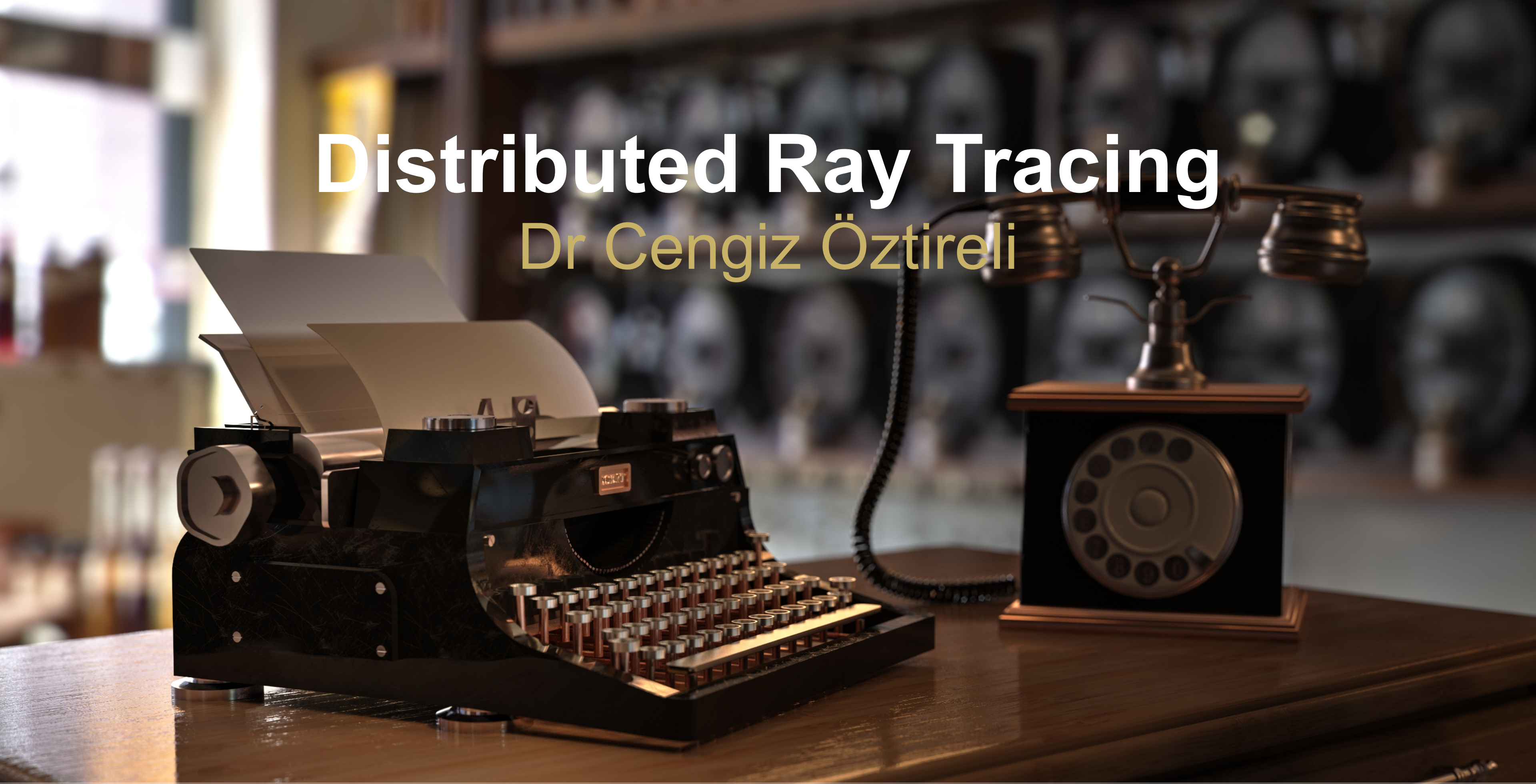

Distributed Ray Tracing

Dr Cengiz Öztireli

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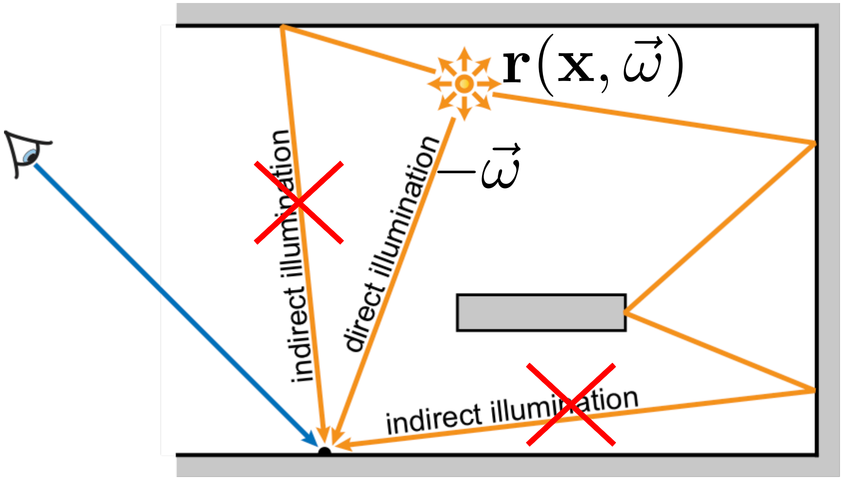
Direct Illumination

- All light comes directly from emitters, i.e. light sources

$$L_o(\mathbf{x},\vec{\omega}_o) = L_e(\mathbf{x},\vec{\omega}_o) + \int_{H^2} f_r(\mathbf{x},\vec{\omega}_i,\vec{\omega}_o) L_i(\mathbf{x},\vec{\omega}_i)\cos\theta_i \, d\vec{\omega}_i$$

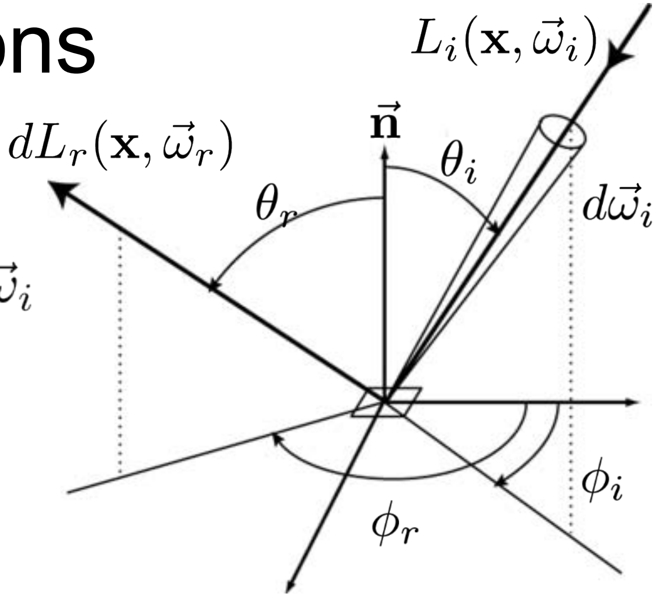


$$L_i(\mathbf{x},\vec{\omega}) = L_e(\mathbf{r}(\mathbf{x},\vec{\omega}), -\vec{\omega})$$

Direct Illumination

- The reflected radiance due to incident illumination from all directions

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$



Problem: estimating this integral

Quadrature: estimating integrals

- Importance sampling

function to integrate

$$\langle F^N \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{p(x_i)}$$

estimator

#samples

density of samples

Quadrature: estimating integrals

- Importance sampling

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

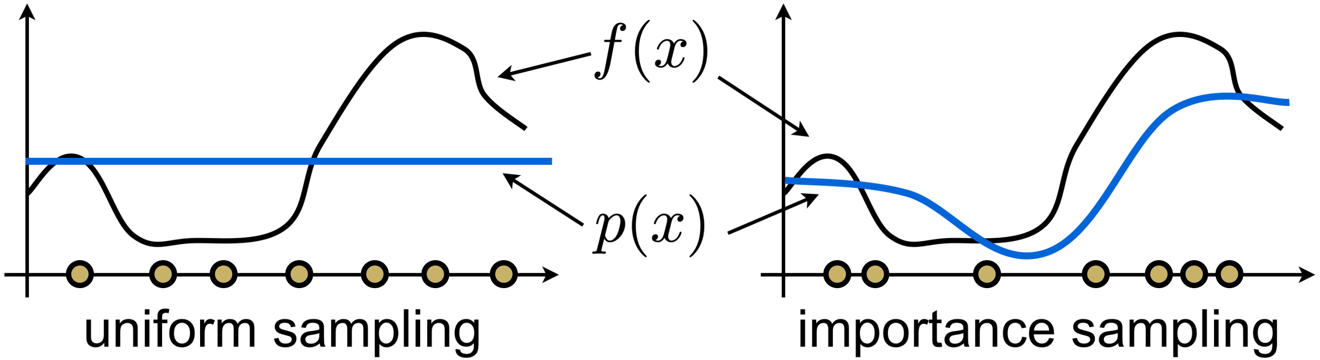
$$\langle L_r(\mathbf{x}, \vec{\omega}_r)^N \rangle = \frac{1}{N} \sum_{k=1}^{N} \frac{f_r(\mathbf{x}, \vec{\omega}_{i,k}, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_{i,k}) \cos\theta_{i,k} \, d\vec{\omega}_{i,k}}{p_\Omega(\vec{\omega}_{i,k})}$$

estimator #samples density of samples

Quadrature: estimating integrals

- Importance sampling

$$\langle L_r(\mathbf{x}, \vec{\omega}_r)^N \rangle = \frac{1}{N} \sum_{k=1}^{N} \frac{f_r(\mathbf{x}, \vec{\omega}_{i,k}, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_{i,k}) \cos\theta_{i,k} \, d\vec{\omega}_{i,k}}{p_\Omega(\vec{\omega}_{i,k})}$$



Sampling Terms

- What to importance sample?

