3: Statistical Properties of Language

Machine Learning and Real-world Data (MLRD)

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Last session: You implemented a Naive Bayes classifier

- Smoothed vs Unsmoothed
- The accuracy of the un-smoothed classifier was seriously affected by unseen words.
- We implemented add-one (Laplace) smoothing:

$$\hat{P}(w_i|c) = \frac{count(w_i, c) + 1}{\sum_{w \in V}(count(w, c) + 1)} = \frac{count(w_i, c) + 1}{(\sum_{w \in V} count(w, c)) + |V|}$$

- Smoothing helped!

Today: frequency distributions in language

Questions:

- Why did smoothing help? (or in other words:)
- What is it about the distribution of words in a language that affected the performance of the un-smoothed classifier?
- Two Laws: Zipf's Law and Heap's Law

Zipf's Law: Word frequency distributions obey a power law

- There are a small number of very high-frequency words
- There are a large number of low-frequency words
- Zipf's law: the nth most frequent word has a frequency proportional to $1/n$

"a word's frequency in a corpus is inversely proportional to its rank"

The parameters of Zipf's law are language-dependent

Zipf's law:

$$f_w \approx \frac{k}{{r_w}^{\alpha}}$$

where

f_w: frequency of word w

r_w: frequency rank of word w

α, k: constants (which vary with the language)

e.g. α is around 1 for English but 1.3 for German

The parameters of Zipf's law are language-dependent

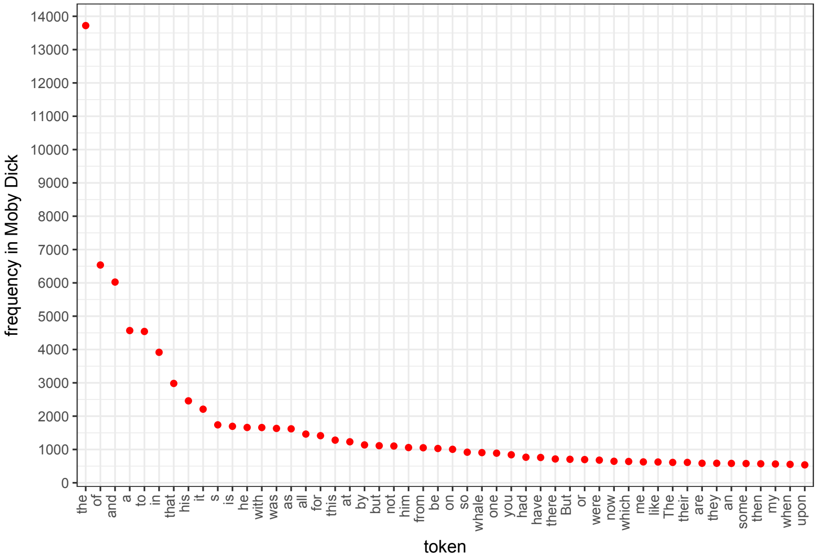
Actually...

$$f_w \approx \frac{k}{(r_w + \beta)^\alpha}$$

where

- β: a shift in the rank
- see summary paper by Piantadosi `https://link.springer.com/article/10.3758/s13423-014-0585-6`
- we won't worry about the rank-shift today

There are a small number of high-frequency words...



Moby Dick has 206,052 words in total.

Similar sorts of high-frequency words across languages

Top 10 most frequent words in some large language samples:

Similar sorts of high-frequency words across languages

Top 10 most frequent words in some large language samples:

English

1	the	6,187,267
2	of	2,941,444
3	and	2,682,863
4	a	2,126,369
5	in	1,812,609
6	to	1,620,850
7	it	1,089,186
8	is	998,389
9	was	923,948
10	to	917,579

BNC,
100Mw

Similar sorts of high-frequency words across languages

Top 10 most frequent words in some large language samples:

	English			German	
1	the	6,187,267	1	der	7,377,879
2	of	2,941,444	2	die	7,036,092
3	and	2,682,863	3	und	4,813,169
4	a	2,126,369	4	in	3,768,565
5	in	1,812,609	5	den	2,717,150
6	to	1,620,850	6	von	2,250,642
7	it	1,089,186	7	zu	1,992,268
8	is	998,389	8	das	1,983,589
9	was	923,948	9	mit	1,878,243
10	to	917,579	10	sich	1,680,106

