Introduction to Neural Networks

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Parametric approach

Image

W * H * 3 numbers

$$f(x, W)$$

Cat	0.1
Dog	0.8
Person	0.02
Road	0.01
Tree	0.01
Airplane	0.01
Apple	0.01
Cell phone	0.03
Cup	0
Table	0.01

10 numbers for class scores

Parametric approach

If W = 32, H = 32,
x = 32 * 32 * 3 = 3072

$$f(x, W) = Wx$$

Image

W * H * 3 numbers

$$f(x, W)$$

Cat	0.1
Dog	0.8
Person	0.02
Road	0.01
Tree	0.01
Airplane	0.01
Apple	0.01
Cell phone	0.03
Cup	0
Table	0.01

10 numbers for class scores

Parametric approach

If W = 32, H = 32,
x = 32 * 32 * 3 = 3072

$$f(x, W) = Wx \quad 3072 \times 1$$

$$10 \times 1 \qquad 10 \times 3072$$

Image

W * H * 3 numbers

$$f(x, W)$$

Cat	0.1
Dog	0.8
Person	0.02
Road	0.01
Tree	0.01
Airplane	0.01
Apple	0.01
Cell phone	0.03
Cup	0
Table	0.01

10 numbers for class scores

Parametric approach

If W = 32, H = 32,
x = 32 * 32 * 3 = 3072

$$f(x, W) = Wx + b$$

10×1 $\qquad$ 10×1

Image

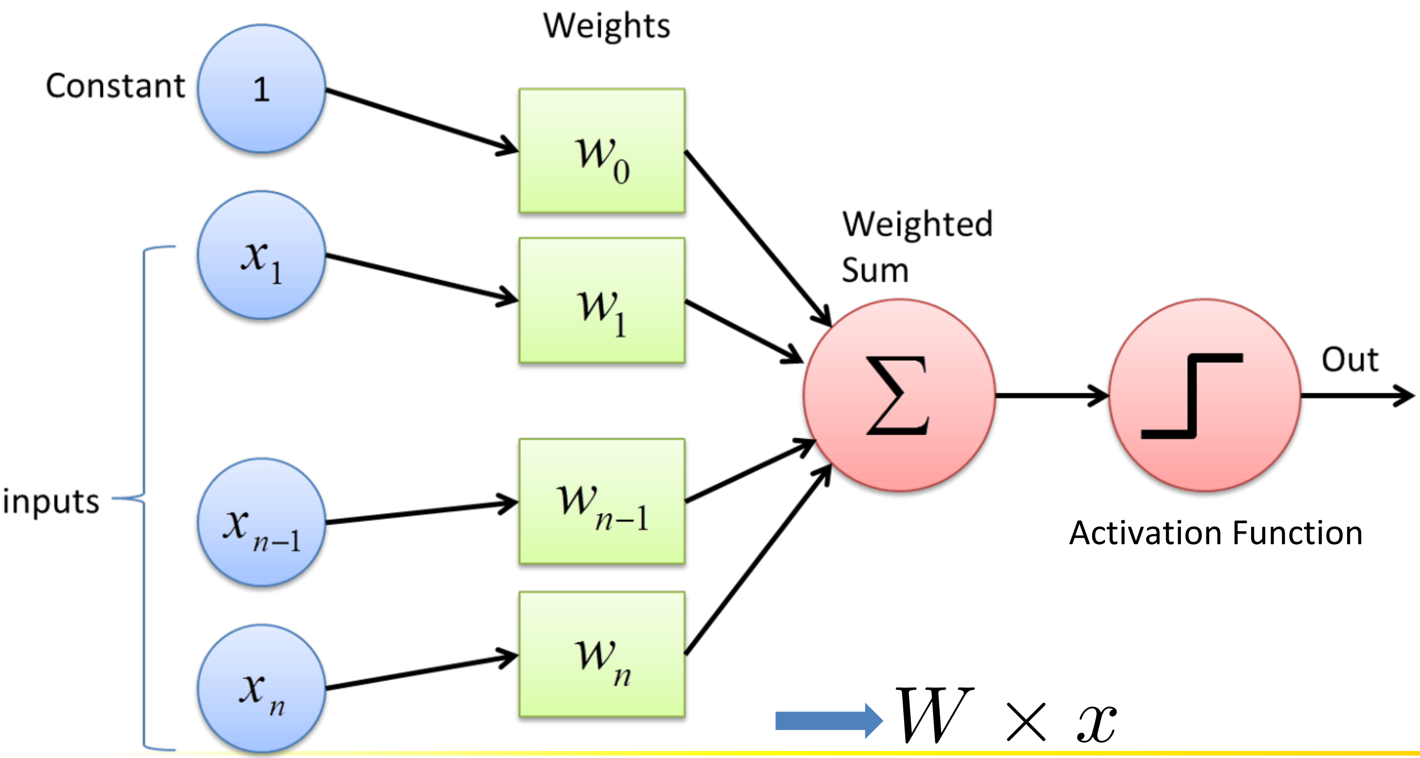
W * H * 3 numbers

$$f(x, W)$$

Cat	0.1
Dog	0.8
Person	0.02
Road	0.01
Tree	0.01
Airplane	0.01
Apple	0.01
Cell phone	0.03
Cup	0
Table	0.01