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \boxed{\cos\theta_i} d\vec{\omega}_i$$

Sampling the Cosine Term

- Example: diffuse objects illuminated by an ambient white sky (ambient occlusion)

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

$$\downarrow$$

$$L_r(\mathbf{x}) = \frac{\rho}{\pi} \int_{H^2} \underbrace{V(\mathbf{x}, \vec{\omega}_i)}_{\text{visibility function}} \cos\theta_i \, d\vec{\omega}_i$$

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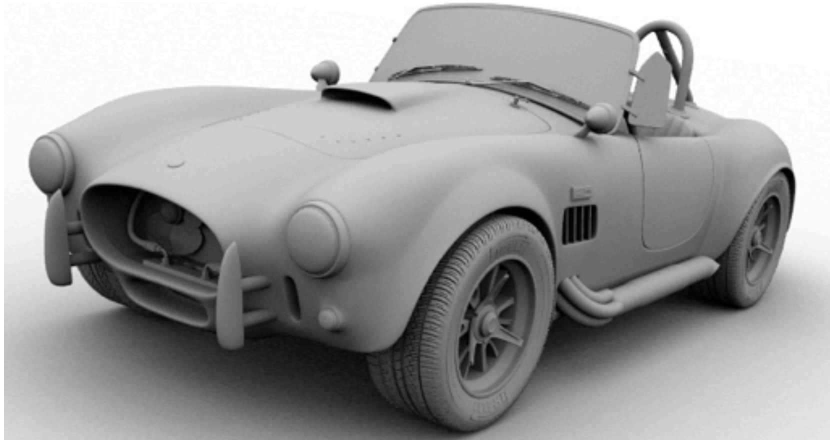
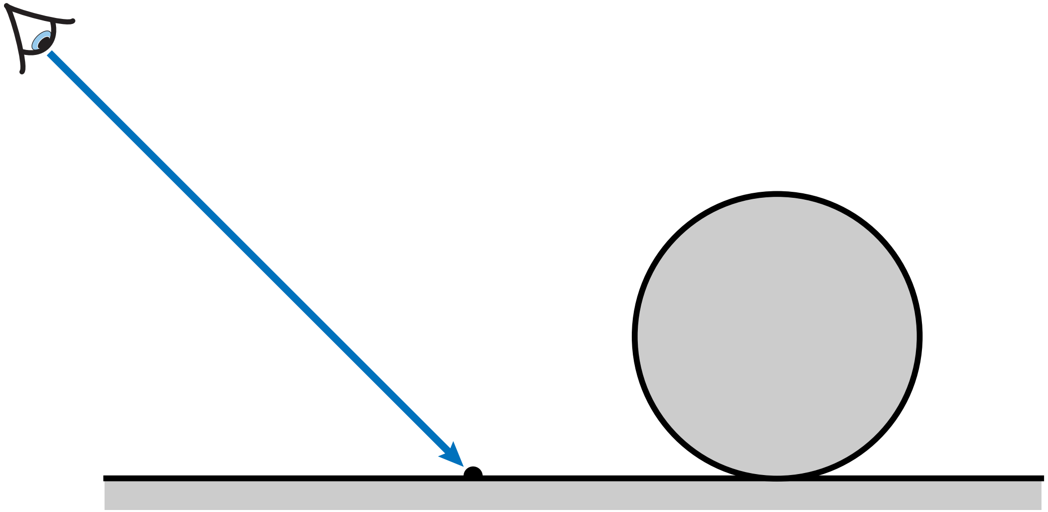
Sampling the Cosine Term

- Ambient occlusion examples



Sampling the Cosine Term

- Ambient occlusion $L_r(\mathbf{x}) = \frac{\rho}{\pi} \int_{H^2} V(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$



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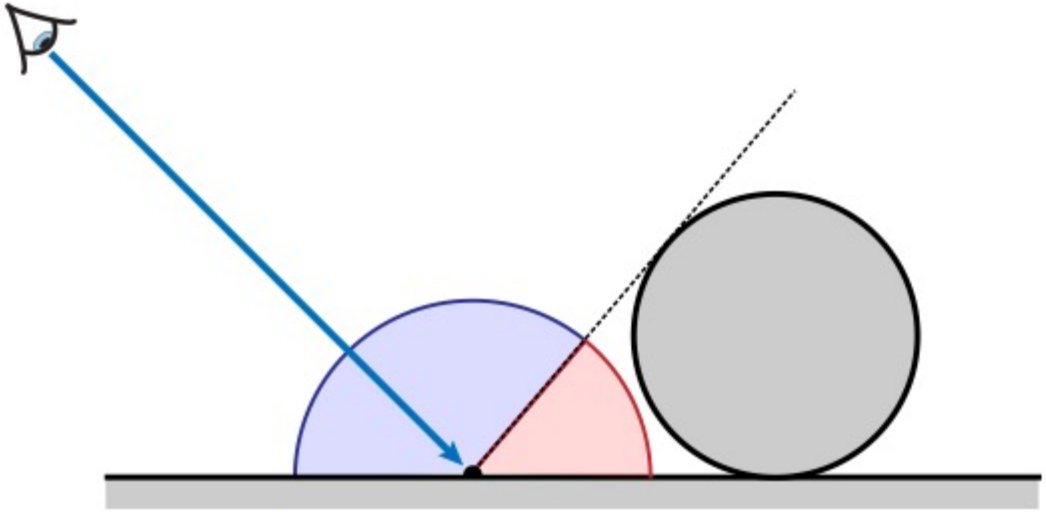
Sampling the Cosine Term

- Ambient occlusion $L_r(\mathbf{x}) = \frac{\rho}{\pi} \int_{H^2} V(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$

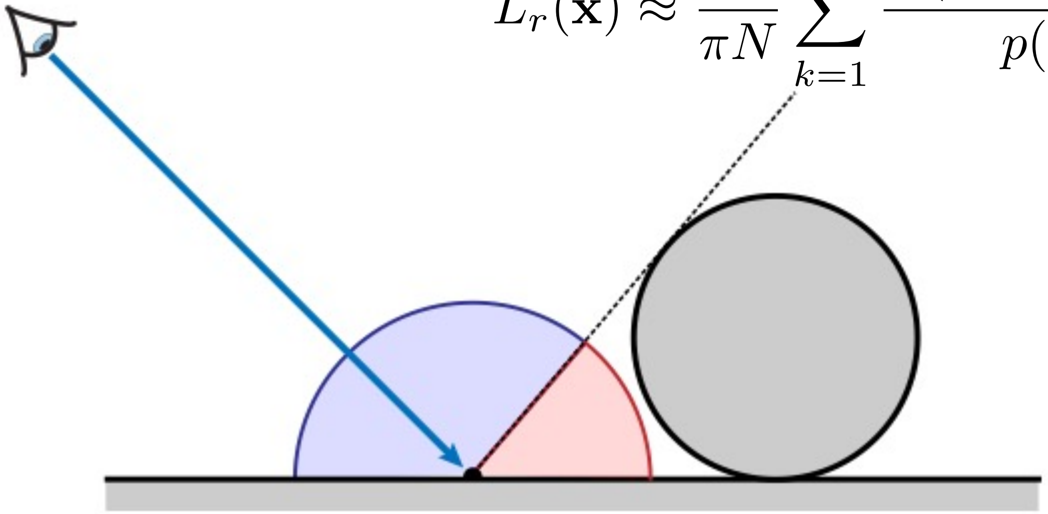


Sampling the Cosine Term

- Ambient occlusion

$$L_r(\mathbf{x}) = \frac{\rho}{\pi} \int_{H^2} V(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

$$L_r(\mathbf{x}) \approx \frac{\rho}{\pi N} \sum_{k=1}^{N} \frac{V(\mathbf{x}, \vec{\omega}_{i,k}) \cos\theta_{i,k}}{p(\vec{\omega}_{i,k})}$$



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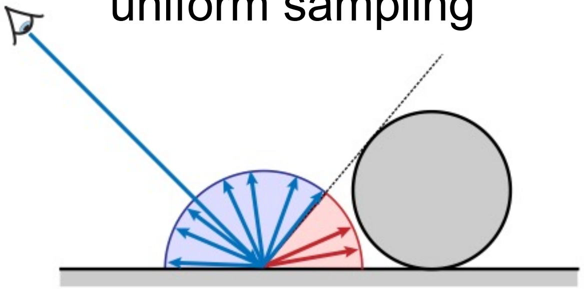
Sampling the Cosine Term

- Ambient occlusion $L_r(\mathbf{x}) \approx \frac{\rho}{\pi N} \sum_{k=1}^{N} \frac{V(\mathbf{x}, \vec{\omega}_{i,k}) \cos \theta_{i,k}}{p(\vec{\omega}_{i,k})}$

