3D Geometry Capture

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Sources of Geometry

Acquisition from the real world

Modeling applications
The West Cambridge Digital Twin project
Shape Acquisition

• Digitizing real world objects
3D Scanning

Touch Probes
+ Precise
- Small objects

Optical Scanning
+ Fast
- Glossy objects

Active

Passive
Active Systems

• Triangulation Laser
  – Laser beam and camera
  – Laser dot is photographed
  – The location of the dot in the image allows triangulation: we get the distance to the object
Active Systems

Structured light
Active Systems

Structured light
Active Systems

• Structured light
  – Pattern of visible or *infrared* light is projected onto the object
  – The distortion of the pattern (recorded by the camera) provides geometric information
  – Very fast – 2D pattern at once
  – Complex distance calculation → prone to noise
Active Systems

• LIDAR
  – Measures the time it takes the laser beam to hit the object and come back
Passive Systems

Multi-view Stereo

(x, y, z)

(x_L, y_L)

(x_R, y_R)

Left camera focal point

Right camera focal point

Left camera projection plane

Right camera projection plane

Epipolar line
3D Imaging

- Ultrasound, CT, MRI
- Discrete volume of density data
- First need to segment the desired object (contouring)
3D Scanning

- Challenges

Noise, outliers, irregularity
Incompleteness
Inconsistency
Shape Acquisition

• Digitizing real world objects

3D Scanning → Registration → Pre-processing → Reconstruction
Registration

• Bringing scans into a common coordinate frame
Registration

\[ M_1 \approx T(M_2), T: \text{ translation + rotation} \]
Registration

\[ M_1 \approx T_2(M_2) \approx \cdots \approx T_n(M_n) \]
• How many points are needed to define a unique rigid transformation?

• The first problem is finding pairs

\[ p_1 \rightarrow q_1 \]
\[ p_2 \rightarrow q_2 \]
\[ p_3 \rightarrow q_3 \]

\[ Rp_i + t \approx q_i \]
Registration

• ICP: Iterative Closest Point
• Idea: Iterate
  – (1) Find correspondences
  – (2) Use them to find a transformation
Registration

- ICP: Iterative Closest Point
- Intuition:
  - With the right correspondences, problem solved
  - If you don’t have the right ones, can still make progress
Registration

• ICP: Iterative Closest Point
Registration

- ICP: Iterative Closest Point -- algorithm
  - Select (e.g., 1000) random points
  - Match each to closest point on other scan
  - Reject pairs with distance too big
  - Construct error function:

\[
E := \sum_i (Rp_i + t - q_i)^2
\]

- Minimize
  - closed form solution in: [http://dl.acm.org/citation.cfm?id=250160](http://dl.acm.org/citation.cfm?id=250160)
Shape Acquisition

• Digitizing real world objects

3D Scanning ➔ Registration ➔ Pre-processing ➔ Reconstruction
Pre-processing

• Cleaning, repairing, resampling
Pre-processing

• Sampling for accurate reconstructions
Reconstruction

• Mathematical representation for a shape
Reconstruction

Connect-the-points Methods

+ Theoretical error bounds
– Expensive
– Not robust to noise

Approximation-based Methods

+ Efficient to compute
+ Robust to noise
– No theoretical error bounds
Reconstruction

• Approximating an implicit function

\[ f : \mathbb{R}^3 \rightarrow \mathbb{R} \]

with value > 0 outside the shape and < 0 inside
Reconstruction

• Approximating an implicit function

\[ f : \mathbb{R}^3 \rightarrow \mathbb{R} \]

with value \( > 0 \) outside the shape and \( < 0 \) inside

\( \{ \mathbf{x} : f(\mathbf{x}) = 0 \} \)

extract zero set
Least Squares

- Problem

\[ f(x) = ax + b \]
Least Squares

- Problem

\[ f(x) = ax + b \]

\[ \min_{a,b} \sum_{i=1}^{n} (f(x_i) - y_i)^2 \]

