
[0-9]+ **as** i	⇒	INT i
[\t\n]	⇒	*skip*

tokens

Token data type:

```
type token =
    INT of int
  | IDENT of string
  | EQUAL
  | IF
  | THEN
  | ELSE
  | ...
```

Today's Q: how can we turn this declarative specification into a program?

Regular expressions

(“regexes”)

Regular expression syntax

Regular expressions e over alphabet Σ are written:

$$e \rightarrow \emptyset \mid \epsilon \mid a \mid e \vee e \mid ee \mid e* \qquad (a \in \Sigma)$$

A regular expression e denotes a language (set of strings) $L(e)$. For example,

$$\begin{aligned} L((a \vee b) * abb) \quad = \quad \{ & abb, \\ & aabb, \\ & babb, \\ & aaabb, \\ & ababb, \\ & baabb, \\ & bbabb, \\ & aaaabb, \\ & \ldots\} \end{aligned}$$

The regular language problem

The $L(-)$ function can be defined inductively:

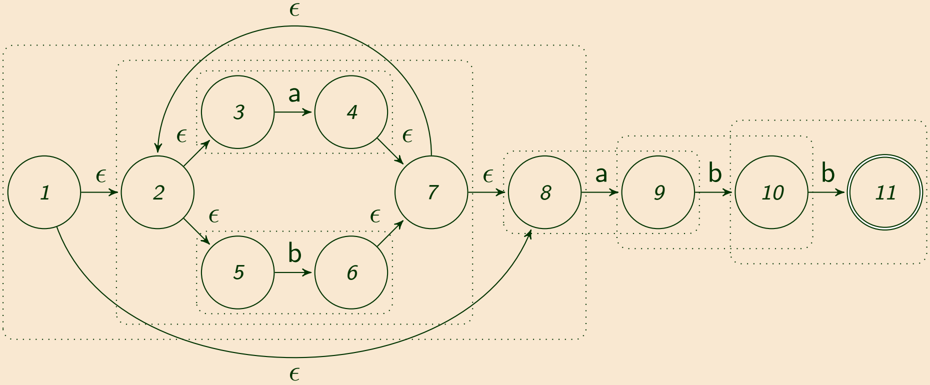
$$\begin{aligned}
L(e) &\subseteq \Sigma* \\
L(\emptyset) &= \{\} \\
L(\epsilon) &= \{\epsilon\} \\
L(a) &= \{a\} \\
L(e_1 \vee e_2) &= L(e_1) \cup L(e_2) \\
L(e_1 e_2) &= \{w_1 w_2 \mid w_1 \in L(e_1), w_2 \in L(e_2)\} \\
L(e^0) &= \{\epsilon\} \\
L(e^{n+1}) &= L(ee^n) \\
L(e*) &= \cup_{n \geq 0} L(e^n)
\end{aligned}$$

The regular language problem: is $w \in L(e)$? This is insufficient for lexing.

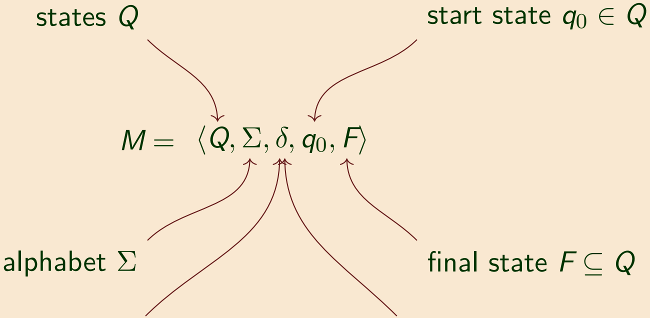
Finite-state automata

An NFA example

A nondeterministic finite-state automaton for recognising $L((a \vee b) * abb)$:



Review of Finite Automata (FA)



states Q

start state $q_0 \in Q$

$M = \langle Q, \Sigma, \delta, q_0, F \rangle$

alphabet Σ

final state $F \subseteq Q$

For NFAs:
(**nondeterministic**)
$\forall q \in Q$
  $\forall a \in (\Sigma \cup \{\epsilon\})$
    $\delta(q, a) \subseteq Q$

For DFAs:
(**deterministic**)
$\forall q \in Q$
  $\forall a \in \Sigma$
    $\delta(q, a) \in Q$