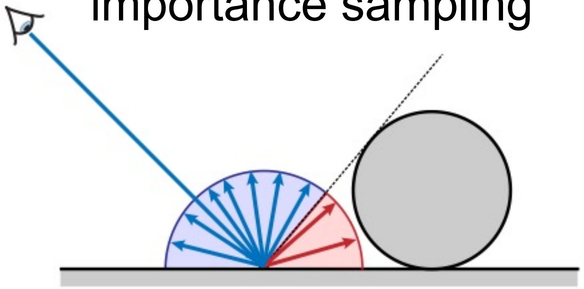
uniform sampling



$$p(\vec{\omega}_{i,k}) = 1/2\pi$$

$$L_r(\mathbf{x}) \approx \frac{2\rho}{N} \sum_{k=1}^{N} V(\mathbf{x}, \vec{\omega}_{i,k}) \cos \theta_{i,k}$$

importance sampling

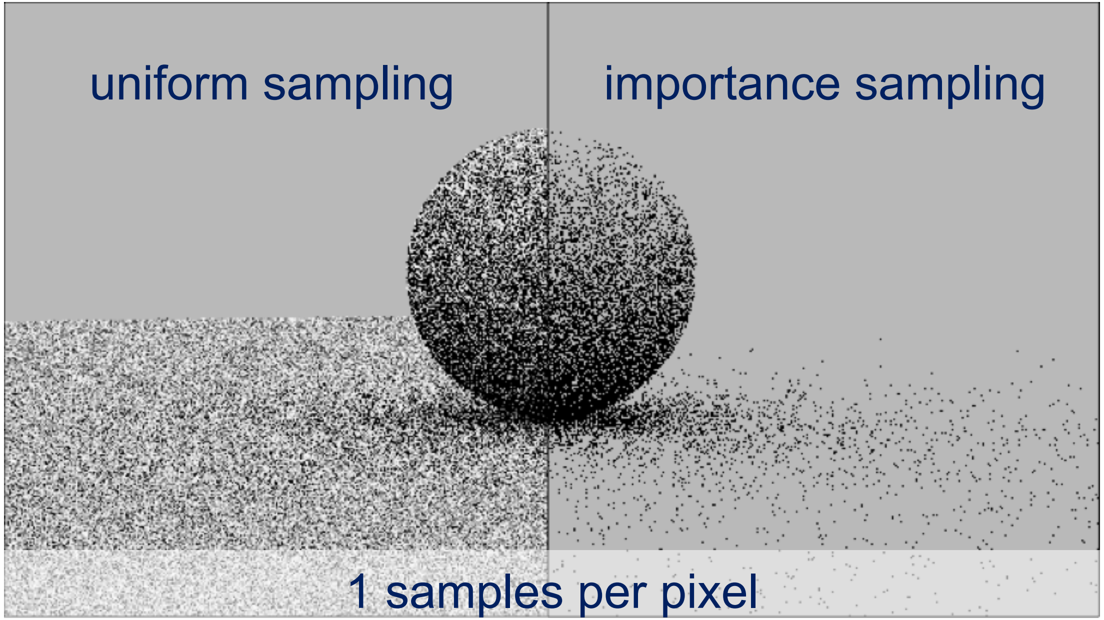


$$p(\vec{\omega}_{i,k}) = \cos \theta_{i,k} / \pi$$

$$L_r(\mathbf{x}) \approx \frac{\rho}{N} \sum_{k=1}^{N} V(\mathbf{x}, \vec{\omega}_{i,k})$$

