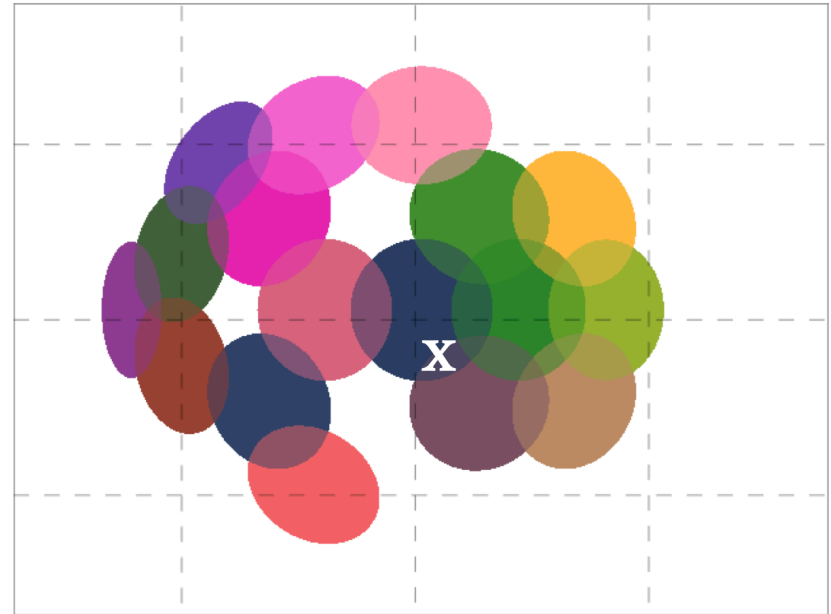
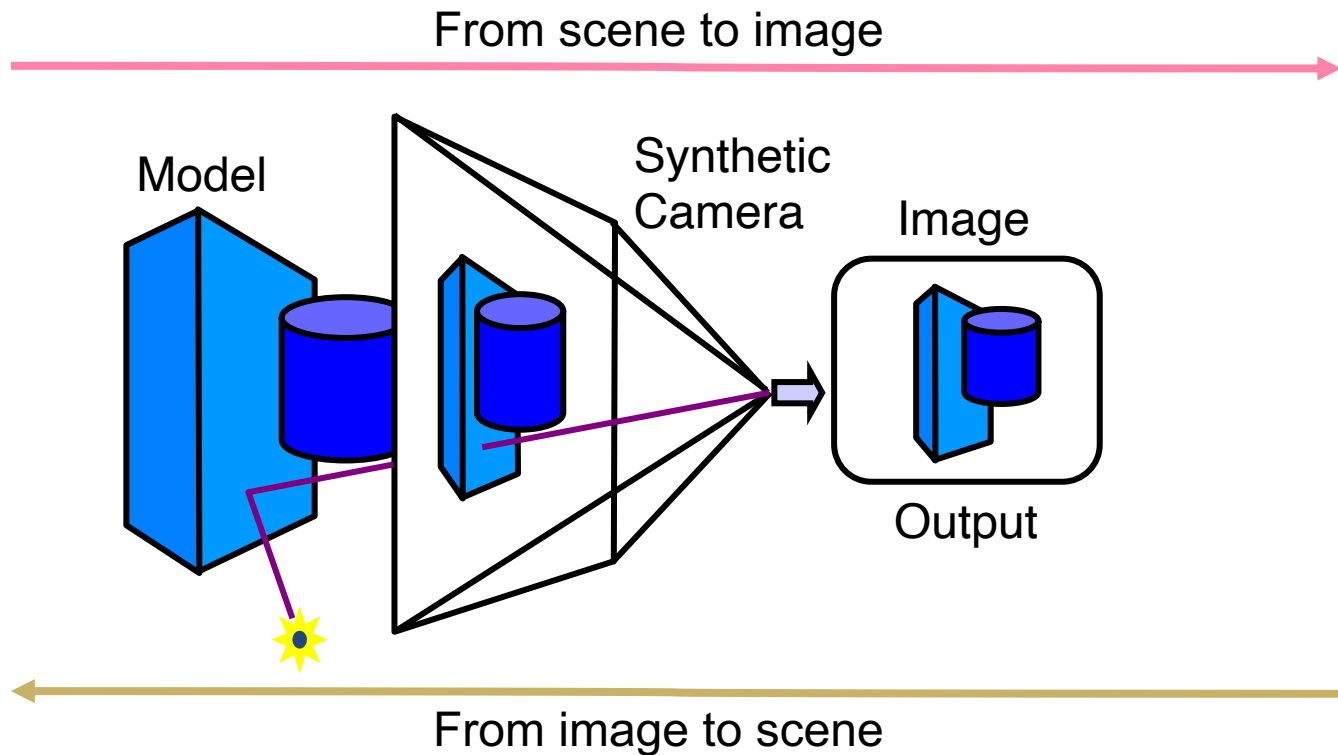