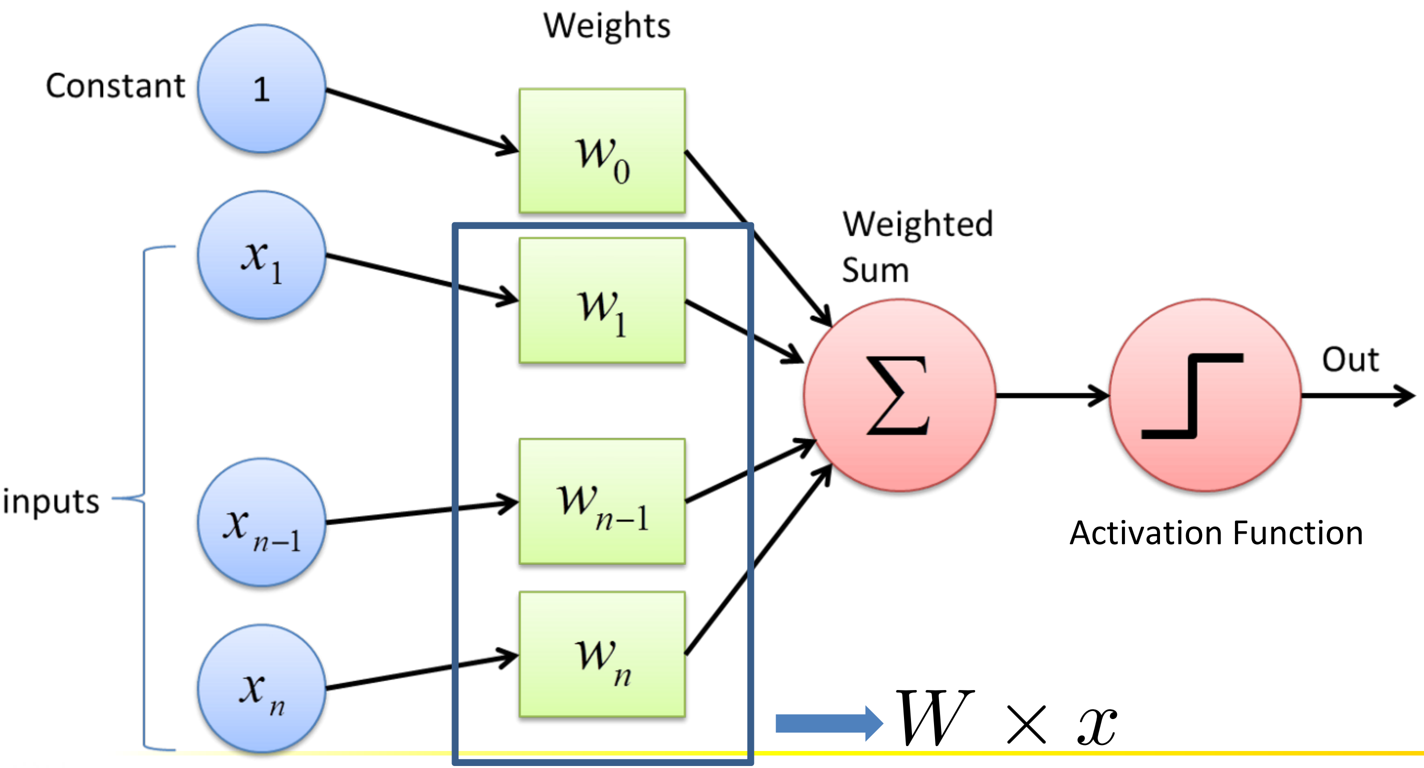
10 numbers for class scores

Perceptron

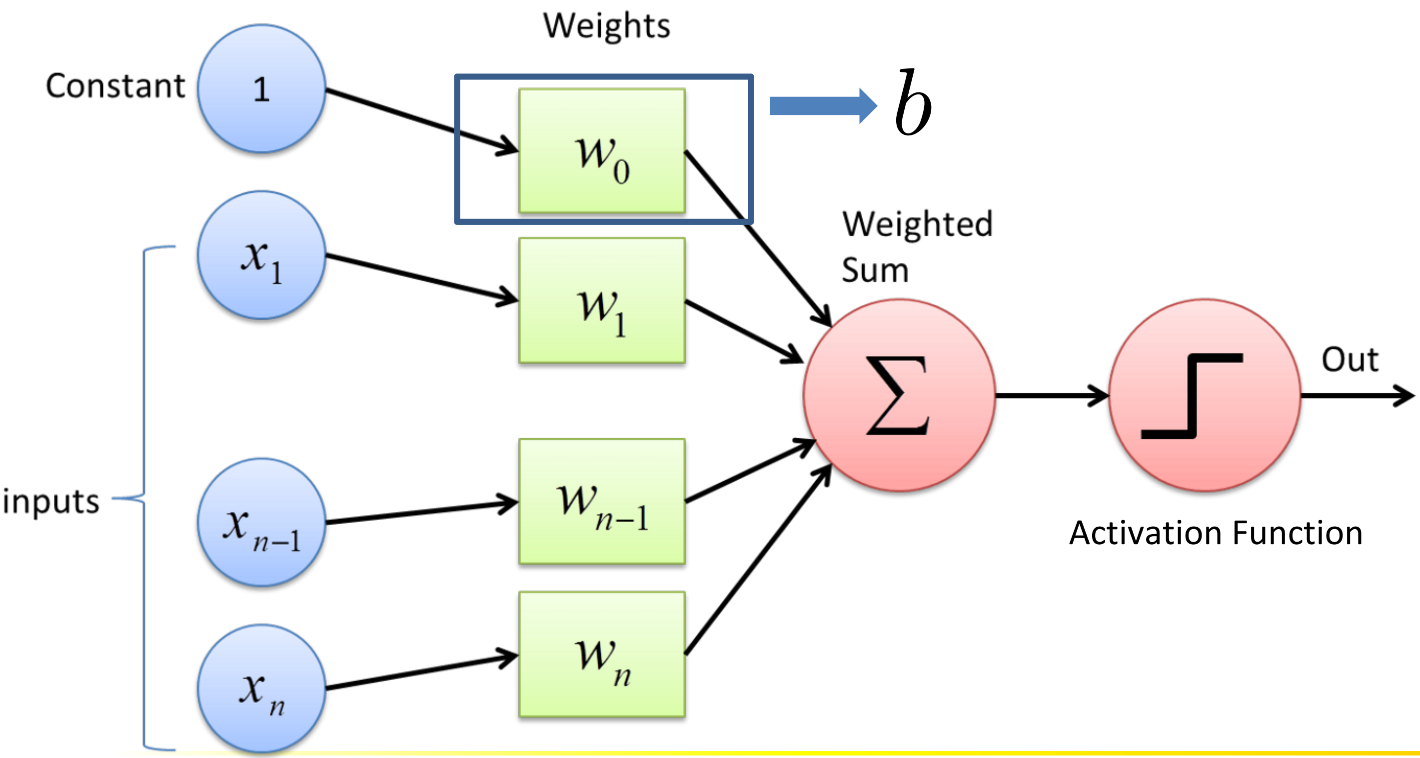


$$W \times x$$

Perceptron


Weights
Constant
1
w_0
x_1
w_1
Weighted
Sum
Σ
Out
inputs
x_{n-1}
w_{n-1}
x_n
w_n
Activation Function
$W \times x$