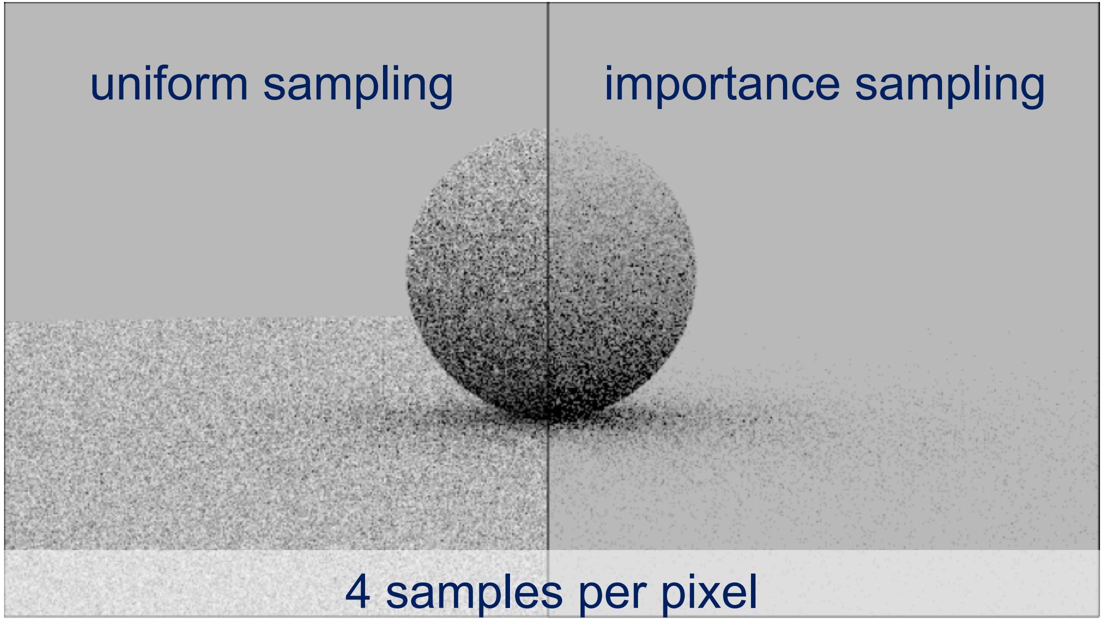
Sampling the Cosine Term

- Ambient occlusion



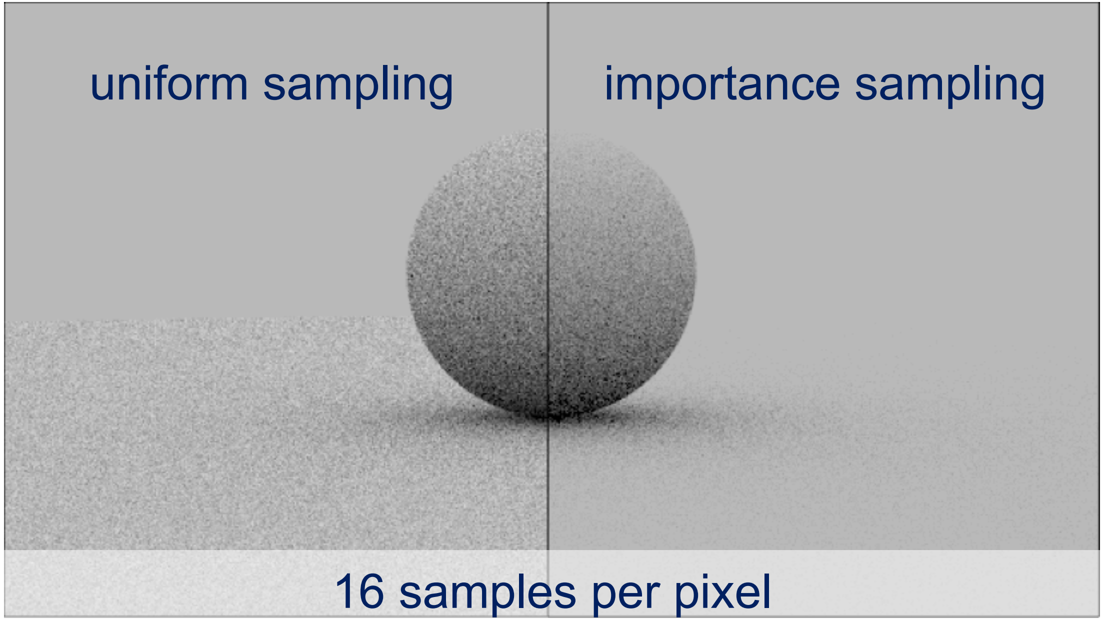
Sampling the Cosine Term

- Ambient occlusion



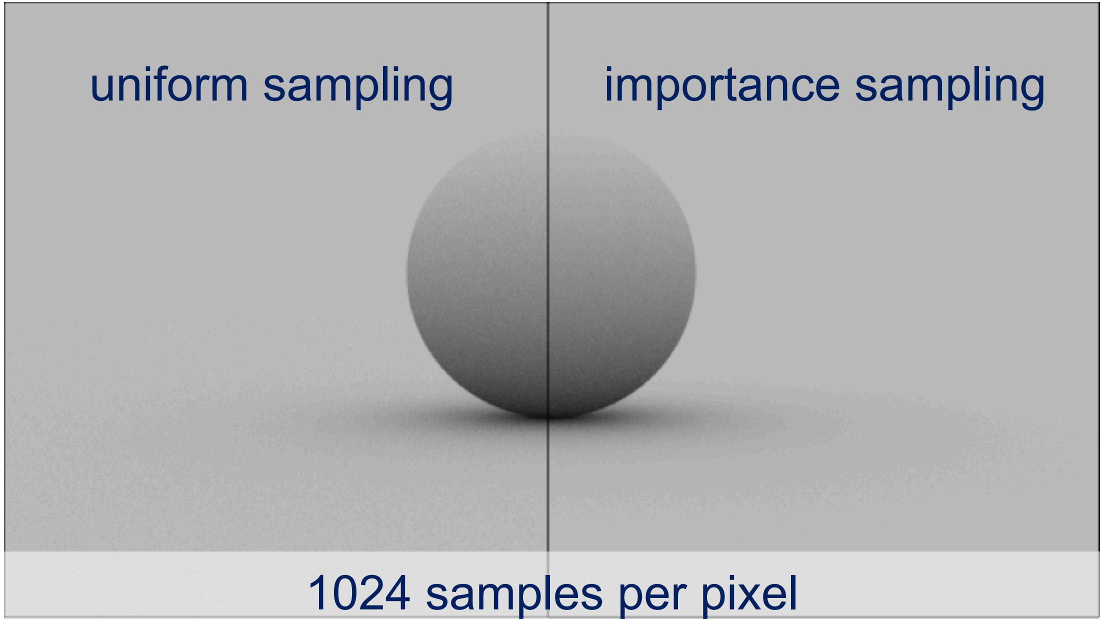
Sampling the Cosine Term

- Ambient occlusion



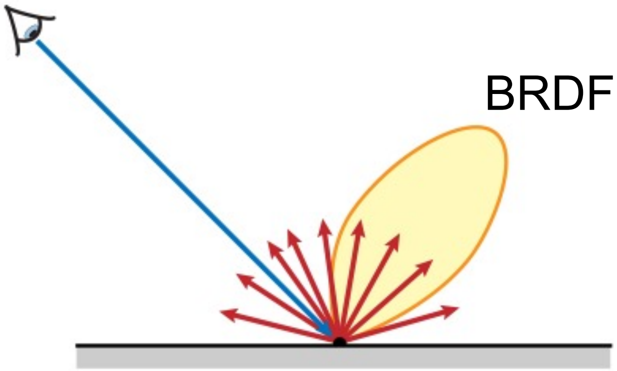
Sampling the Cosine Term

- Ambient occlusion

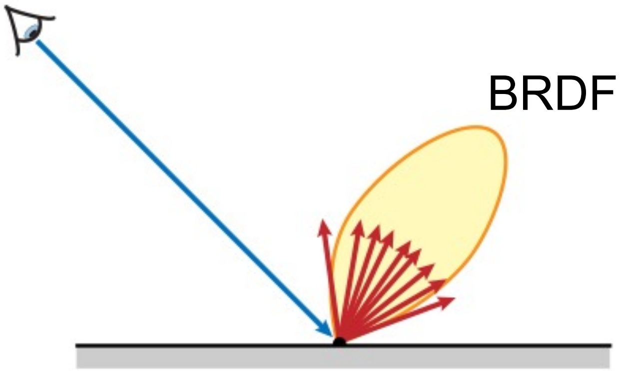


Sampling BRDFs

- What to importance sample?



Cosine-weighted
Importance sampling

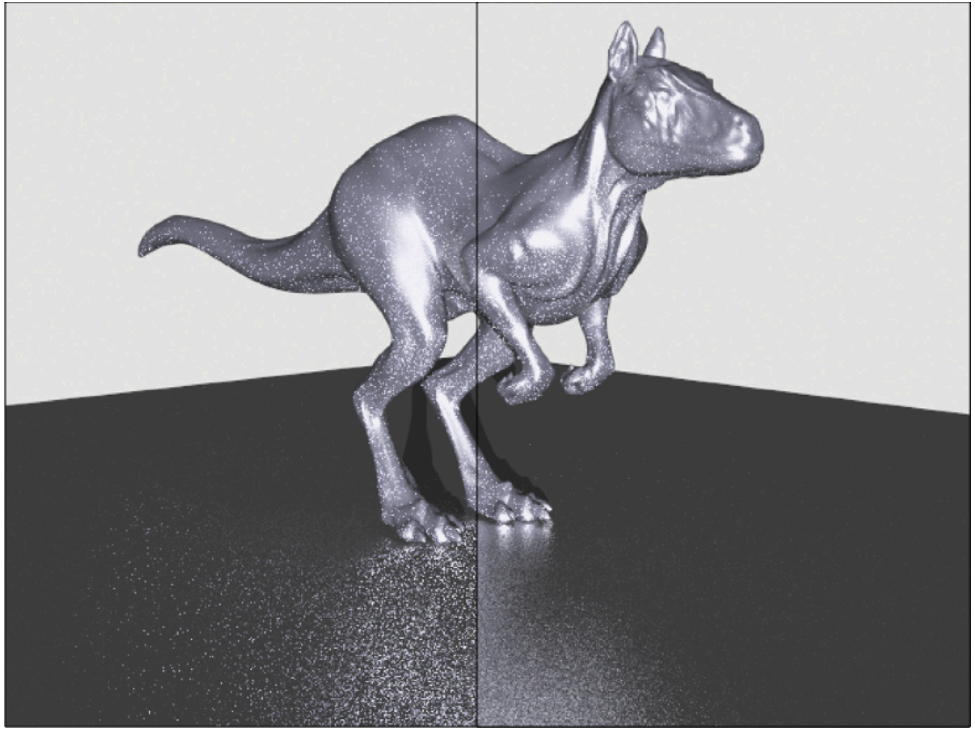


BRDF Importance sampling

$$p(\vec{\omega}_i) \propto f(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r)$$

Sampling BRDFs

uniform hemispherical sampling

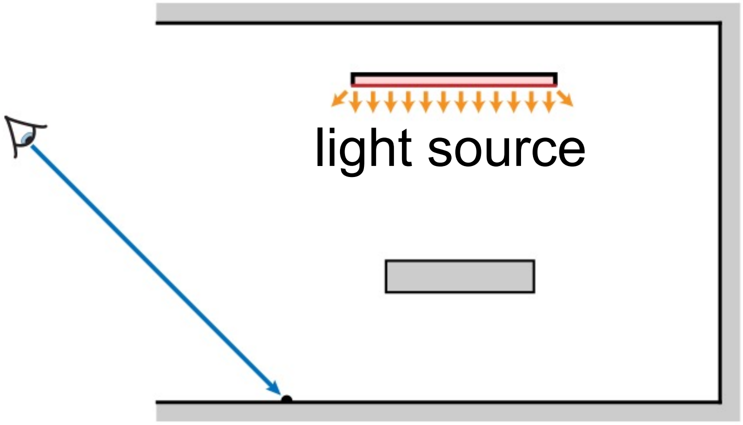


BRDF importance sampling

Sampling Lights

- What to importance sample?

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$



sample emissive surfaces only

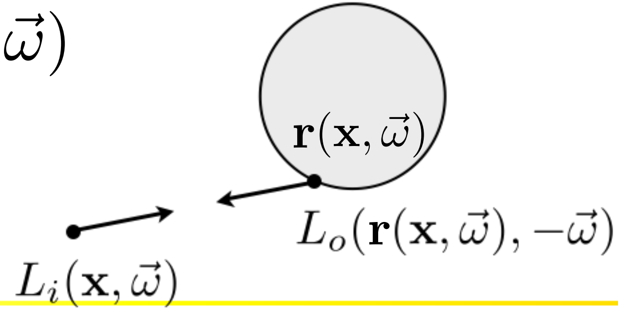
Global Illumination

- The rendering equation

$$L_o(\mathbf{x}, \vec{\omega}_o) = L_e(\mathbf{x}, \vec{\omega}_o) + \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_o) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

- No participating media → radiance is constant along rays
- We can relate incoming radiance to outgoing radiance

$$L_i(\mathbf{x}, \vec{\omega}) = L_o(\mathbf{r}(\mathbf{x}, \vec{\omega}), -\vec{\omega})$$



Global Illumination

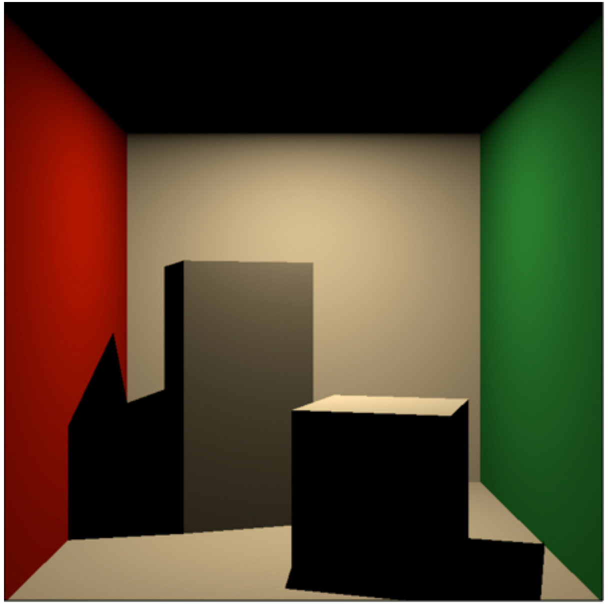
- The rendering equation

$$L(\mathbf{x},\vec{\omega}) = L_e(\mathbf{x},\vec{\omega}) + \int_{H^2} f_r(\mathbf{x},\vec{\omega}',\vec{\omega}) L(\mathbf{r}(\mathbf{x},\vec{\omega}'),-\vec{\omega}') \cos\theta' d\vec{\omega}'$$