# Inverse Rendering

Dr Cengiz Öztireli



# Forward / inverse rendering



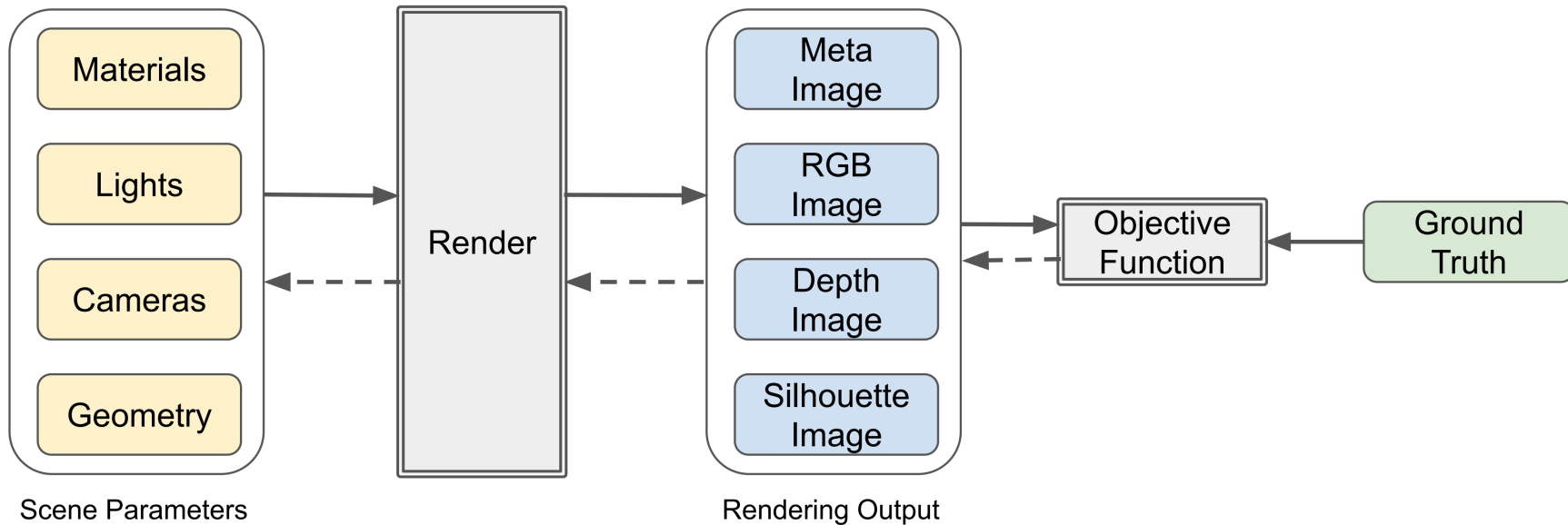


# Forward / inverse rendering

Models for  
Geometry  
Motion  
Appearance  
Lighting  
Camera

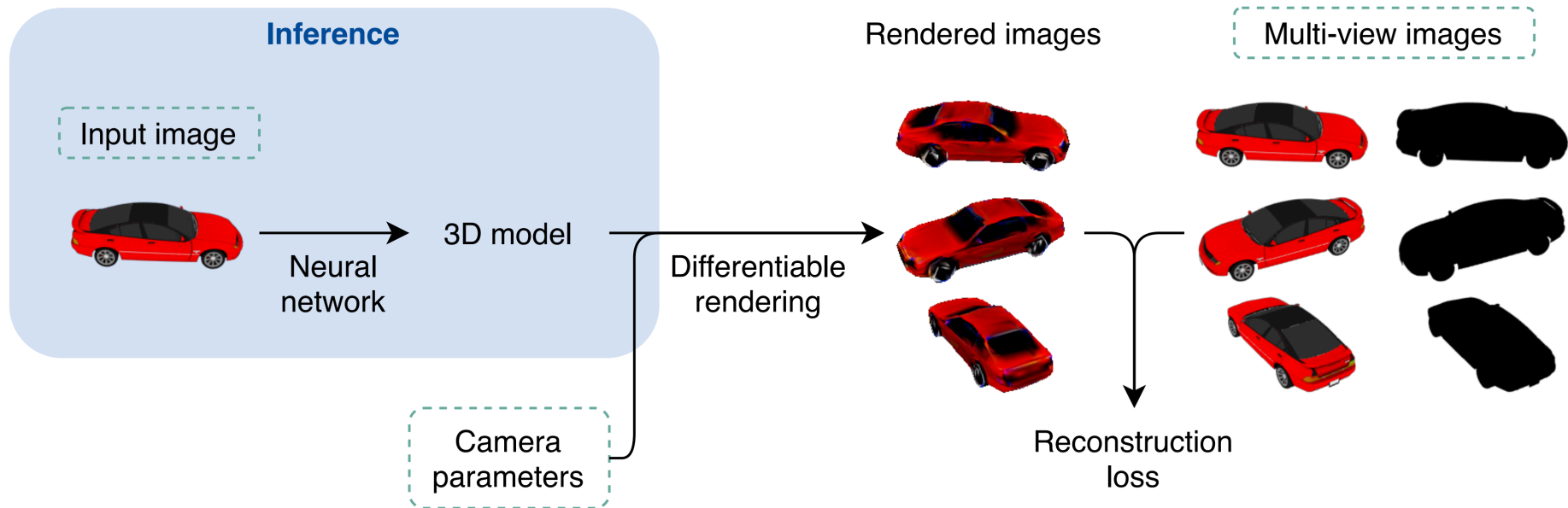
...