Perceptron


Weights
Constant
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w_0
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Weighted Sum
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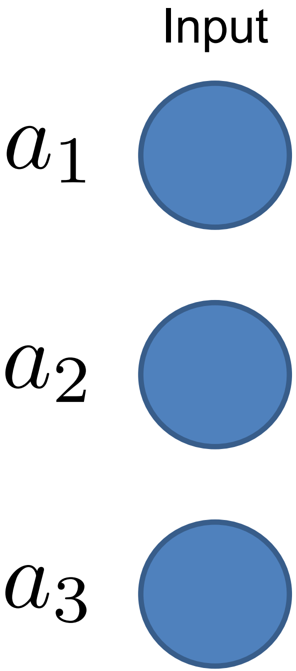
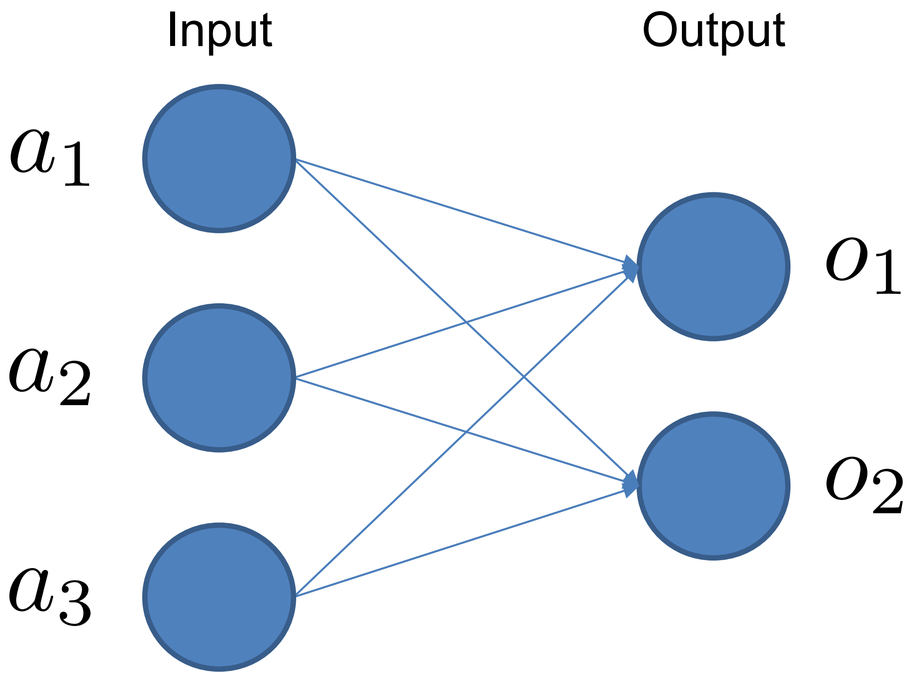
inputs
x_{n-1}
w_{n-1}
x_n
w_n
Activation Function