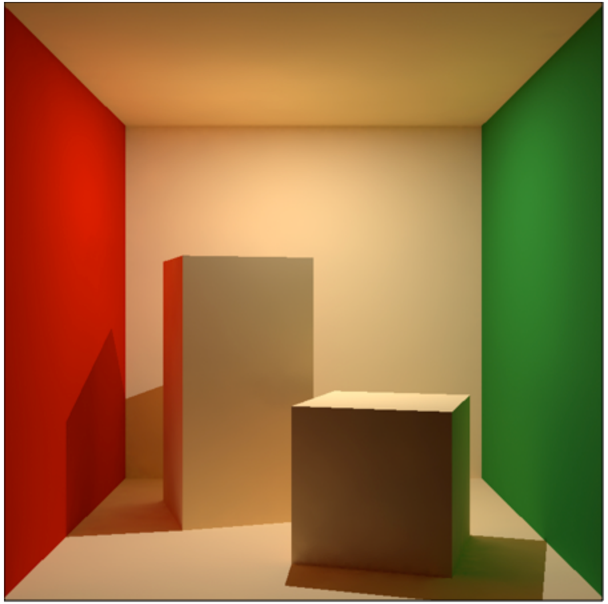
ray tracing function

- Only outgoing radiance functions, we drop o subscript
- Fredholm equation of the second kind (recursive)

Global Illumination



local illumination



global illumination

Global Illumination

- Subsurface scattering

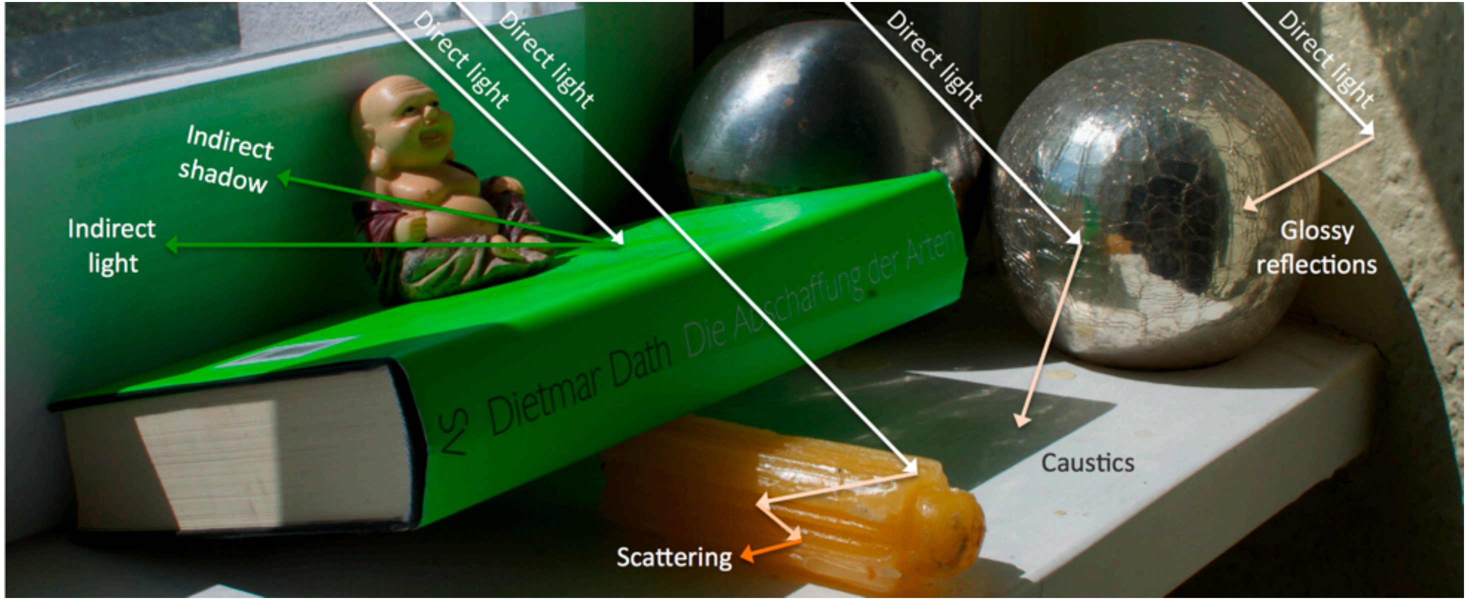
Global Illumination

- Caustics



Global Illumination

- And more



Solving the rendering equation

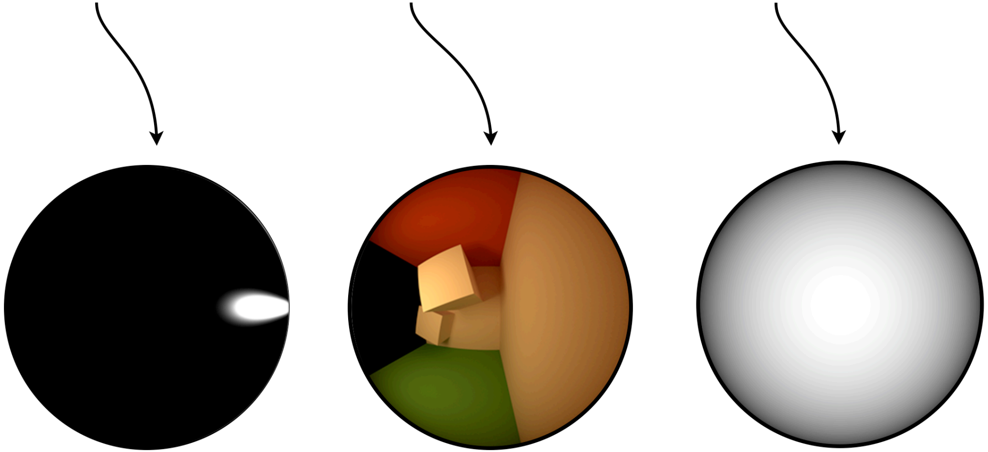
$$L(\mathbf{x},\vec{\omega}) = L_e(\mathbf{x},\vec{\omega}) + \int_{H^2} f_r(\mathbf{x},\vec{\omega}',\vec{\omega}) L(\mathbf{r}(\mathbf{x},\vec{\omega}'),-\vec{\omega}') \cos\theta' d\vec{\omega}'$$

Monte Carlo methods

- **Unbiased methods**
  - Recursive ray tracing
  - Path tracing and light tracing
  - Bidirectional path tracing
- **Biased methods**
  - Many-light algorithms
  - Density estimation
  - Photon mapping
  - Irradiance caching

Solving the rendering equation

$$L(\mathbf{x},\vec{\omega}) = L_e(\mathbf{x},\vec{\omega}) + \int_{H^2} f_r(\mathbf{x},\vec{\omega}',\vec{\omega})L(\mathbf{r}(\mathbf{x},\vec{\omega}'),-\vec{\omega}')\cos\theta' d\vec{\omega}'$$



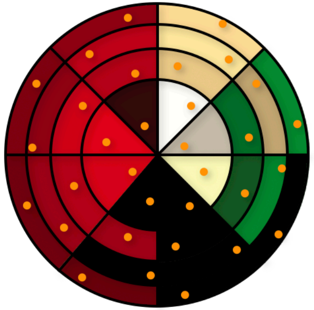
How do we get this?

Recursive Ray Tracing

$$L(\mathbf{x},\vec{\omega}) = L_e(\mathbf{x},\vec{\omega}) + \int_{H^2} f_r(\mathbf{x},\vec{\omega}',\vec{\omega}) L(\mathbf{r}(\mathbf{x},\vec{\omega}'),-\vec{\omega}') \cos\theta' d\vec{\omega}'$$

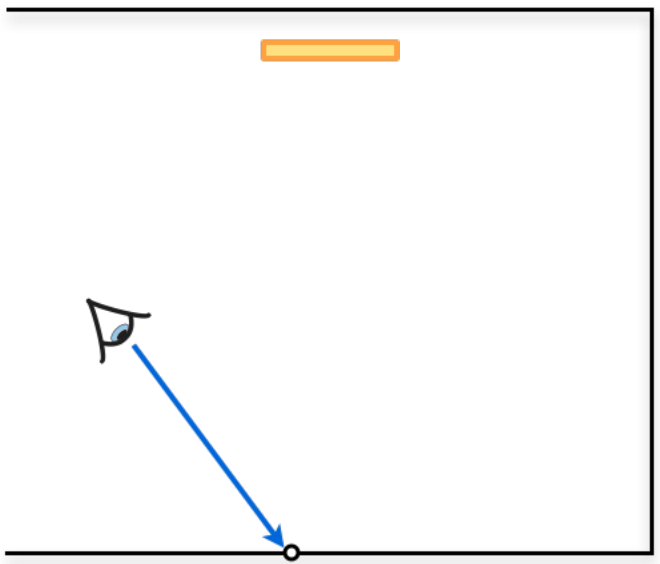
$$L(\mathbf{x},\vec{\omega}) \approx L_e(\mathbf{x},\vec{\omega}) + \frac{1}{N}\sum_{i=1}^{N} \frac{f_r(\vec{\mathbf{x}},\vec{\omega}',\vec{\omega}) L(\mathbf{r}(\mathbf{x},\vec{\omega}'),-\vec{\omega}') \cos\theta'}{p(\vec{\omega}')}$$