Transition notation

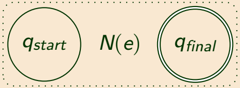
Notation for DFAs:	Notation for NFAs:
$q \xrightarrow{\epsilon} q$	$q \xrightarrow{\epsilon} q$
	$q_1 \xrightarrow{w} q_3$ if $q_2 \in \delta(q_1, \epsilon)$ and $q_2 \xrightarrow{w} q_3$
$q_1 \xrightarrow{aw} q_3$ if $\delta(q_1, a) = q_2$ and $q_2 \xrightarrow{w} q_3$	$q_1 \xrightarrow{aw} q_3$ if $q_2 \in \delta(q_1, a)$ and $q_2 \xrightarrow{w} q_3$
$L(M) = \{w \mid \exists q \in F, q_0 \xrightarrow{w} q\}$	$L(M) = \{w \mid \exists q \in F, q_0 \xrightarrow{w} q\}$

Regular expressions ⟶ NFAs

Review of RE ⟶ NFA

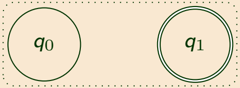
$N(-)$ takes a regex e to an NFA $N(e)$ accepting $L(e)$ with a single final state.

$N(e) =$

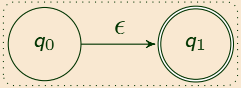


$N(-)$ is defined by induction on e.

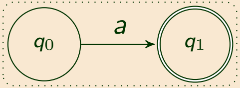
$N(\emptyset) =$



$N(\epsilon) =$

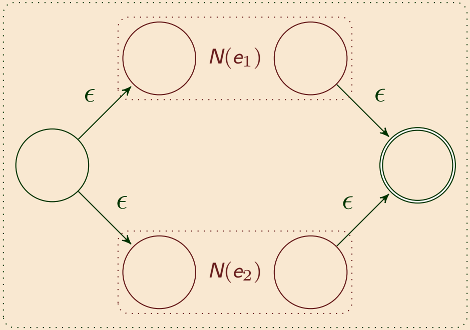


$N(a) =$

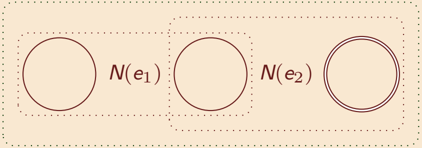


Review of RE ⟶ NFA

$$N(e_1 \vee e_2) \quad =$$

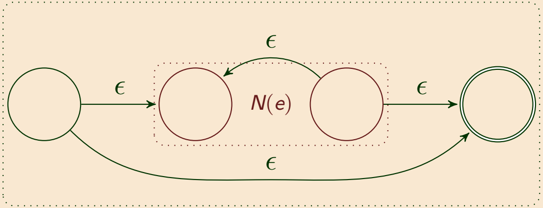


$$N(e_1 e_2) \quad =$$



Review of RE ⟶ NFA

$N(e*) \quad =$



Note: an alternative to this simple construction is Glushkov's (1961) algorithm, which produces an equivalent automaton without the ϵ *transitions.*

NFAs $\longrightarrow$ DFAs

Review of NFA $\longrightarrow$ DFA

The *powerset construction* takes a NFA

$$M = \langle Q, \Sigma, \delta, q_0, F \rangle$$

and constructs an DFA

$$M' = \langle Q', \Sigma', \delta', q_0', F' \rangle$$

where the components of M' are calculated as follows:

$$\begin{array}{rcl} Q' & = & \{S \mid S \subseteq Q\} \\ \delta'(S, a) & = & \epsilon\text{-closure}(\{q' \in \delta(q, a) \mid q \in S\}) \\ q_0' & = & \epsilon\text{-closure}\{q_0\} \\ F' & = & \{S \subseteq Q \mid S \cap F \neq \emptyset\} \end{array}$$

and the ϵ-*closure* is:

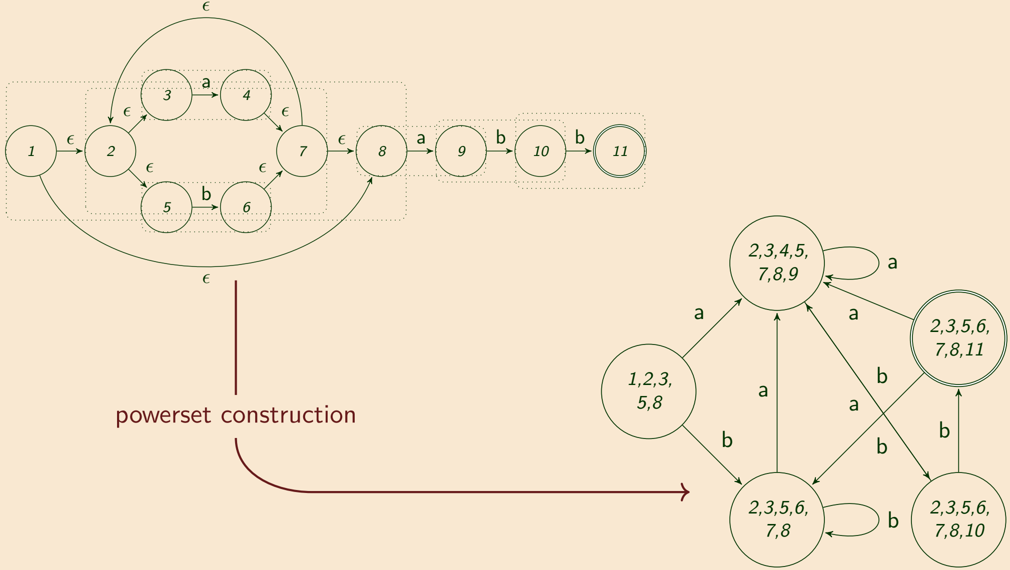
$$\epsilon\text{-closure}(S) \quad = \quad \{q' \in Q \mid \exists q \in S, q \xrightarrow{\epsilon} q'\}$$

How do we compute ϵ-closure(S)?

```
$\epsilon$-closure:
    push all elements of S onto a stack
    result := S
    while stack not empty
        pop $q$ off the stack
        for each $u \in \delta(q, \epsilon)$
            if $u \notin$ result
            then result := $\{u\} \cup$ result
                push $u$ on stack
    return result
```

(NB: just an instance of transitive closure)

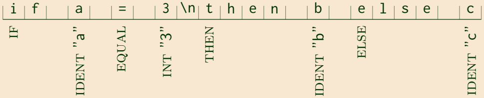
DFA(N($(a \vee b) * abb$))


powerset construction

The lexing problem

The lexing problem

The regular language problem (i.e. “is $w \in L(e)$?”) is insufficient for lexing.
We need to tokenize a string using a lexer specification



if ⇒ IF
. . .
[a-zA-Z]+ **as** s ⇒ IDENT s
[0-9]+ **as** i ⇒ INT i
[\t\n] ⇒ *skip*

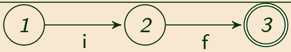
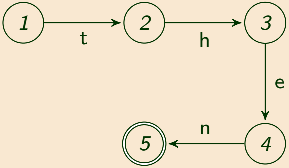
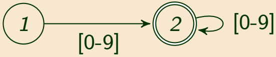
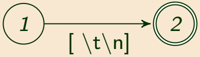
taking into account that

Expressions are ordered by priority.
(treat `if` as a keyword because the IF rule comes before the IDENT rule)

We should find the longest match.
(treat `ifif` as a variable, not two keywords)

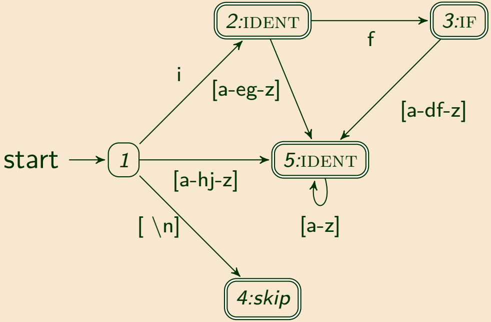
We should skip whitespace.
(because whitespace is irrelevant to the parser)

Define tokens with regexes (automata)

Regex	Finite automaton	Token
`if`		IF
`then`		THEN
`[a-zA-Z][a-zA-Z0-9]*`		IDENT s
`[0-9][0-9]*`		INT n
`[ \t\n]`		*skip* (not really a token)

Constructing a Lexer

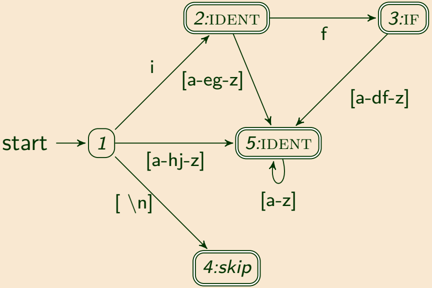
Input: $e_1 \Rightarrow t_1, e_2 \Rightarrow t_2, \ldots, e_k \Rightarrow t_k$, priority-ordered lexing rules, highest first
$\Rightarrow$ Tagged NFA for $e = e_1 \vee e_2 \vee \ldots \vee e_k$
$\Rightarrow$ DFA with each accepting state tagged for the e_i of highest priority.



lexer rules
if ⇒ IF
[a-z]+ **as** s ⇒ IDENT s
[\n] ⇒ *skip*

State 3 could be either an IDENT or the keyword IF. The priority rule eliminates this ambiguity, associating state 3 with the keyword.