# Forward / inverse rendering



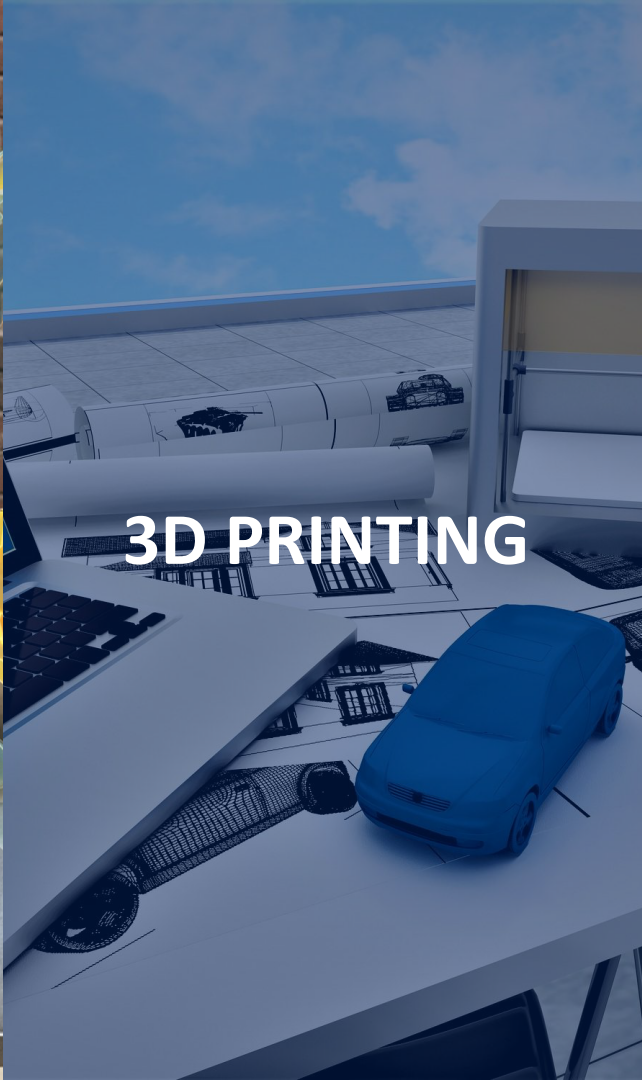
# Forward / inverse rendering

Example: 3D geometry reconstruction

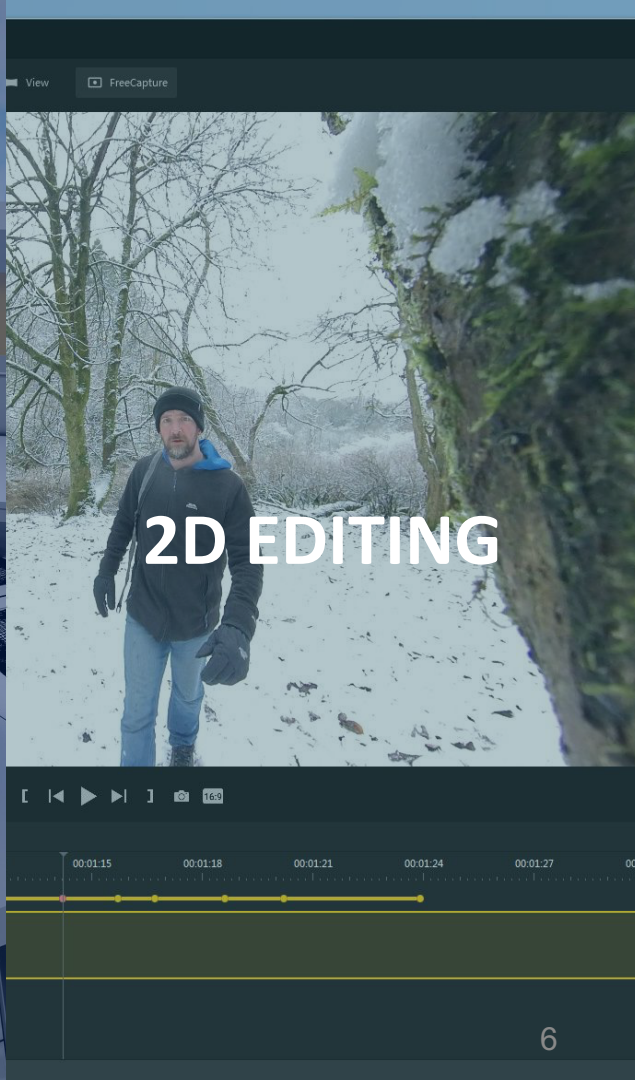




AR / VR



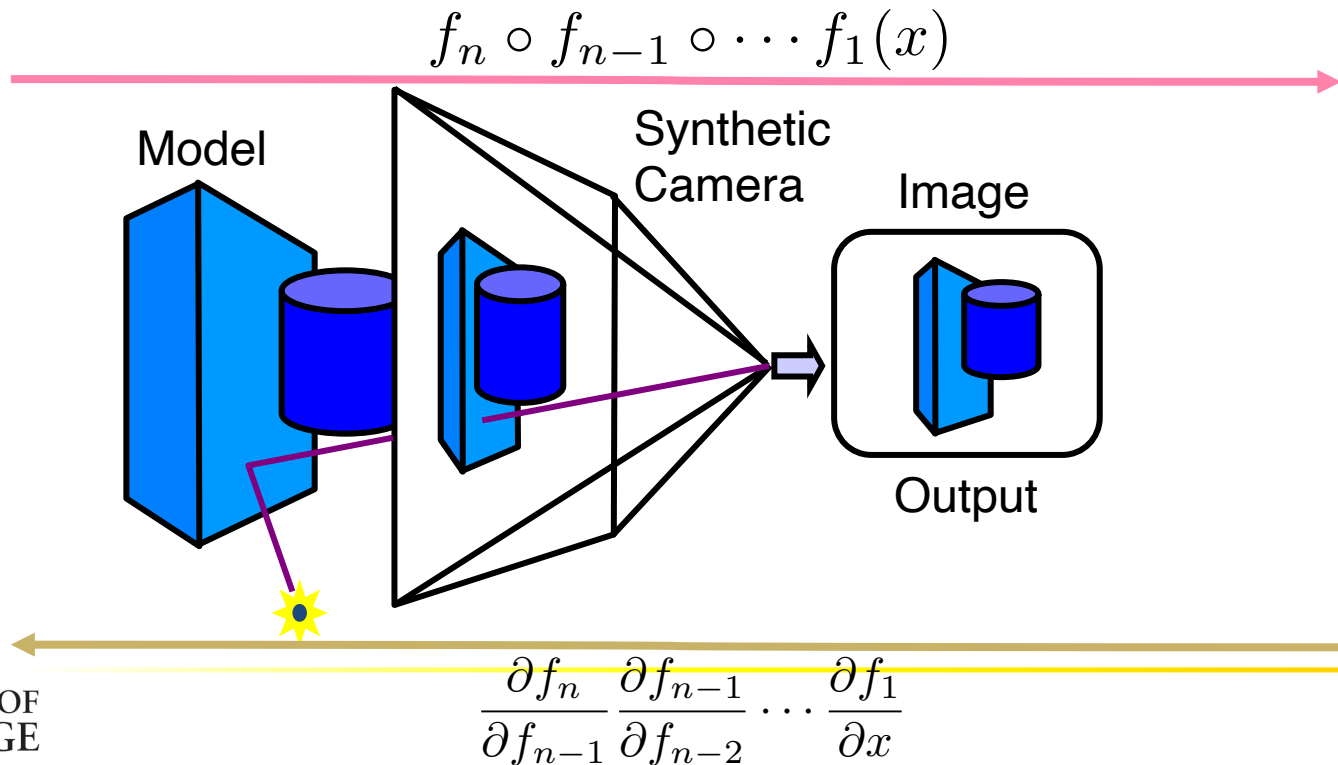
3D PRINTING



2D EDITING

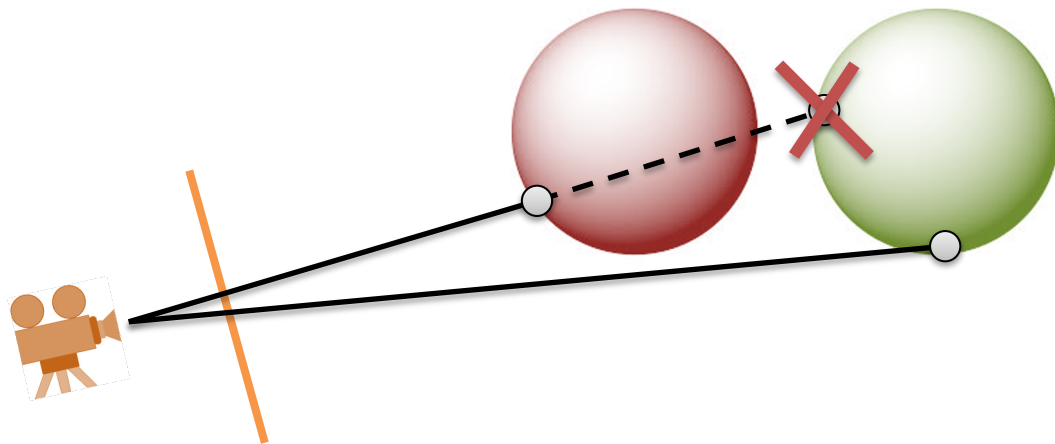
# Inverse rendering

- Deep learning to help: make everything differentiable