Sample the hemisphere -- recursively



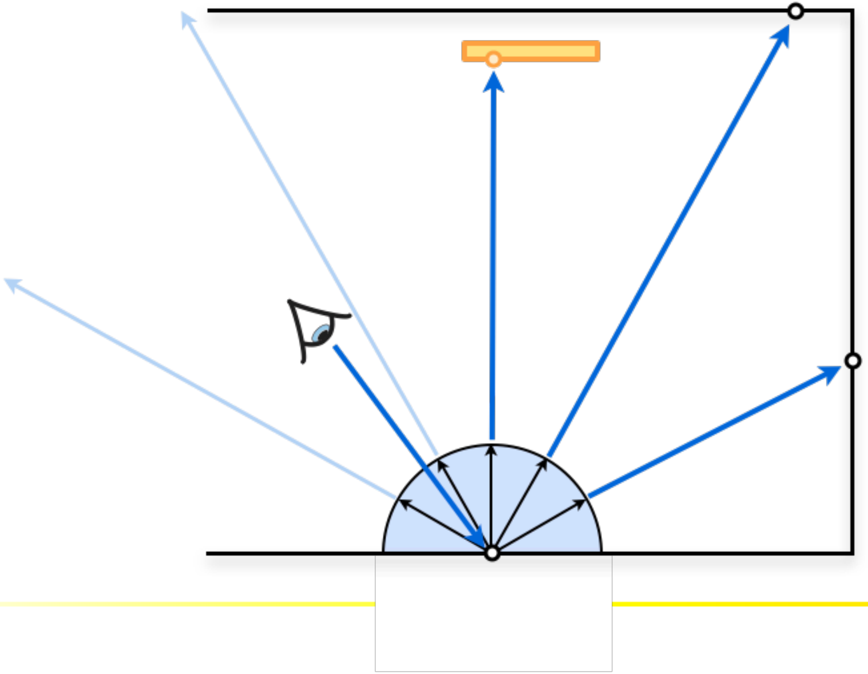
Recursive Ray Tracing

$$L(\mathbf{x},\vec{\omega}) \approx L_e(\mathbf{x},\vec{\omega}) + \frac{1}{N}\sum_{i=1}^{N} \frac{f_r(\vec{\mathbf{x}},\vec{\omega}',\vec{\omega})L(\mathbf{r}(\mathbf{x},\vec{\omega}'),-\vec{\omega}')\cos\theta'}{p(\vec{\omega}')}$$



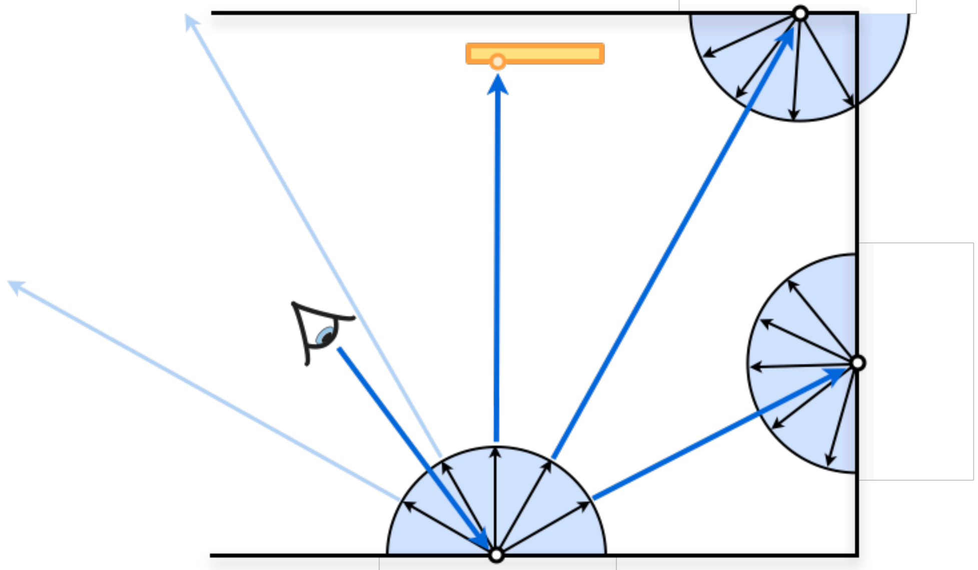
Recursive Ray Tracing

$$L(\mathbf{x},\vec{\omega}) \approx L_e(\mathbf{x},\vec{\omega}) + \frac{1}{N}\sum_{i=1}^{N} \frac{f_r(\vec{\mathbf{x}},\vec{\omega}',\vec{\omega})L(\mathbf{r}(\mathbf{x},\vec{\omega}'),-\vec{\omega}')\cos\theta'}{p(\vec{\omega}')}$$



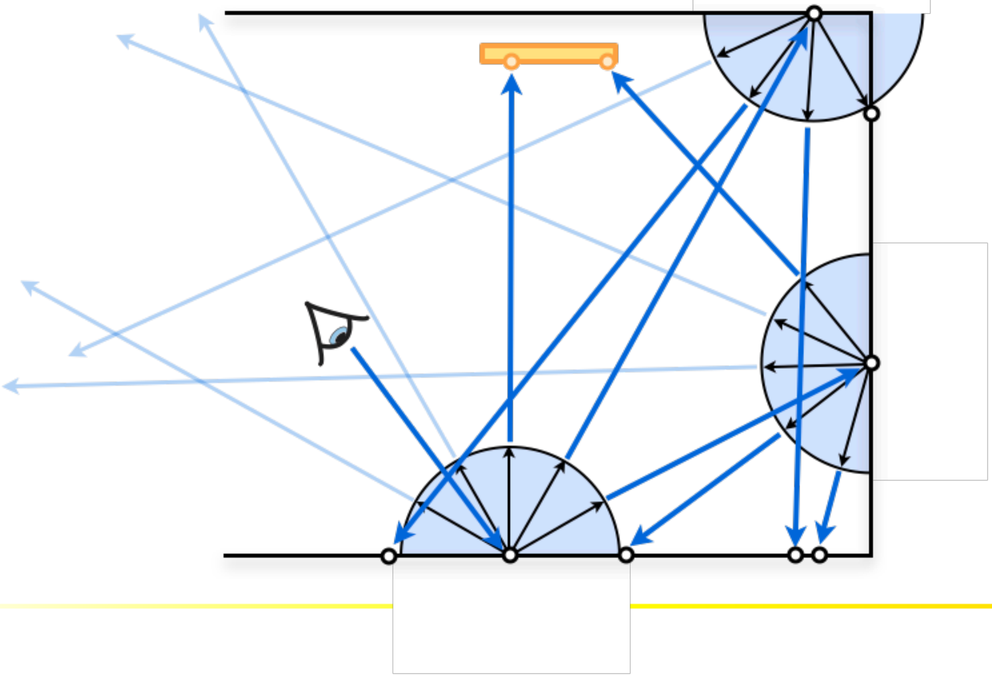
Recursive Ray Tracing

$$L(\mathbf{x},\vec{\omega}) \approx L_e(\mathbf{x},\vec{\omega}) + \frac{1}{N}\sum_{i=1}^{N} \frac{f_r(\vec{\mathbf{x}},\vec{\omega}',\vec{\omega})L(\mathbf{r}(\mathbf{x},\vec{\omega}'),-\vec{\omega}')\cos\theta'}{p(\vec{\omega}')}$$



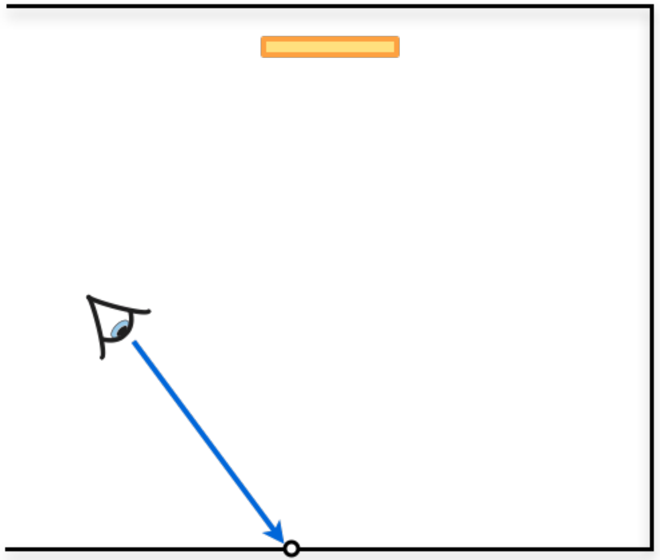
Recursive Ray Tracing

$$L(\mathbf{x}, \vec{\omega}) \approx L_e(\mathbf{x}, \vec{\omega}) + \frac{1}{N} \sum_{i=1}^{N} \frac{f_r(\vec{\mathbf{x}}, \vec{\omega}', \vec{\omega}) L(\mathbf{r}(\mathbf{x}, \vec{\omega}'), -\vec{\omega}') \cos\theta'}{p(\vec{\omega}')}$$



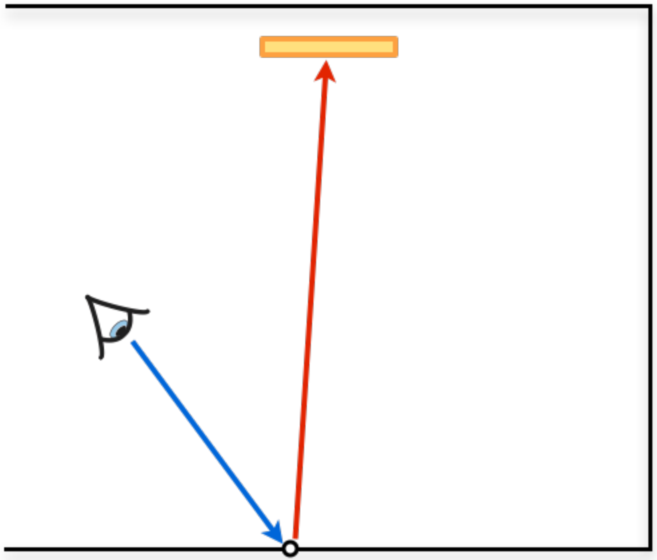
Recursive Ray Tracing

$$L(\mathbf{x},\vec{\omega}) \approx L_e(\mathbf{x},\vec{\omega}) + \frac{1}{N}\sum_{i=1}^{N}\frac{f_r(\vec{\mathbf{x}},\vec{\omega}',\vec{\omega})L(\mathbf{r}(\mathbf{x},\vec{\omega}'),-\vec{\omega}')\cos\theta'}{p(\vec{\omega}')}$$ with shadow rays