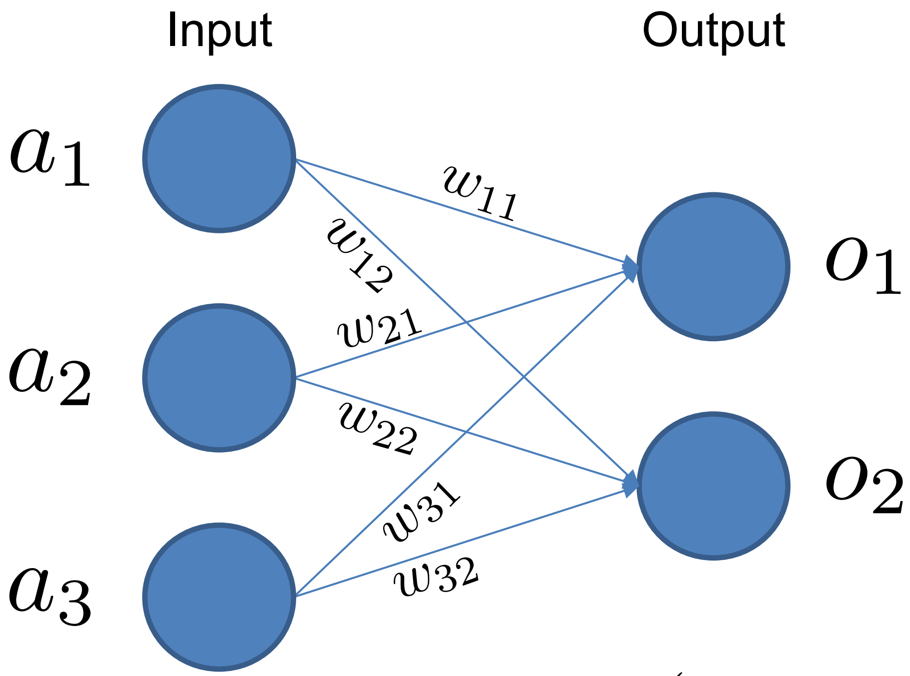
Two-layer Perceptron



Two-layer Perceptron

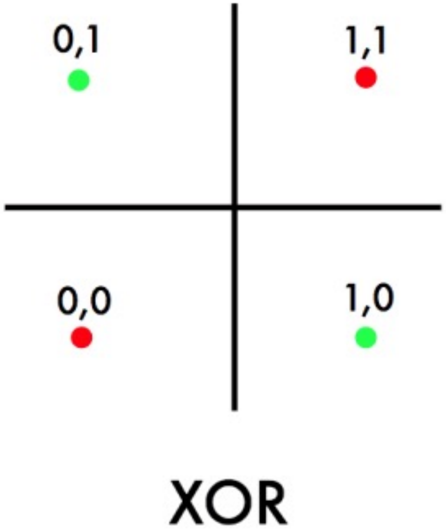


Two-layer Perceptron

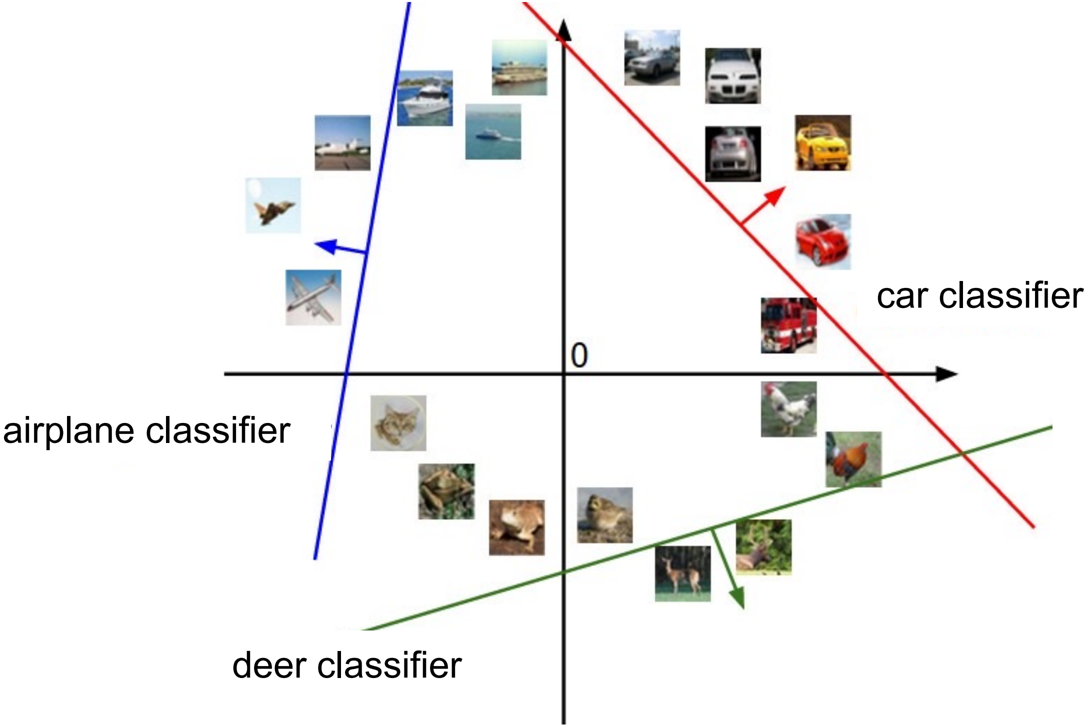


$$o_1 = g(w_{11} * a_1 + w_{12} * a_2 + w_{13} * a_3)$$

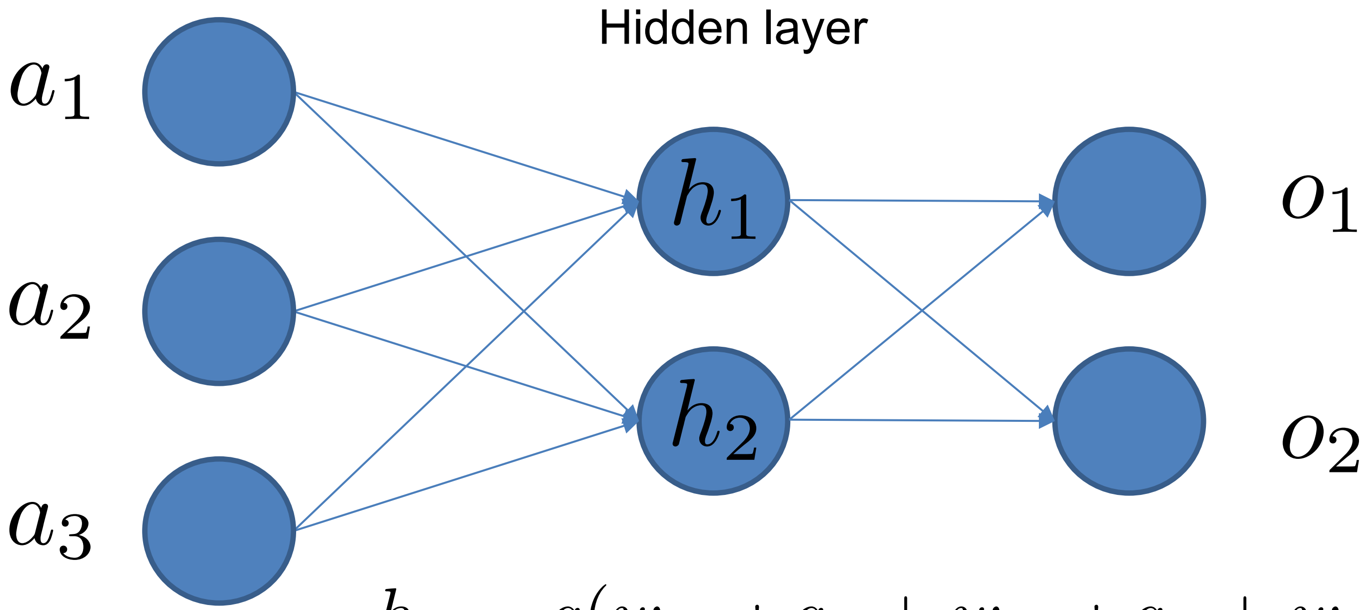
AI Winter



Can not solve!