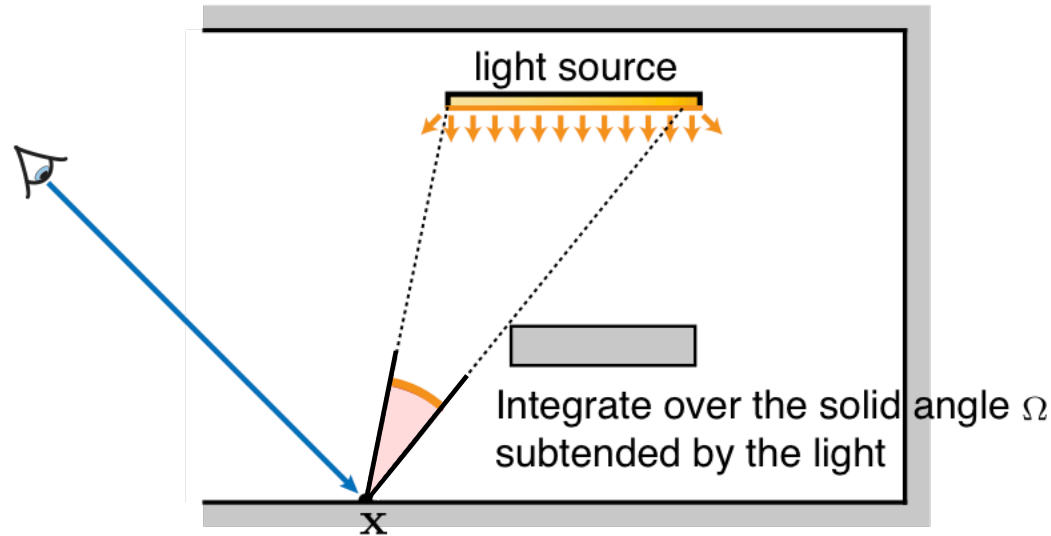
# Inverse rendering

- Problem: Visibility is not differentiable



# Surface Area Form

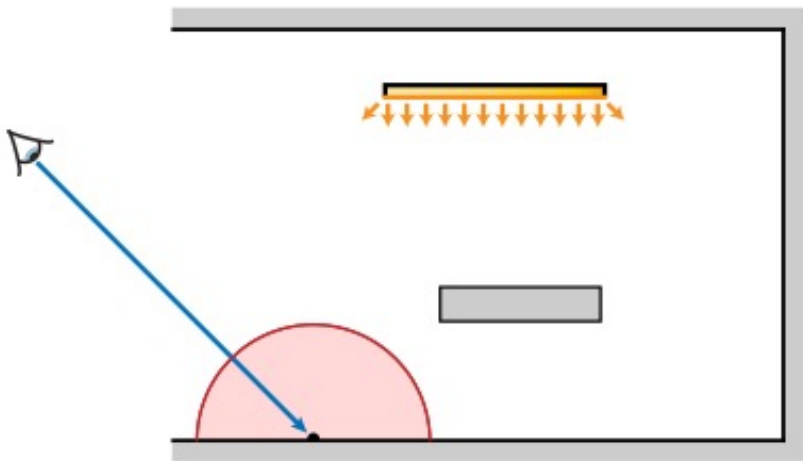
$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{\Omega} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_e(\mathbf{r}(\mathbf{x}, \vec{\omega}_i), -\vec{\omega}_i) |\cos \theta_i| d\vec{\omega}_i$$