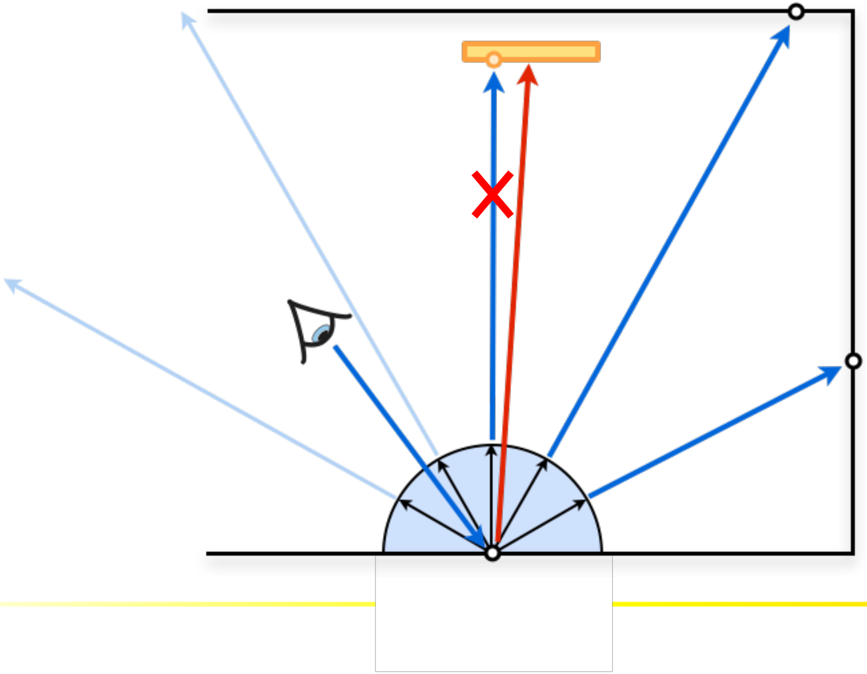
Recursive Ray Tracing

$$L(\mathbf{x}, \vec{\omega}) \approx L_e(\mathbf{x}, \vec{\omega}) + \frac{1}{N}\sum_{i=1}^{N} \frac{f_r(\vec{\mathbf{x}}, \vec{\omega}', \vec{\omega}) L(\mathbf{r}(\mathbf{x}, \vec{\omega}'), -\vec{\omega}') \cos\theta'}{p(\vec{\omega}')}$$ with shadow rays



Recursive Ray Tracing

$$L(\mathbf{x},\vec{\omega}) \approx L_e(\mathbf{x},\vec{\omega}) + \frac{1}{N}\sum_{i=1}^{N} \frac{f_r(\vec{\mathbf{x}},\vec{\omega}',\vec{\omega})L(\mathbf{r}(\mathbf{x},\vec{\omega}'),-\vec{\omega}')\cos\theta'}{p(\vec{\omega}')}$$ with shadow rays



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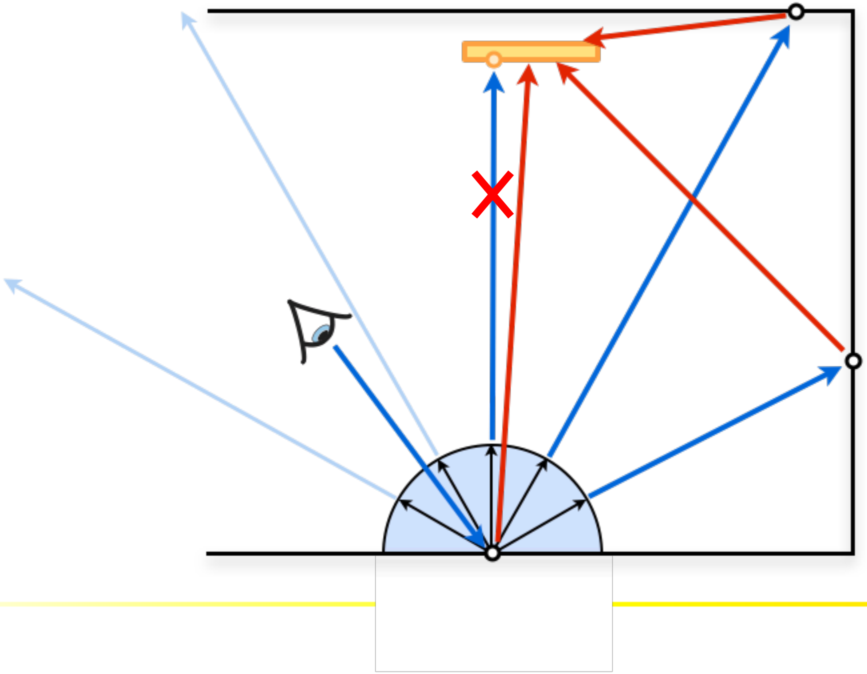
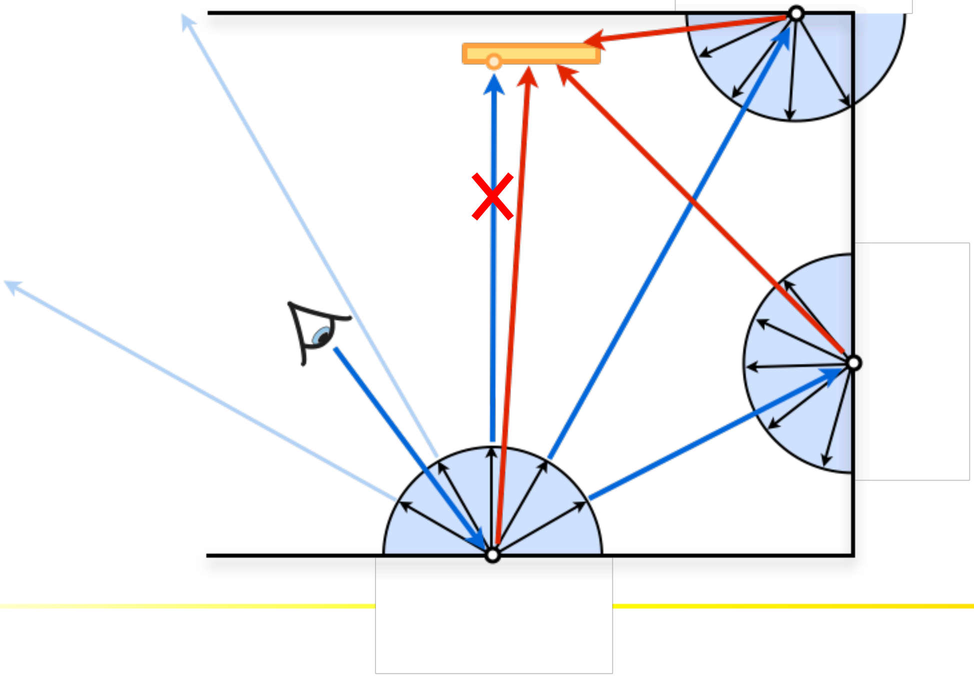
Recursive Ray Tracing

$$L(\mathbf{x},\vec{\omega}) \approx L_e(\mathbf{x},\vec{\omega}) + \frac{1}{N}\sum_{i=1}^{N}\frac{f_r(\vec{\mathbf{x}},\vec{\omega}',\vec{\omega})L(\mathbf{r}(\mathbf{x},\vec{\omega}'),-\vec{\omega}')\cos\theta'}{p(\vec{\omega}')}$$ with shadow rays