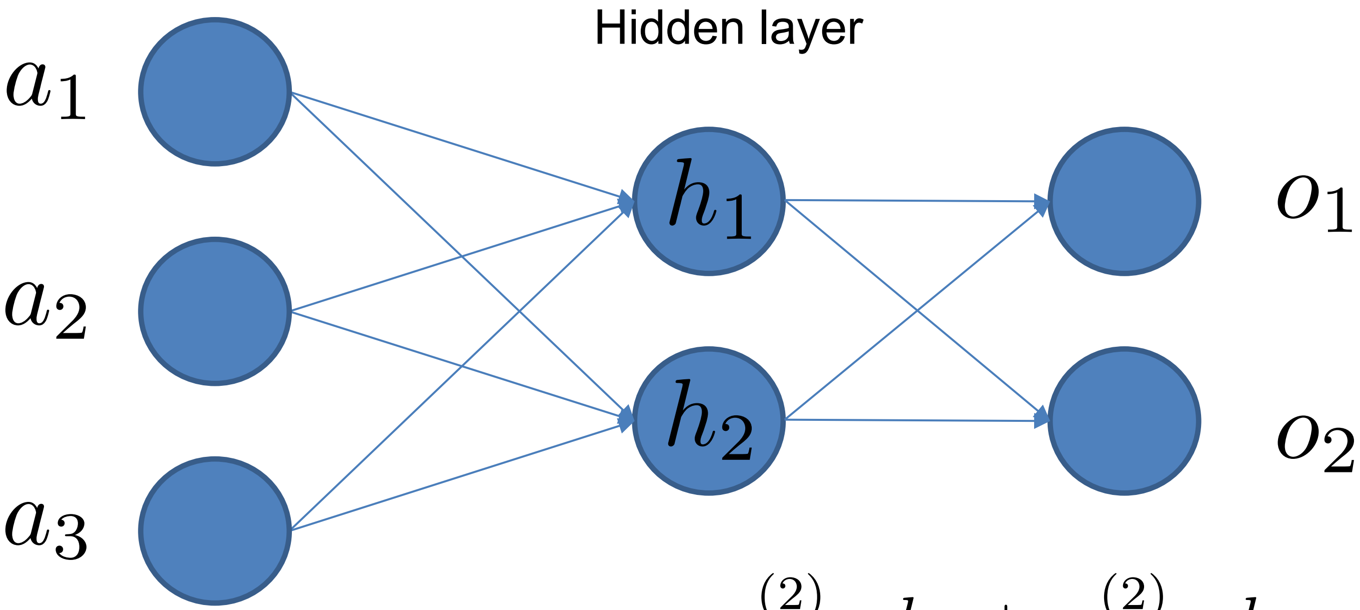


Multi-layer Perceptron



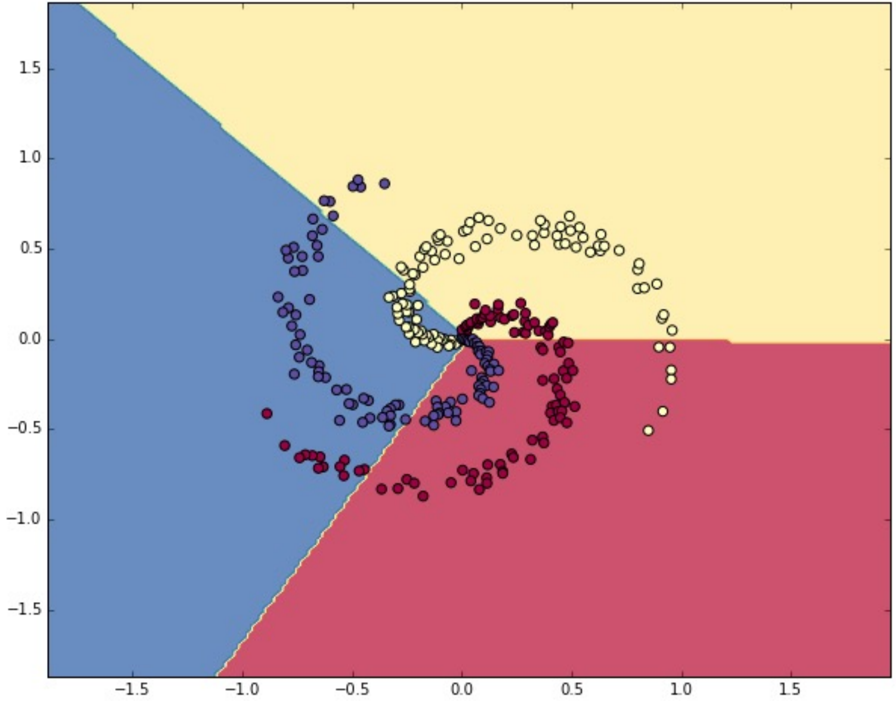
$$h_1 = g(w_{11} * a_1 + w_{12} * a_2 + w_{13} * a_3)$$