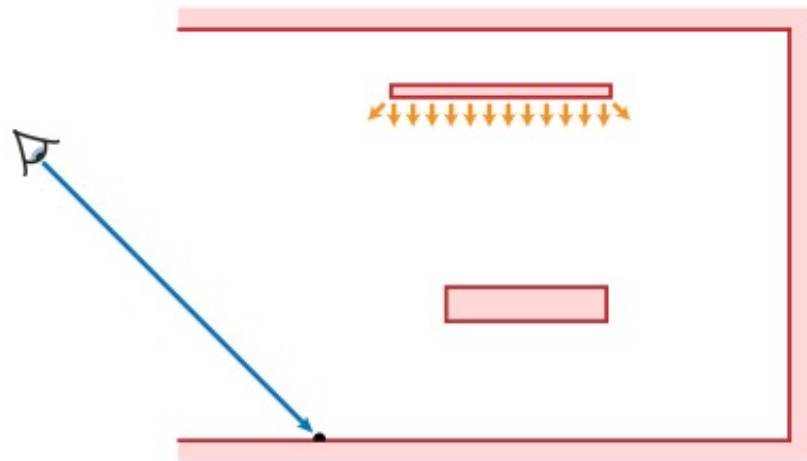
# Surface Area Form

Hemispherical integration



$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

Surface area integration



$$L_r(\mathbf{x}, \mathbf{z}) = \int_A f_r(\mathbf{x}, \mathbf{y}, \mathbf{z}) L_i(\mathbf{x}, \mathbf{y}) G(\mathbf{x}, \mathbf{y}) dA(\mathbf{y})$$