Recursive Ray Tracing

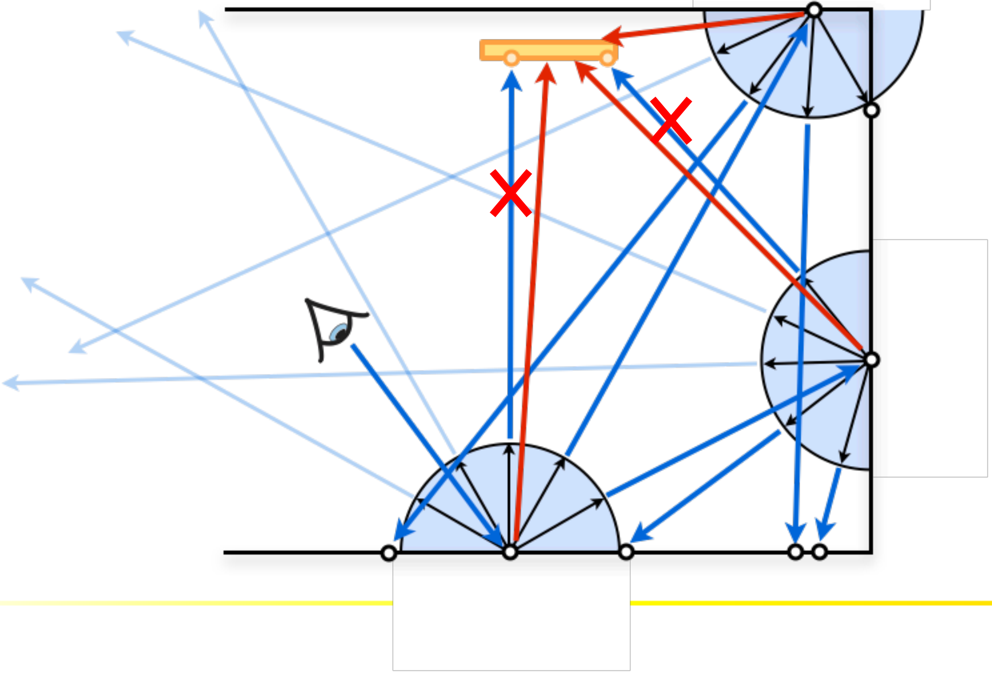
$$L(\mathbf{x},\vec{\omega}) \approx L_e(\mathbf{x},\vec{\omega}) + \frac{1}{N}\sum_{i=1}^{N}\frac{f_r(\vec{\mathbf{x}},\vec{\omega}',\vec{\omega})L(\mathbf{r}(\mathbf{x},\vec{\omega}'),-\vec{\omega}')\cos\theta'}{p(\vec{\omega}')}$$ with shadow rays



Recursive Ray Tracing

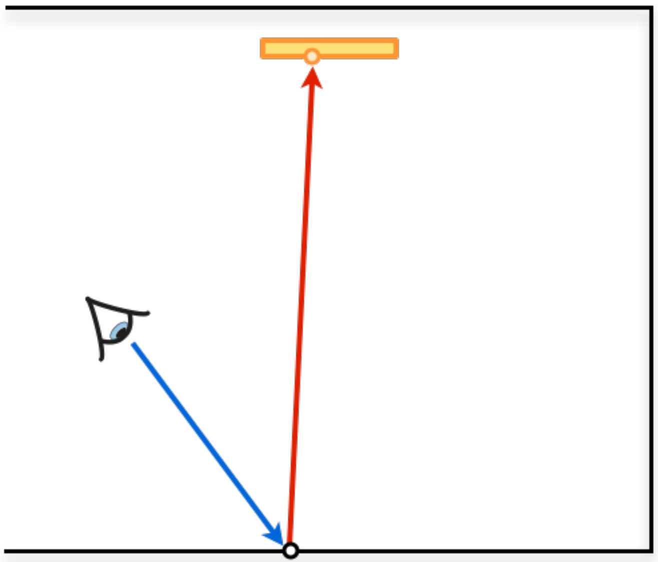
$$L(\mathbf{x},\vec{\omega}) \approx L_e(\mathbf{x},\vec{\omega}) + \frac{1}{N}\sum_{i=1}^{N}\frac{f_r(\vec{\mathbf{x}},\vec{\omega}',\vec{\omega})L(\mathbf{r}(\mathbf{x},\vec{\omega}'),-\vec{\omega}')\cos\theta'}{p(\vec{\omega}')}$$

with shadow rays



Path Tracing

$$L(\mathbf{x},\vec{\omega}) \approx L_e(\mathbf{x},\vec{\omega}) + \frac{1}{N}\sum_{i=1}^{N}\frac{f_r(\vec{\mathbf{x}},\vec{\omega}',\vec{\omega})L(\mathbf{r}(\mathbf{x},\vec{\omega}'),-\vec{\omega}')\cos\theta'}{p(\vec{\omega}')}$$ with shadow rays



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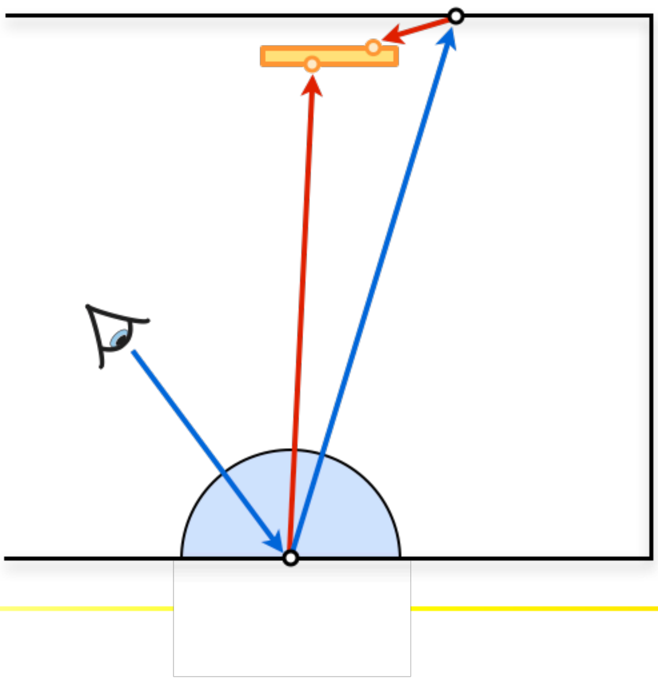
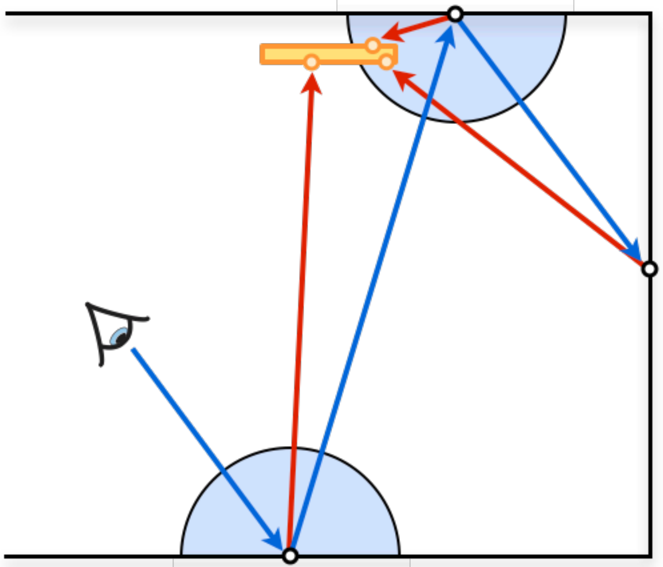
Path Tracing

$$L(\mathbf{x},\vec{\omega}) \approx L_e(\mathbf{x},\vec{\omega}) + \frac{1}{N}\sum_{i=1}^{N} \frac{f_r(\vec{\mathbf{x}},\vec{\omega}',\vec{\omega})L(\mathbf{r}(\mathbf{x},\vec{\omega}'),-\vec{\omega}')\cos\theta'}{p(\vec{\omega}')}$$ with shadow rays



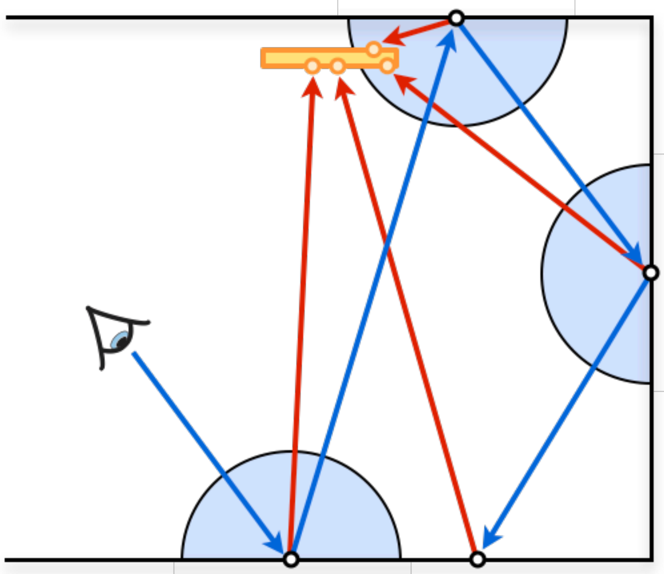
Path Tracing

$$L(\mathbf{x},\vec{\omega}) \approx L_e(\mathbf{x},\vec{\omega}) + \frac{1}{N}\sum_{i=1}^{N}\frac{f_r(\vec{\mathbf{x}},\vec{\omega}',\vec{\omega})L(\mathbf{r}(\mathbf{x},\vec{\omega}'),-\vec{\omega}')\cos\theta'}{p(\vec{\omega}')}$$ with shadow rays



Path Tracing

$$L(\mathbf{x},\vec{\omega}) \approx L_e(\mathbf{x},\vec{\omega}) + \frac{1}{N}\sum_{i=1}^{N}\frac{f_r(\vec{\mathbf{x}},\vec{\omega}',\vec{\omega})L(\mathbf{r}(\mathbf{x},\vec{\omega}'),-\vec{\omega}')\cos\theta'}{p(\vec{\omega}')}$$ with shadow rays



Path Tracing

- Full solution to the rendering equation
- Simple to implement
- Slow convergence
    - requires 4x more samples to half the error
- Robustness issues
    - does not handle some light paths well, e.g. caustics
    - No reuse or caching of computation
- General sampling issue
    - makes only locally optimal decisions

Path Tracing

Nature ~ **2 . 10^{33}** / second

Fastest GPU ray tracer ~ **2 . 10^{8}** / second