Quantum Computing: Exercise Sheet 4

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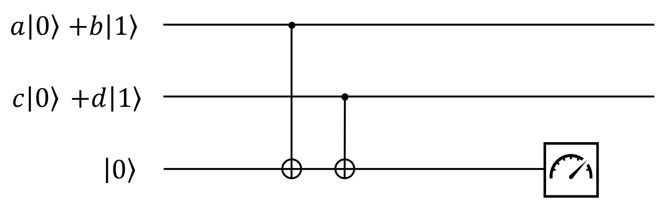
1. If a repetition code is used to encode a single bit, which is then sent through a binary symmetric channel with error probability p_e, following which a majority vote is used for correction, what is the probability of error after this correction when:
   (a) A five-bit repetition code is used.
   (b) A seven-bit repetition code is used.
   (c) A n-bit repetition code is used (for some odd n).

2. If a qubit experiences a bit-flip and a phase-flip, then show that the order in which these occur doesn't matter by showing that the Pauli-X (bit-flip) and Pauli-Z (phase-flip) matrices commute (up to a global phase difference).

3. For the circuit shown below, what are the possible post-measurement states, and with what probability does each occur?



4. Let a three-qubit system be prepared in the state $\alpha|000\rangle + \beta|111\rangle$. Suppose all three qubits experience a bit-flip in the noisy channel of circuit shown on Slide 7 of lecture 13, which aims to detect and correct a single bit-flip:
   (a) What is the final state?
   (b) What is the final state if only the first two qubits experience a bit-flip?

5. (a) Show that the $(7,4)$ Hamming code parity-check matrix (shown on Slide 21 of lecture 13) outputs 000 when applied to any valid codeword.
   (b) Show that if the $(7,4)$ Hamming code parity-check is applied to a corrupted codeword that differs from a valid codeword by a single bit, then the output of the parity-check is not 000. For each of the seven possible single bit-flips give the output of the parity-check, and comment on your result.

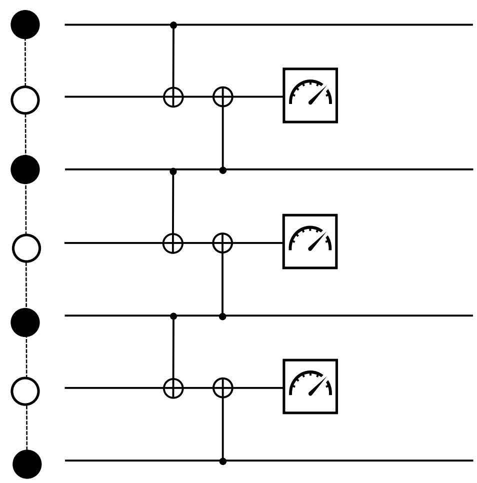
6. Noting that the Shor code encodes an arbitrary qubit $\alpha|0\rangle + \beta|1\rangle$ in 9 qubits as:

$$\frac{1}{2\sqrt{2}}\Big(\alpha(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)$$
$$+ \beta(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)\Big)$$

   show that the circuit on Slide 13 of lecture 13 can detect and correct a phase flip on the first qubit.

7. Show that, if a transversal implementation of the T gate for the Steane code is attempted by applying n T gates to each of the seven qubits in the code, then no value of n will correctly implement the gate.

8. Shown below is a line of a surface code which performs bit-flip checks. The four black circles represent data-qubits, which encode the state $\alpha|0000\rangle + \beta|1111\rangle$, and the three white circles represent parity-check ancillas, which check for bit-flips (i.e., by the parity-check circuit also shown.



Give: the post-error state; parity check bits; the detected error; and corrected state (if correction is possible) in the following scenarios:
(a) The top qubit experiences a bit-flip?
(b) The top two qubits experience bit-flips?
(c) The top and bottom qubits experience bit-flips?
(d) All four qubits experience bit-flips?

9. One of the criticisms of D-Wave is that its qubits have very poor coherence. Is this likely to be a major problem for the typical applications of D-Wave?

10. Which physical realisation of a qubit do you think will come to dominate quantum computing in the future?