Multi-layer Perceptron

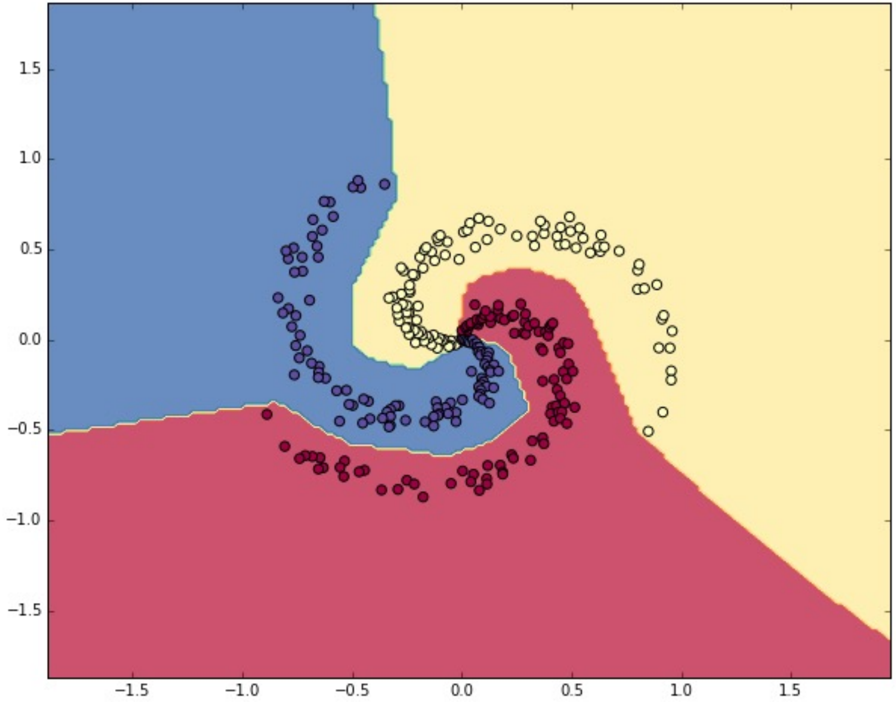


$$o_1 = w_{11}^{(2)} * h_1 + w_{21}^{(2)} * h_2$$

Comparison

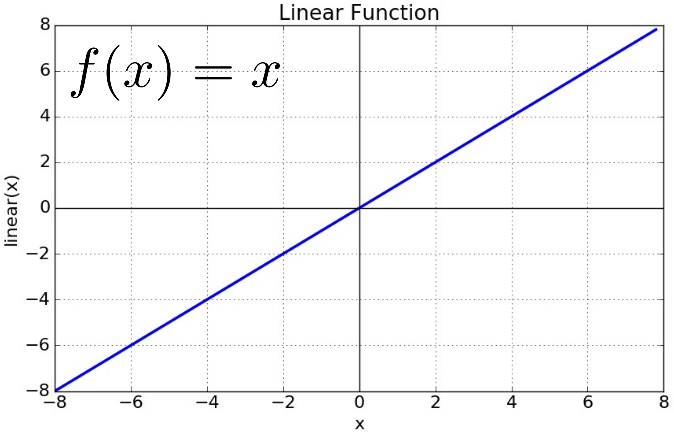


Without hidden layers

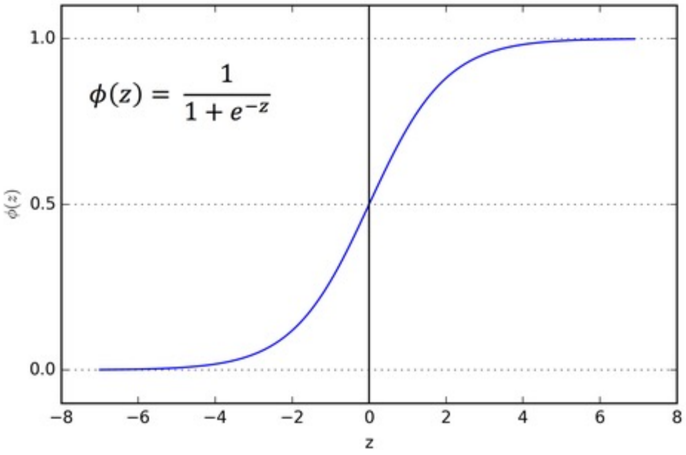


With hidden layers

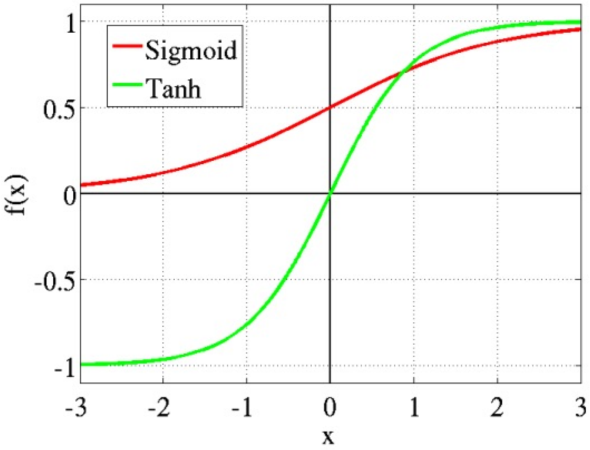
Activation Functions



Sigmoid

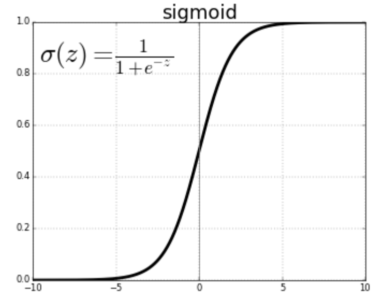


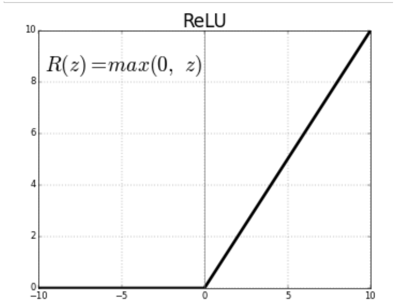
Tanh



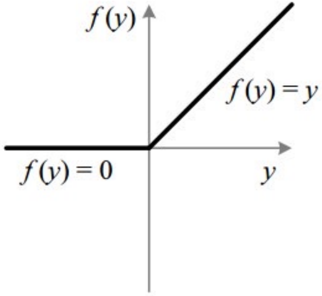
Activation Functions

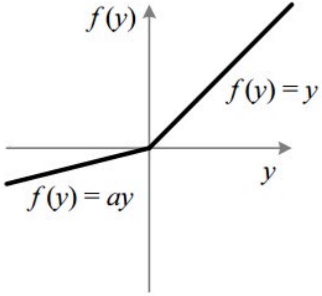
Sigmoid and ReLU





ReLU and Leaky ReLU





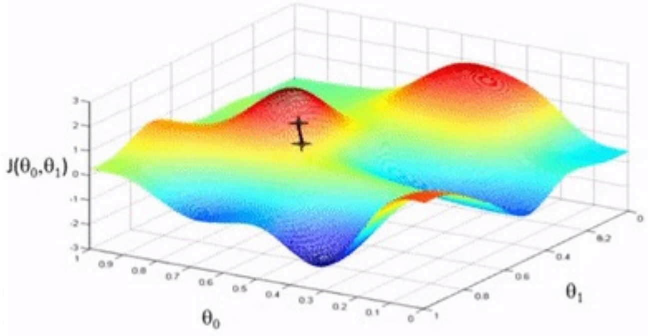
Gradient Decent

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

Parameters: θ_0, θ_1

Cost function: $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$

Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$



Gradient Descent

$$\theta_i = \theta_i - \underbrace{\boxed{\alpha}}_{\text{Learning rate}} * \underbrace{\boxed{\frac{\partial L}{\partial \theta_i}}}_{\text{Gradient}}$$

Momentum & RMSProp

Momentum: accelerates the training process

$$v_i = \gamma v_i + \alpha * \frac{\partial L}{\partial \theta_i}$$

$$\theta_i = \theta_i - v_i$$

RMSProp: adjust the learning rate

$$v_i = \beta v_i + (1 - \beta) * \frac{\partial L}{\partial \theta_i}$$

$$\theta_i = \theta_i - \alpha \frac{\frac{\partial L}{\partial \theta_i}}{\sqrt{v_i} + \epsilon}$$

Source

Adam

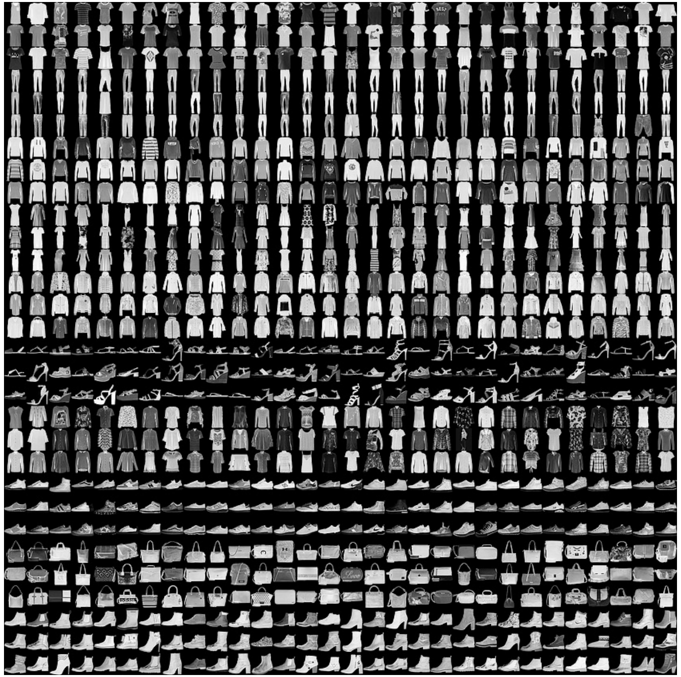
Require: α: Stepsize
Require: $\beta_1, \beta_2 \in [0, 1)$: Exponential decay rates for the moment estimates
Require: $f(\theta)$: Stochastic objective function with parameters θ
Require: θ_0: Initial parameter vector

$m_0 \leftarrow 0$ (Initialize 1st moment vector)
$v_0 \leftarrow 0$ (Initialize 2nd moment vector)
$t \leftarrow 0$ (Initialize timestep)
while θ_t not converged **do**
  $t \leftarrow t + 1$
  $g_t \leftarrow \nabla_\theta f_t(\theta_{t-1})$ (Get gradients w.r.t. stochastic objective at timestep t)
  $m_t \leftarrow \beta_1 \cdot m_{t-1} + (1 - \beta_1) \cdot g_t$ (Update biased first moment estimate)
  $v_t \leftarrow \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g_t^2$ (Update biased second raw moment estimate)
  $\widehat{m}_t \leftarrow m_t / (1 - \beta_1^t)$ (Compute bias-corrected first moment estimate)
  $\widehat{v}_t \leftarrow v_t / (1 - \beta_2^t)$ (Compute bias-corrected second raw moment estimate)
  $\theta_t \leftarrow \theta_{t-1} - \alpha \cdot \widehat{m}_t / (\sqrt{\widehat{v}_t} + \epsilon)$ (Update parameters)
end while
return θ_t (Resulting parameters)

Kingma, Diederik P., and Jimmy Ba. "Adam: A method for stochastic optimization." *ICLR* (2015).