BNC, 100Mw

“Deutscher Wortschatz”, 500Mw

Similar sorts of high-frequency words across languages

Top 10 most frequent words in some large language samples:

English			German			Spanish		
1	the	6,187,267	1	der	7,377,879	1	que	32,894
2	of	2,941,444	2	die	7,036,092	2	de	32,116
3	and	2,682,863	3	und	4,813,169	3	no	29,897
4	a	2,126,369	4	in	3,768,565	4	a	22,313
5	in	1,812,609	5	den	2,717,150	5	la	21,127
6	to	1,620,850	6	von	2,250,642	6	el	18,112
7	it	1,089,186	7	zu	1,992,268	7	es	16,620
8	is	998,389	8	das	1,983,589	8	y	15,743
9	was	923,948	9	mit	1,878,243	9	en	15,303
10	to	917,579	10	sich	1,680,106	10	lo	14,010
BNC, 100Mw			"Deutscher Wortschatz", 500Mw			subtitles, 27.4Mw		

Similar sorts of high-frequency words across languages

Top 10 most frequent words in some large language samples:

	English			German			Spanish			Italian	
1	the	6,187,267	1	der	7,377,879	1	que	32,894	1	non	25,757
2	of	2,941,444	2	die	7,036,092	2	de	32,116	2	di	22,868
3	and	2,682,863	3	und	4,813,169	3	no	29,897	3	che	22,738
4	a	2,126,369	4	in	3,768,565	4	a	22,313	4	è	18,624
5	in	1,812,609	5	den	2,717,150	5	la	21,127	5	e	17,600
6	to	1,620,850	6	von	2,250,642	6	el	18,112	6	la	16,404
7	it	1,089,186	7	zu	1,992,268	7	es	16,620	7	il	14,765
8	is	998,389	8	das	1,983,589	8	y	15,743	8	un	14,460
9	was	923,948	9	mit	1,878,243	9	en	15,303	9	a	13,915
10	to	917,579	10	sich	1,680,106	10	lo	14,010	10	per	10,501
	BNC, 100Mw			"Deutscher Wortschatz", 500Mw			subtitles, 27.4Mw			subtitles, 5.6Mw	

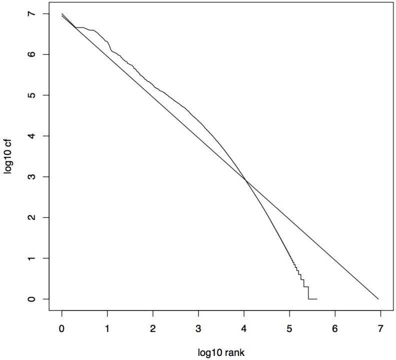
Similar sorts of high-frequency words across languages

Top 10 most frequent words in some large language samples:

	English		German		Spanish		Italian		Dutch	
1	the	6,187,267	der	7,377,879	que	32,894	non	25,757	de	4,770
2	of	2,941,444	die	7,036,092	de	32,116	di	22,868	en	2,709
3	and	2,682,863	und	4,813,169	no	29,897	che	22,738	het/'t	2,469
4	a	2,126,369	in	3,768,565	a	22,313	è	18,624	van	2,259
5	in	1,812,609	den	2,717,150	la	21,127	e	17,600	ik	1,999
6	to	1,620,850	von	2,250,642	el	18,112	la	16,404	te	1,935
7	it	1,089,186	zu	1,992,268	es	16,620	il	14,765	dat	1,875
8	is	998,389	das	1,983,589	y	15,743	un	14,460	die	1,807
9	was	923,948	mit	1,878,243	en	15,303	a	13,915	in	1,639
10	to	917,579	sich	1,680,106	lo	14,010	per	10,501	een	1,637
	BNC, 100Mw		"Deutscher Wortschatz", 500Mw		subtitles, 27.4Mw		subtitles, 5.6Mw		subtitles, 800Kw	