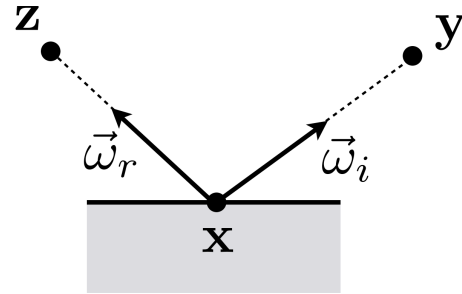
# Surface Area Form

- Change in notation

$$L_i(\mathbf{x}, \vec{\omega}_i) = L_i(\mathbf{x}, \mathbf{y})$$

$$L_r(\mathbf{x}, \vec{\omega}_r) = L_r(\mathbf{x}, \mathbf{z})$$

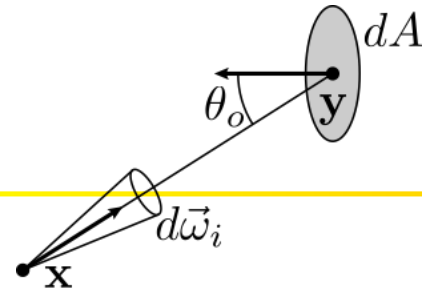
$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = f_r(\mathbf{x}, \mathbf{y}, \mathbf{z})$$



- Transform integral over directions to over surface area

Jacobian of the transform:

$$d\vec{\omega}_i = \frac{|\cos \theta_o|}{\|\mathbf{x} - \mathbf{y}\|^2} dA$$



# Surface Area Form

- Surface area form

$$L_r(\mathbf{x}, \mathbf{z}) = \int_A f_r(\mathbf{x}, \mathbf{y}, \mathbf{z}) L_i(\mathbf{x}, \mathbf{y}) G(\mathbf{x}, \mathbf{y}) dA(\mathbf{y})$$

Geometry term

$$G(\mathbf{x}, \mathbf{y}) = V(\mathbf{x}, \mathbf{y}) \frac{|\cos \theta_i| |\cos \theta_o|}{\|\mathbf{x} - \mathbf{y}\|^2}$$

Visibility term

Original foreshortening term

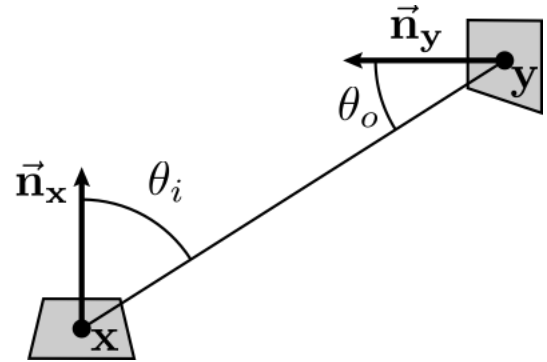
Jacobian determinant of the transform

# Surface Area Form

- Interpreting the new cosine term

$$G(\mathbf{x}, \mathbf{y}) = V(\mathbf{x}, \mathbf{y}) \frac{|\cos \theta_i| |\cos \theta_o|}{\|\mathbf{x} - \mathbf{y}\|^2}$$

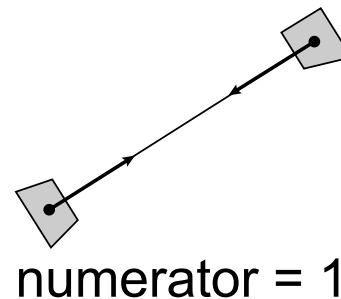
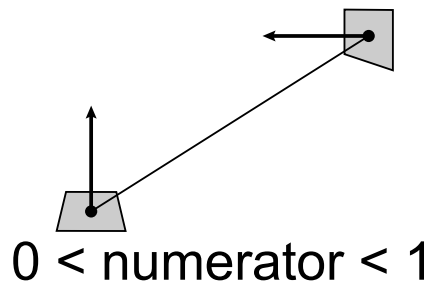
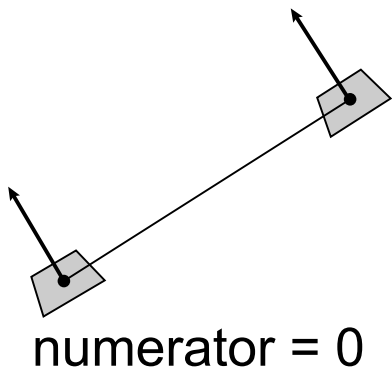
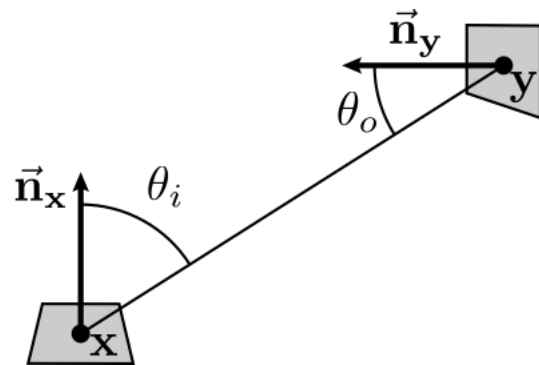
- The chance that a photon emitted from a differential patch will hit another differential patch decreases as:
  - the patches **face away** from each other (numerator)
  - the patches **move away** from each other (denominator)