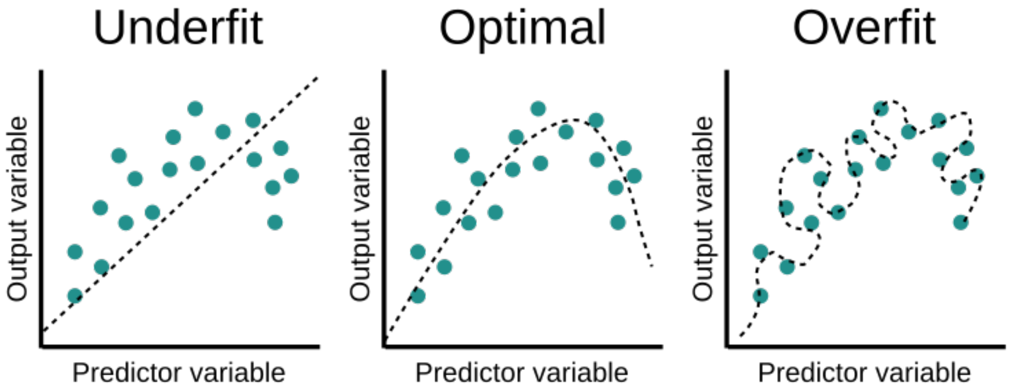
Example: Training a Classifier

- Data
  - MNIST Fashion
  - 70,000 grayscale images
  - 10 categories
  - 28 * 28 pixels

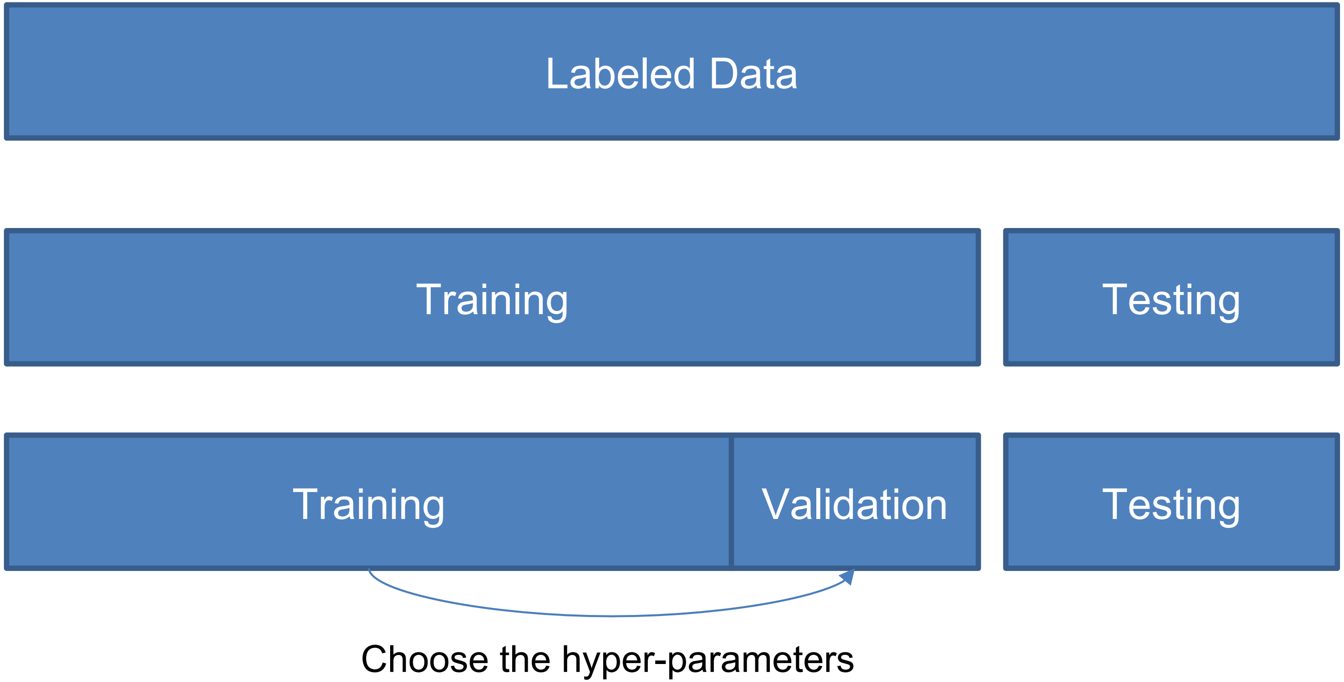


Code: https://colab.research.google.com/drive/1ETGBYYFFSp3RAerKE9WZkBXFxPa5GfYv?usp=sharing