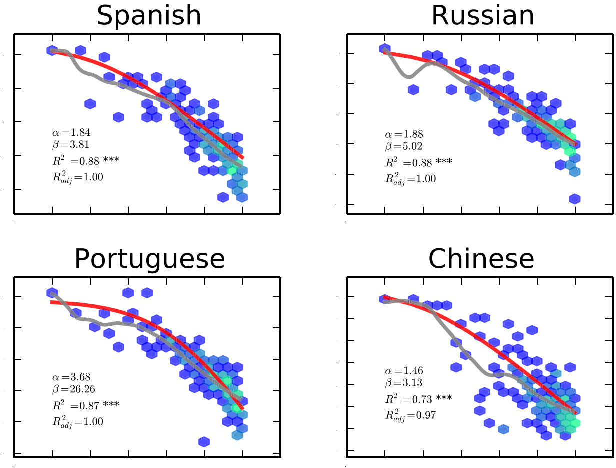
It is helpful to plot Zipf curves in log-space

Reuters dataset: taken from `https://nlp.stanford.edu/IR-book/pdf/irbookonlinereading.pdf` – chapter 5



By fitting a simple line to the data in log-space we can estimate the language specific parameters α and k (we will do this today!)

In log-space we can more easily estimate the language specific parameters

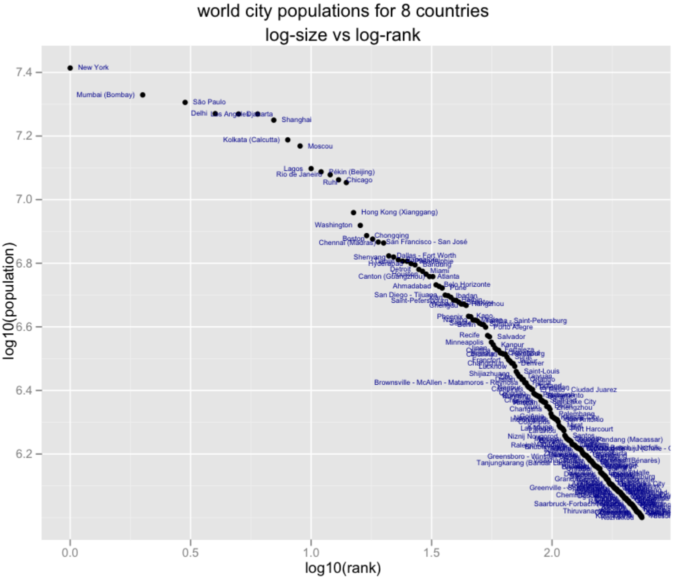


From Piantadosi https://link.springer.com/article/10.3758/s13423-014-0585-6

Zipfian (or near-Zipfian) distributions occur in many collections

- Sizes of settlements
- Frequency of access to web pages
- Size of earthquakes
- Word senses per word
- Notes in musical performances
- machine instructions
- ...

Zipfian (or near-Zipfian) distributions occur in many collections



There is a relationship between vocabulary size and text length

So far we have been thinking about frequencies of particular words:

- we call any unique word a type: *the* is a word type
- we call an instance of a type a token: there are 13721 *the* tokens in Moby Dick
- the number of types in a text is the size of the vocabulary (also called dictionary)

Today we will explore the relationship between vocabulary size and the length of a text.

Heaps' law describes the vocabulary / text-length relationship

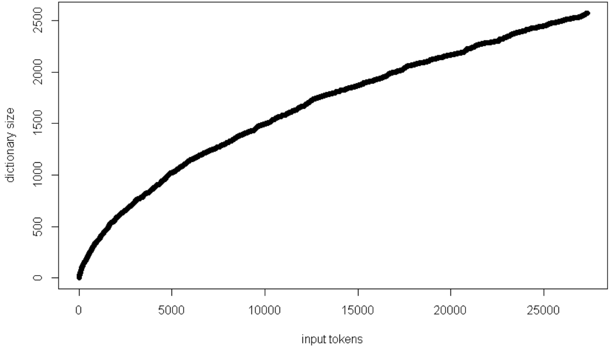
Heaps' Law:
Describes the relationship between the size of a vocabulary and the size of text that gave rise to it:

$$u_n = kn^\beta$$

where

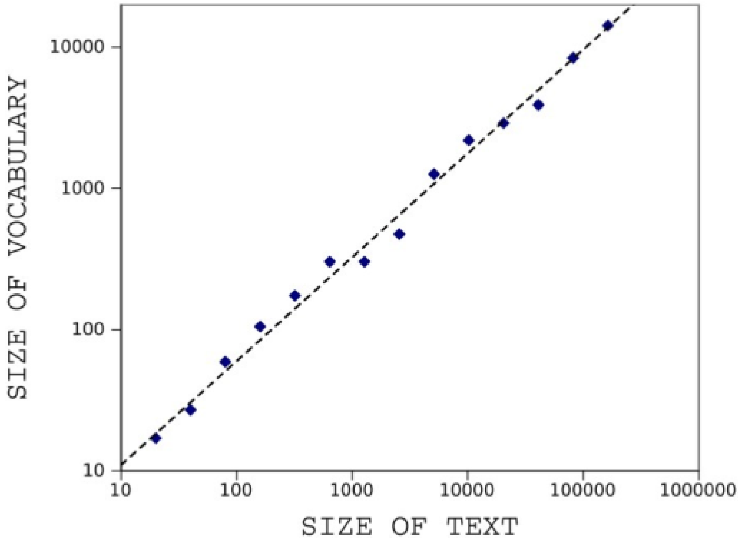
- u_n: number of types (unique items)—i.e. vocabulary size
- n: total number of tokens—i.e.text size
- β, k: constants (language-dependent)
  - β is around $\frac{1}{2}$
  - $30 \leq k \leq 100$

Heaps' Law



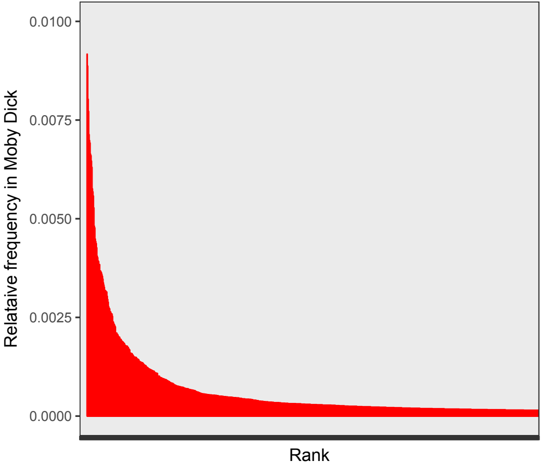
- No saturation: there will always be more new types
- As we progress through a text it takes longer and longer to encounter a new type

It is helpful to plot Heaps' law in log-space

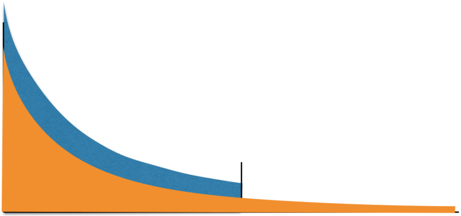


Zipf's law and Heaps' law affected our classifier

- Zipf curve has a lot of probability mass in the long tail.
- By Heaps' law, we need increasing amounts of text to see new word types in the tail



Zipf's law and Heaps' law affected our classifier



- With MLE, only seen types receive a probability estimate:

e.g. we used:

$$\hat{P_{MLE}}(w_i|c) = \frac{count(w_i, c)}{\sum_{w \in V_{training}} count(w, c)}$$

- True probability (e.g. for NEG class): orange; MLE: blue
- Total probabilities must sum to 1; in MLE all that probability mass is given to seen types
- MLE overestimates the probability of seen types (as opposed to unseen)

Smoothing redistributes the probability mass

- Add-one smoothing redistributes the probability mass.

e.g. we used:

$$\hat{P}(w_i|c) = \frac{count(w_i, c) + 1}{\sum_{w \in V}(count(w, c) + 1)} = \frac{count(w_i, c) + 1}{(\sum_{w \in V} count(w, c)) + |V|}$$

- It takes some portion away from the MLE overestimate.
- It redistributes this portion to the unseen types.
- Better estimate; still not perfect.

Today we will investigate Zipf's and Heaps' law in movie reviews

Follow task instructions on moodle to:

- Plot a frequency vs rank graph for larger set of movie reviews (you are given chart plotting code)
- Plot a log frequency vs log rank graph
- Indicate the location of your 10 chosen words from Tick 1, e.g. in colour, on this plot.
- Use least-squares algorithm to fit a line to the log-log plot (you are given `best-fit` code)
- Estimate the parameters of the Zipf equation
- Plot type vs token graph for the movie reviews

Lecture Etiquette for LT1 and LT2

- If you are late for a lecture, **don't** use the doors on the ground floor
- Use the doors on **first floor** instead
- This is about disruption of lectures
- Thanks!

Ticking for Task 3

There is no automatic ticker for Task 3

- Write everything in your lab book
- Save all your graphs (as screenshots or otherwise)