The Rendering Equation

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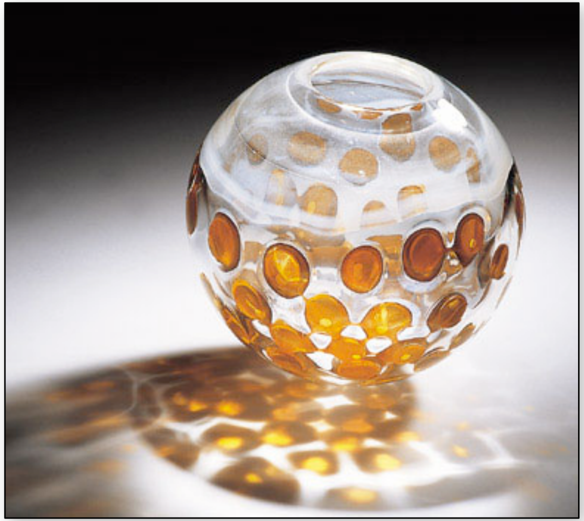
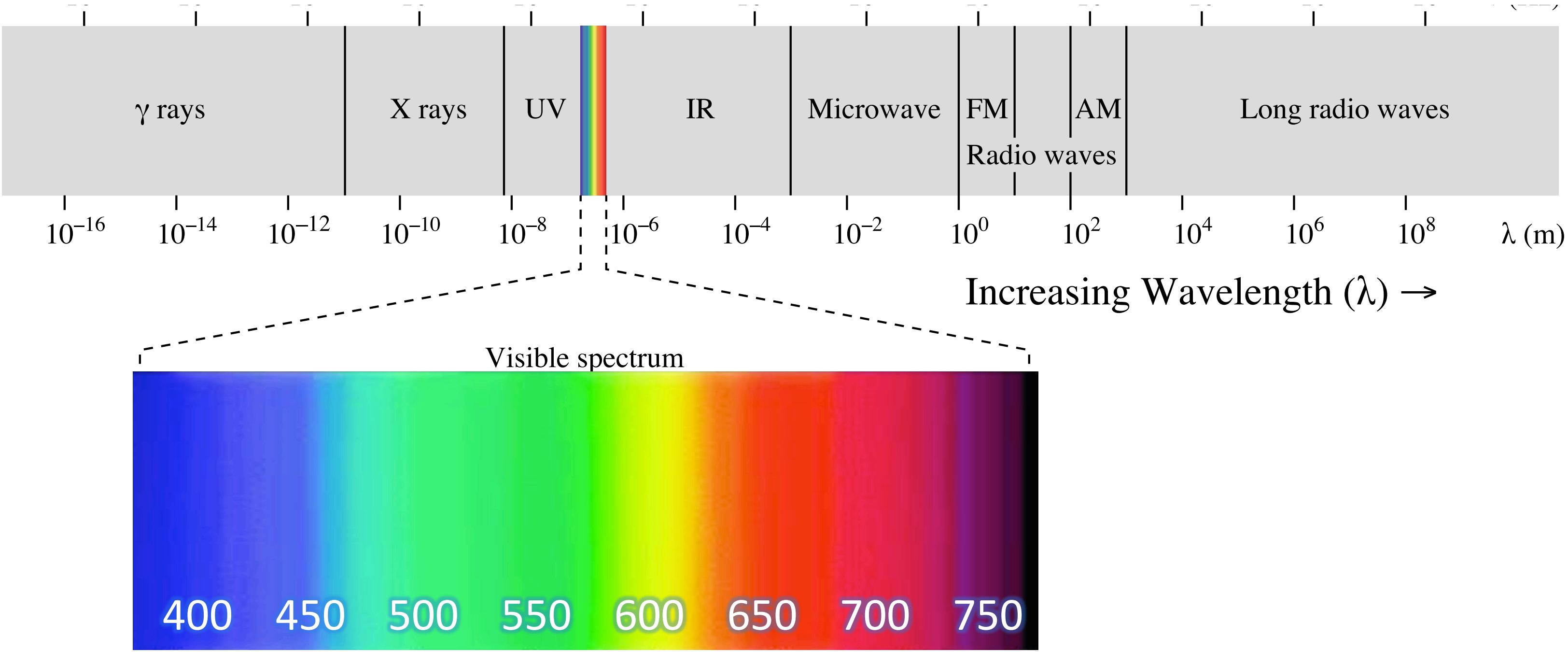
Rendering – Simulating Light