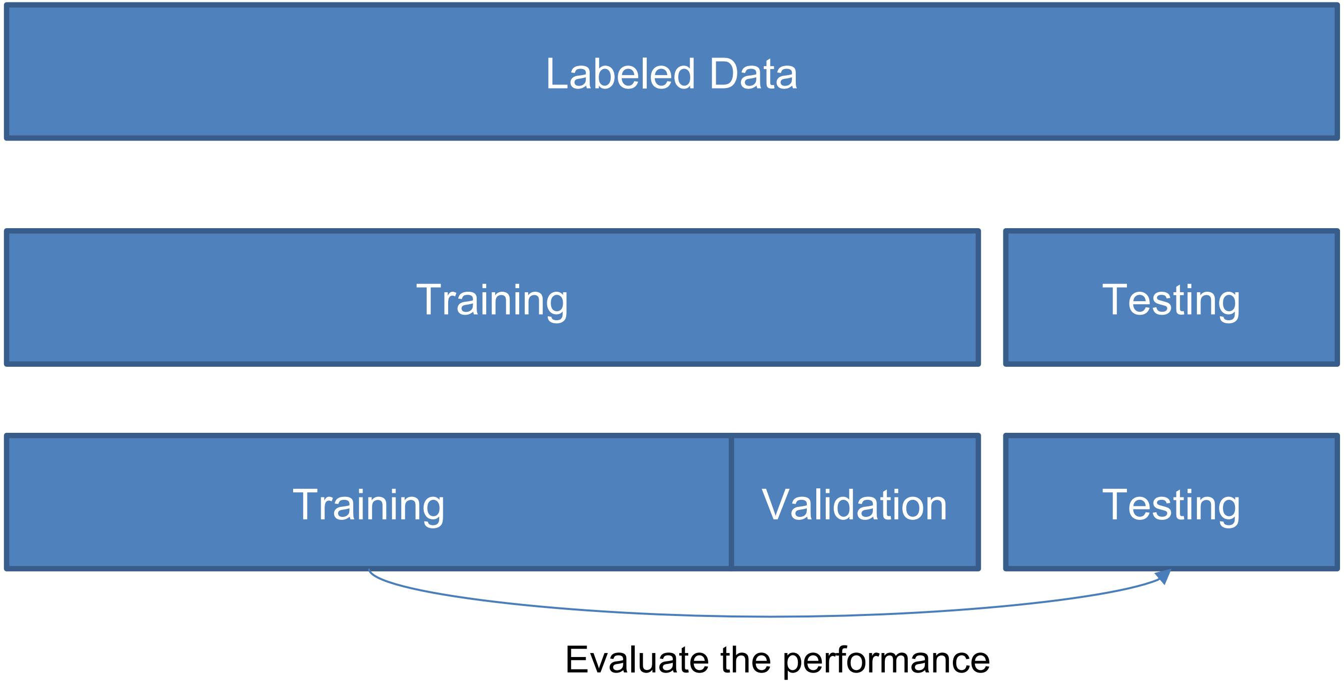
Overfitting vs. Underfitting



Train / Test / Val



Train / Test / Val



Example: Prepare the Data

```
fashion_mnist = tf.keras.datasets.fashion_mnist

(train_images, train_labels), (test_images, test_labels) =
fashion_mnist.load_data()
```

```
train_images.shape
```

output: (60000, 28, 28)

```
test_images.shape
```

output: (10000, 28, 28)

```
train_labels
```

output: array([9, 0, 0, ..., 3, 0, 5], dtype=uint8)

Example: Prepare the Data

```
fashion_mnist = tf

(train_images, tra                    t_labels) =
fashion_mnist.load
```

```
train_images.shape
```

output: (60000, 28, 28)

```
test_images.shape
```

output: (10000, 28, 28)

```
train_labels
```

output: array([9, 0, 0, ...,

Label	Class
0	T-shirt/top
1	Trouser
2	Pullover
3	Dress
4	Coat
5	Sandal
6	Shirt
7	Sneaker
8	Bag
9	Ankle boot

Example: Pre-process

```
train_images = train_images / 255.0

test_images = test_images / 255.0
```

Example: Build the Model

```
model = tf.keras.Sequential([
    tf.keras.layers.Flatten(input_shape=(28, 28)),
    tf.keras.layers.Dense(128, activation='relu'),
    tf.keras.layers.Dense(10)
])
```

Input layer

Example: Build the Model

```
model = tf.keras.Sequential([
    tf.keras.layers.Flatten(input_shape=(28, 28)),
    tf.keras.layers.Dense(128, activation='relu'),
    tf.keras.layers.Dense(10)
])
```

Hidden layer

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Example: Build the Model

```
model = tf.keras.Sequential([
    tf.keras.layers.Flatten(input_shape=(28, 28)),
    tf.keras.layers.Dense(128, activation='relu'),
    tf.keras.layers.Dense(10)
])
```

Output layer

Example: Optimizer

```
model.compile(
  optimizer=tf.keras.optimizers.Adam(learning_rate=1e-3),
  loss=tf.keras.losses.SparseCategoricalCrossentropy(from_logits=True),
  metrics=['accuracy'])
```

Learning rate:

- too large: unstable, hard to find the minimal
- too small: local minimal, training is too slow

Example: Optimizer

```
model.compile(
  optimizer=tf.keras.optimizers.Adam(learning_rate=1e-3),
  loss=tf.keras.losses.SparseCategoricalCrossentropy(from_logits=True),
  metrics=['accuracy'])
```

Cross-entropy is usually used to train a classification model:

$$\ell = \sum_{c=1}^{C} y_c * \log(y_c^{'})$$

Example: Train the Model

```
model.fit(train_images, train_labels, batch_size=128, epochs=10)
```

Batch size: the number of samples in one iteration

Example: Train the Model

```
model.fit(train_images, train_labels, batch_size=128, epochs=10)
```

Batch size: the number of samples in one iteration

Epoch: the number of passes over the entire training dataset

Example: Training Log

```
model.fit(train_images, train_labels, batch_size = 128, epochs=10)
```

```
Epoch 1/10
469/469 [==============================] - 2s 3ms/step - loss: 0.2359 - accuracy: 0.9127
Epoch 2/10
469/469 [==============================] - 1s 3ms/step - loss: 0.2282 - accuracy: 0.9161
Epoch 3/10
469/469 [==============================] - 1s 3ms/step - loss: 0.2226 - accuracy: 0.9181
Epoch 4/10
469/469 [==============================] - 1s 3ms/step - loss: 0.2158 - accuracy: 0.9213
Epoch 5/10
469/469 [==============================] - 1s 3ms/step - loss: 0.2113 - accuracy: 0.9223
Epoch 6/10
469/469 [==============================] - 1s 3ms/step - loss: 0.2103 - accuracy: 0.9224
Epoch 7/10
469/469 [==============================] - 1s 3ms/step - loss: 0.2024 - accuracy: 0.9254
Epoch 8/10
469/469 [==============================] - 2s 3ms/step - loss: 0.1983 - accuracy: 0.9265
Epoch 9/10
469/469 [==============================] - 1s 3ms/step - loss: 0.1938 - accuracy: 0.9290
Epoch 10/10
469/469 [==============================] - 1s 3ms/step - loss: 0.1885 - accuracy: 0.9301
<keras.callbacks.History at 0x7f40c8b42890>
```

Example: Evaluation

```
test_loss, test_acc = model.evaluate(test_images, test_labels, verbose=2)
print('\nTest accuracy:', test_acc)
```

output: 313/313 - 0s - loss: 0.3369 - accuracy: 0.8880

Test accuracy: 0.8880000114440918

```
probability_model = tf.keras.Sequential(
  [model, tf.keras.layers.Softmax()])
predictions = probability_model.predict(test_images)
predictions[0]
```

output: array([9.8546373e-08, 5.0680104e-13, 6.7453760e-07, 1.8696349e-08, 5.5620589e-08, 8.5184647e-04, 1.2255837e-06, 4.7711013e-03, 9.1507090e-06, 9.9436593e-01], dtype=float32)

Example: Get Predictions

```
np.argmax(predictions[0])
```

Output: 9