\[ \min_{a,b} \sum_{i=1}^{n} (ax_i + b - y_i)^2 \]
Least Squares

- Multi-dimensional problem

\[ f(x) : \mathbb{R}^d \rightarrow \mathbb{R} \quad \min_{f \in \Pi^d_m} \sum_{i} (f(x_i) - f_i)^2 \]

\[ \Pi^d_m : \text{polynomials of degree } m \text{ in } d \text{ dimensions} \]

\[ f(x) = b(x)^T c \]

\[ m = 2, \ d = 2 \quad b(x) = \begin{bmatrix} 1 & x & y & x^2 & y^2 & xy \end{bmatrix}^T \]

\[ f(x) = c_0 + c_1 x + c_2 y + c_3 x^2 + c_4 y^2 + c_5 xy \]
Least Squares

• Multi-dimensional problem

\[ f(x) : \mathbb{R}^d \rightarrow \mathbb{R} \]

\[ \min_{f \in \Pi_m} \sum_{i} (f(x_i) - f_i)^2 \]

\[ f(x) = b(x)^T c \]

\[
\begin{align*}
\min_c E(c) & \quad E(c) = \sum_i (b(x_i)^T c - f_i)^2
\end{align*}
\]
Least Squares

- Multi-dimensional problem

\[
\min_{\mathbf{c}} E(\mathbf{c}) \quad E(\mathbf{c}) = \sum_i (\mathbf{b}(\mathbf{x}_i)^T \mathbf{c} - f_i)^2
\]

\( m = 1, d = 1 \)

\[
E(\mathbf{c}) = \sum_i (c_0 + c_1 x_i - f_i)^2
\]

\( m = 2, d = 1 \)

\[
E(\mathbf{c}) = \sum_i (c_0 + c_1 x_i + c_2 x_i^2 - f_i)^2
\]
Least Squares

- Multi-dimensional problem

\[
\min_c E(c) \quad E(c) = \sum_i (b(x_i)^T c - f_i)^2
\]

\[
E(c) = \sum_i (c_0 + c_1 x + c_2 y + c_3 x^2 + c_4 y^2 + c_5 xy - f_i)^2
\]

\[m = 2, \quad d = 2\]
Least Squares

• Solution of the multi-dimensional problem

\[
\min_{\mathbf{c}} E(\mathbf{c}) \quad E(\mathbf{c}) = \sum_i (\mathbf{b}(\mathbf{x}_i)^T \mathbf{c} - f_i)^2 \quad \mathbf{b}(\mathbf{x}_i) = [b_1(\mathbf{x}_i) \cdots b_m(\mathbf{x}_i)]^T
\]

\[
\frac{\partial E(\mathbf{c})}{\partial c_k} = \sum_i 2b_k(\mathbf{x}_i) [\mathbf{b}(\mathbf{x}_i)^T \mathbf{c} - f_i] = 0
\]

\[
\frac{\partial E(\mathbf{c})}{\partial \mathbf{c}} = 2 \sum_i \mathbf{b}(\mathbf{x}_i) [\mathbf{b}(\mathbf{x}_i)^T \mathbf{c} - f_i] = 0
\]

\[
\sum_i \mathbf{b}(\mathbf{x}_i)\mathbf{b}(\mathbf{x}_i)^T \mathbf{c} = \sum_i \mathbf{b}(\mathbf{x}_i)f_i 
\]

\[
c = \left[ \sum_i \mathbf{b}(\mathbf{x}_i)\mathbf{b}(\mathbf{x}_i)^T \right]^{-1} \sum_i \mathbf{b}(\mathbf{x}_i)f_i
\]
Least Squares

• Solution of the multi-dimensional problem

Example

\[ m = 2, d = 1 \quad E(c) = \sum_i (c_0 + c_1x + c_2x^2 - f_i)^2 \]

\[
\sum_i \begin{bmatrix} 1 & x_i & x_i^2 \\ x_i & x_i^2 & x_i^3 \\ x_i^2 & x_i^3 & x_i^4 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \sum_i \begin{bmatrix} 1 \\ x_i \\ x_i^2 \end{bmatrix} f_i
\]
Weighted Least Squares