# Surface Area Form

- Interpreting the new cosine term

$$G(\mathbf{x}, \mathbf{y}) = V(\mathbf{x}, \mathbf{y}) \frac{|\cos \theta_i| |\cos \theta_o|}{\|\mathbf{x} - \mathbf{y}\|^2}$$



# Visibility Term

- Surface area form

$$L_r(\mathbf{x}, \mathbf{z}) = \int_A f_r(\mathbf{x}, \mathbf{y}, \mathbf{z}) L_i(\mathbf{x}, \mathbf{y}) G(\mathbf{x}, \mathbf{y}) dA(\mathbf{y})$$

Geometry term

$$G(\mathbf{x}, \mathbf{y}) = V(\mathbf{x}, \mathbf{y}) \frac{|\cos \theta_i| |\cos \theta_o|}{\|\mathbf{x} - \mathbf{y}\|^2} \quad V(\mathbf{x}, \mathbf{y}) = \begin{cases} 1 : & \text{visible} \\ 0 : & \text{not visible} \end{cases}$$

Visibility term

**Discontinuous!**

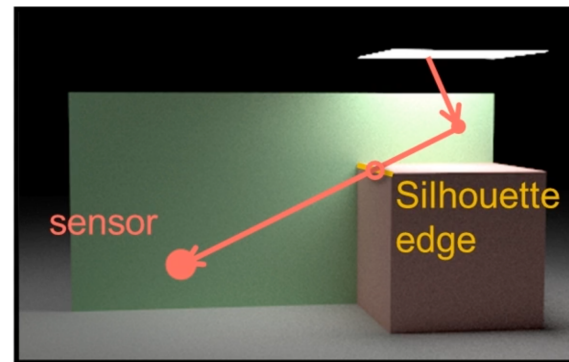
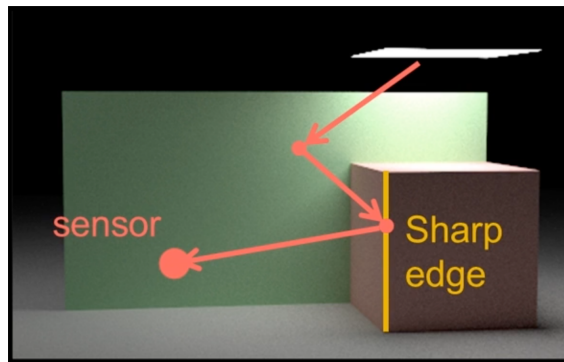
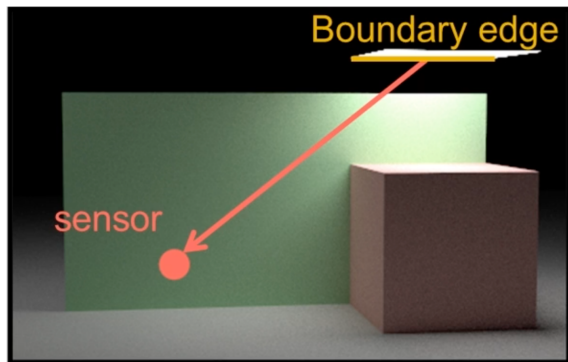
# Visibility Term

- Problem with discontinuities and differentiability

$$\frac{\partial}{\partial \pi} \int_A f(\mathbf{x}) d\mathbf{x} \neq \int_A \frac{\partial}{\partial \pi} f(\mathbf{x}) d\mathbf{x}$$

# Visibility Term

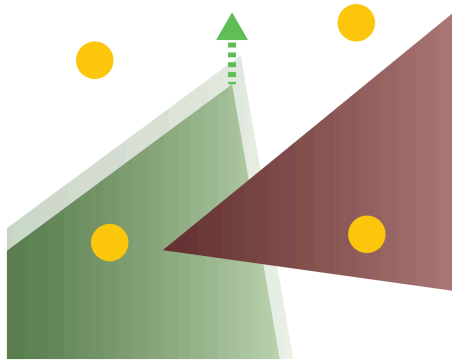
- Sources of discontinuities due to visibility



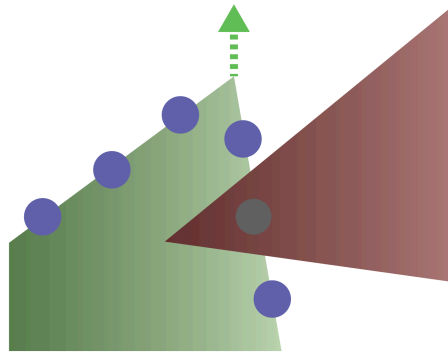
Zhang et al., Path-Space Differentiable Rendering, 2020.