What about longest match?



Start in initial state, and repeatedly:

1. read input until failure (no transition). Emit tag for last accepting state.
2. reset state to start state

(| = current position, $ = EOF)

<table>
<tr><th>Input</th><th>Current state</th><th>Last accepting state</th><th>Action</th></tr>
<tr><td>|if ifx$</td><td>1</td><td>⊥</td><td></td></tr>
<tr><td>i|f ifx$</td><td>2</td><td>2</td><td></td></tr>
<tr><td>if| ifx$</td><td>3</td><td>3</td><td></td></tr>
<tr><td>if |ifx$</td><td>⊥</td><td>3</td><td>emit IF</td></tr>
<tr><td>if| ifx$</td><td>1</td><td>⊥</td><td>reset</td></tr>
<tr><td>if |ifx$</td><td>4</td><td>4</td><td></td></tr>
<tr><td>if i|fx$</td><td>⊥</td><td>4</td><td>*skip*</td></tr>
<tr><td>if |ifx$</td><td>1</td><td>⊥</td><td>reset</td></tr>
<tr><td>if i|fx$</td><td>2</td><td>2</td><td></td></tr>
<tr><td>if if|x$</td><td>3</td><td>3</td><td></td></tr>
<tr><td>if ifx|$</td><td>5</td><td>5</td><td></td></tr>
<tr><td>if ifx$|</td><td>⊥</td><td>5</td><td>emit IDENT "ifx"</td></tr>
</table>

Lexing with derivatives

Matching with derivatives

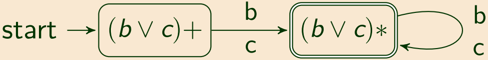
Brzozowski (1964)'s formulation of regex matching, based on *derivatives*.

Derivative of regex r w.r.t. character c is
another regex $\partial_c\, r$ that matches s iff r matches cs.

E.g.: consider $(b \vee c)+$. After matching c, can accept either ϵ or more b/c, so:

$$\partial_c\,(b \vee c)+ \quad = \quad \epsilon \vee (b \vee c)+ \quad = \quad (b \vee c)*$$

Construct DFA for r, taking regexes r as states, adding transition $r_i \xrightarrow{c} r_j$ whenever $\partial_c\, r_i = r_j$. For example, for $(b \vee c)+$:



NB: $\partial_c\,(b \vee c)* = (b \vee c)*$. (Can you see why?) Also: ϵ-matching states are accepting.

Defining ∂_c

∂_c is defined inductively over regexes.

Can you see the similarities with derivatives of numerical functions?
(Hint: read $r_1 r_2$ as $r_1 \times r_2$ and $r_1 \vee r_2$ as $r_1 + r_2$.)

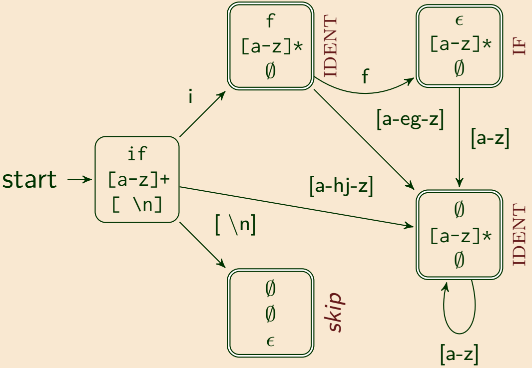
$$\begin{aligned}
\partial_c\, \emptyset &= \emptyset \\
\partial_c\, \epsilon &= \emptyset \\
\partial_c\, b &= \emptyset \\
\partial_c\, c &= \epsilon \\
\partial_c\, (rs) &= (\partial_c\, r)s \mid \nu(r)(\partial_c\, s) \\
\partial_c\, (r \vee s) &= \partial_c\, r \vee \partial_c\, s \\
\partial_c\, r* &= (\partial_c\, r)r*
\end{aligned}$$

$$\begin{aligned}
\nu(r) &= \epsilon \text{ if } \epsilon \in L(r) \\
&= \emptyset \text{ if } \epsilon \notin L(r)
\end{aligned}$$

More information: Regular-expression derivatives re-examined (Owens et al, 2009).

Lexing with derivatives

Lexers match input string against multiple regexes in parallel. Automaton for matching one token; each state corresponds to vector of regexes, one per lexer rule. ∂_c acts pointwise on the regex vector.



Next time: context-free grammars