• Multiply the terms with given weights

\[ \text{LS} \quad \min_c E(c) \quad E(c) = \sum_i \left( (b(x_i)^T c - f_i)^2 \right) \]

\[ \text{WLS} \quad \min_c E(c) \quad E(c) = \sum_i \left( (b(x_i)^T c - f_i)^2 w_i \right) \]
Moving Least Squares

• Idea: make the weights local
Moving Least Squares

- Idea: make the weights local

\[
of(x) = \min_{f_x \in \Pi^d_m} \sum_i \phi(||x - x_i||) (f_x(x_i) - f_i)^2
\]

Local approximation

Weights depend on x
Moving Least Squares

• Idea: make the weights local

\[ c(x) = \operatorname{argmin}_c \quad E_x(c) = \sum_i \phi(||x - x_i||) \left( b(x_i)^T c - f_i \right)^2 \]

\[ f(x) = b(x)^T c(x) \]

In comparison, LS:

\[ c = \operatorname{argmin}_c \quad E(c) = \sum_i \left( b(x_i)^T c - f_i \right)^2 \]

\[ f(x) = b(x)^T c \]
Moving Least Squares

• Local solution

\[ c(x) = \left[ \sum_i \phi_i(x)b(x_i)b(x_i)^T \right]^{-1} \sum_i \phi_i(x)b(x_i)f_i \]

\[ \phi_i(x) = \phi(||x - x_i||) \]

\[ f(x) = b(x)^T c(x) \]
Moving Least Squares

• Local solution

Example $m = 1, d = 1$

$$\min_{c_0, c_1} \sum_i \phi_i(x) (c_0 + c_1 x_i - f_i)^2$$

$$f_x(x) = c_0 + c_1 x$$

$$f(x) = f_x(x)$$
Implicit MLS Surfaces

- Basic problem
  - Given sample points & attributes
  - Compute a function
    \[ f(x) : \mathbb{R}^2 \text{ or } \mathbb{R}^3 \rightarrow \mathbb{R} \]
  - such that the curve/surface is given by
    \[ S = \{x | f(x) = 0, \nabla f(x) \neq 0\} \]
Implicit MLS Surfaces

Sample Points

Query Point

Neighborhood defined by \( \phi(||x - \cdot||) \)

\[
f(x) = f_x(x)
\]
Implicit MLS Surfaces

Example $m = 1, d = 2$

$$f_x(x) = c_0(x) + c_1(x)x + c_2(x)y$$
Implicit MLS Surfaces

How can we avoid the trivial solution

$$f(x) = 0 \ \forall x$$

Gradient constraints

$$\| \nabla f(x) \| = 1 \quad \nabla f(x_i) = n_i$$

Reproduce local functions

$$f_i(x) = n_i^T (x - x_i)$$
Implicit MLS Surfaces

- Example

\[ m = 1, d = 2 \]

\[ f_x(x) = n_x^T x + o_x \quad \|n_x\| = 1 \]

\[ (n_x, o_x) = \arg\min_{n,o} \sum_i \phi_i(x) \left( n_{x_i}^T x_i + o \right)^2 \quad \|n\| = 1 \]
Implicit MLS Surfaces

• Examples in 3D

Feature Preserving Point Set Surfaces based on Non-Linear Kernel Regression, Eurographics 2009
Implicit MLS Surfaces

• Examples in 3D

Spatio-Temporal Geometry Fusion for Multiple Hybrid Cameras using Moving Least Squares Surfaces, Eurographics 2014
Shape Acquisition

• Digitizing real world objects

3D Scanning → Registration → Pre-processing → Reconstruction
Neural Radiance Fields

Representing Scenes as Neural Radiance Fields for View Synthesis
Neural Radiance Fields

\[(x, y, z, \theta, \phi) \rightarrow F_\Theta \rightarrow (RGB, \sigma)\]
Neural Radiance Fields
Neural Radiance Fields

pixelNeRF: Neural Radiance Fields from One or Few Images