# Differentiable Visibility

- Edge-sampling for accurate gradients



Standard sampling



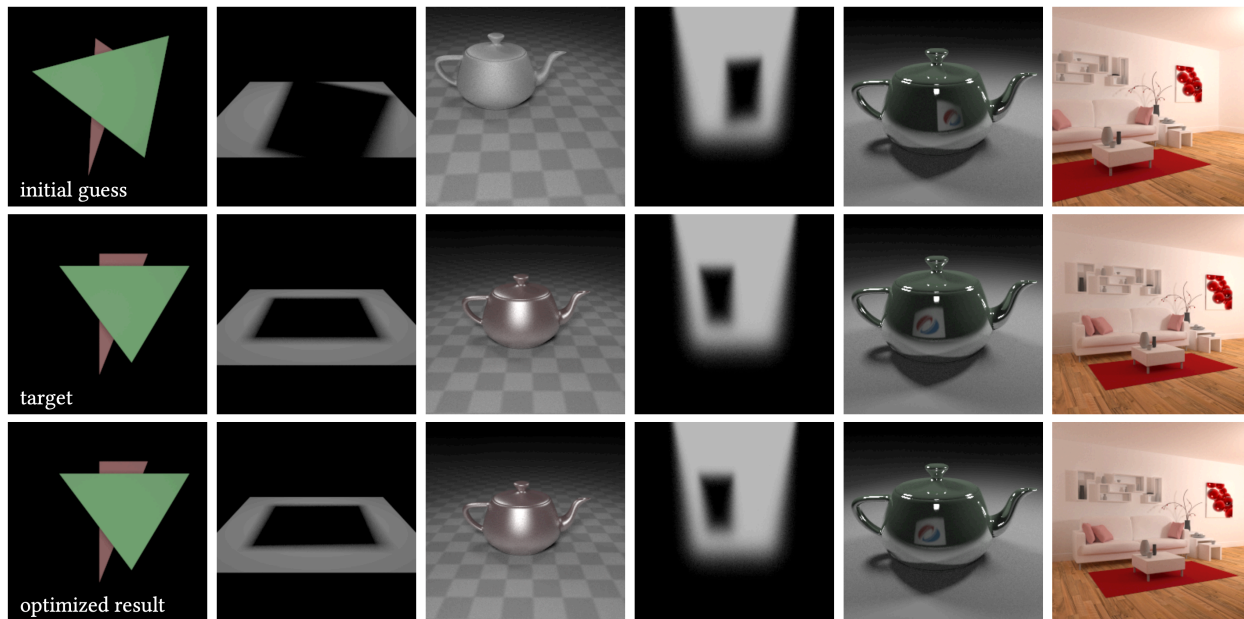
Edge sampling

Li et al., Differentiable Monte Carlo Ray Tracing through Edge Sampling, 2018.



# Differentiable Visibility

- Edge-sampling for accurate gradients



Li et al., Differentiable Monte Carlo Ray Tracing through Edge Sampling, 2018.

# Differentiable Visibility

- Re-parametrizing discontinuities

$$\int \text{integrand} \, d\omega = \int \text{convolved integrand} \, d\omega$$

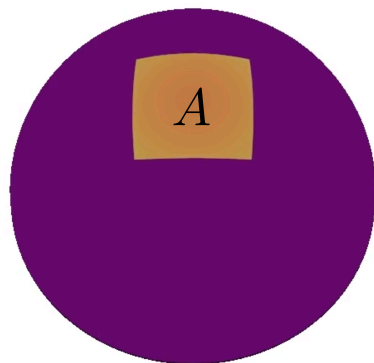
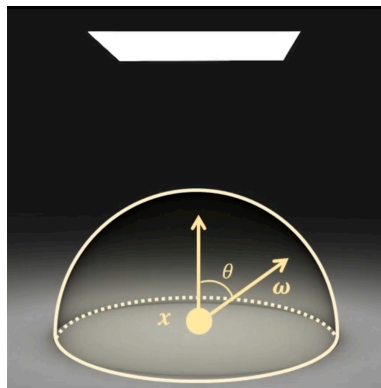
integrand                      convolved integrand

Loubet et al., Reparameterizing discontinuous integrands for differentiable rendering, 2019.

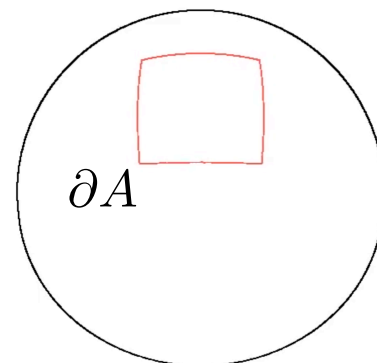
# Differentiable Visibility

- Reynolds transport theorem

$$\frac{\partial}{\partial \pi} \int_A f(\mathbf{x}) d\mathbf{x} = \int_A \frac{\partial}{\partial \pi} f(\mathbf{x}) d\mathbf{x} + \int_{\partial A} g(\mathbf{l}) d\mathbf{l}$$



$f(\mathbf{x})$



Zhang et al., Path-Space Differentiable Rendering, 2020.

# Frameworks

- Automatic differentiation
- Fast & parallel computation
- Active development

**TensorFlow**  
Graphics