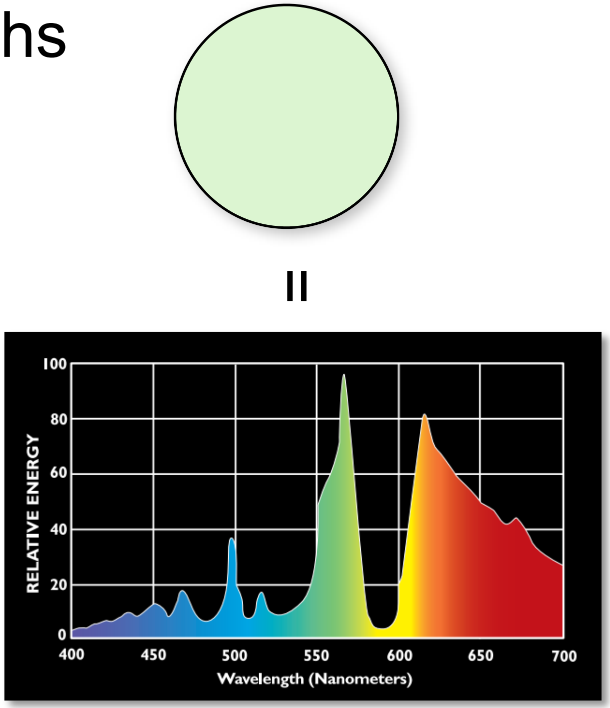


Light and Colors



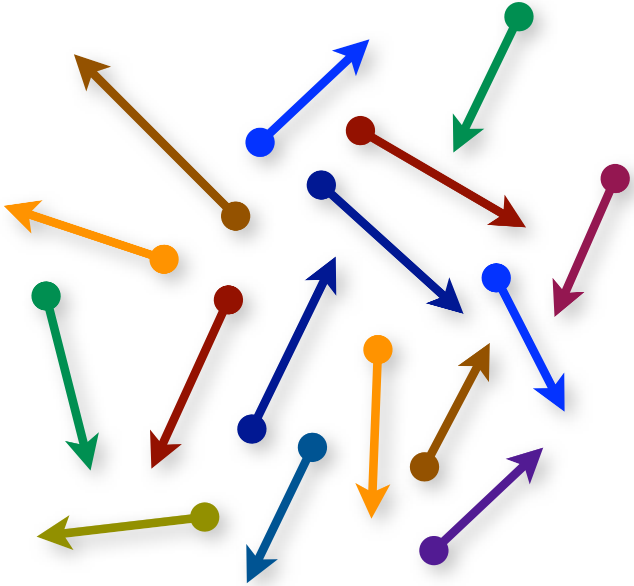
Light and Colors

- Light can be a mixture of many wavelengths
- Spectral power distribution (SPD)
  - $P(\lambda)$ = intensity at wavelength λ
  - intensity as a function of wavelength
- We perceive these distributions as colors



Measuring Light

- How do we measure light



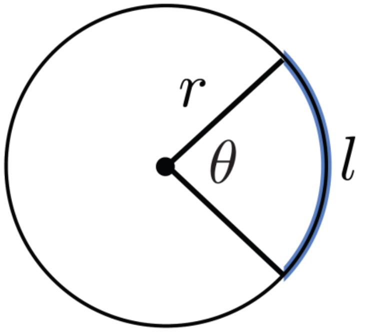
Measuring = Counting photons

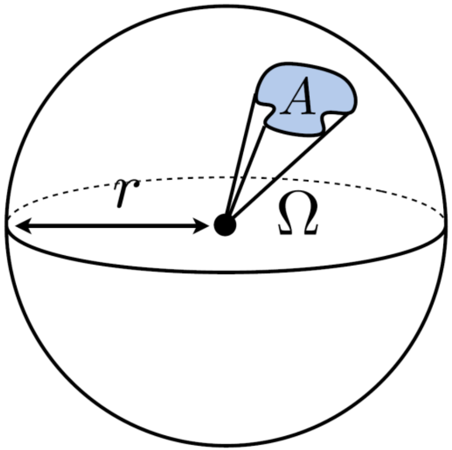
Measuring Light

- Radiometry
  - Studies the measurement of electromagnetic radiation, including visible light

Basic Definitions

- Angle: $\theta = \frac{l}{r}$
  - circle: 2π radians
- Solid angle: $\Omega = \frac{A}{r^2}$
  - sphere: 4π steradians



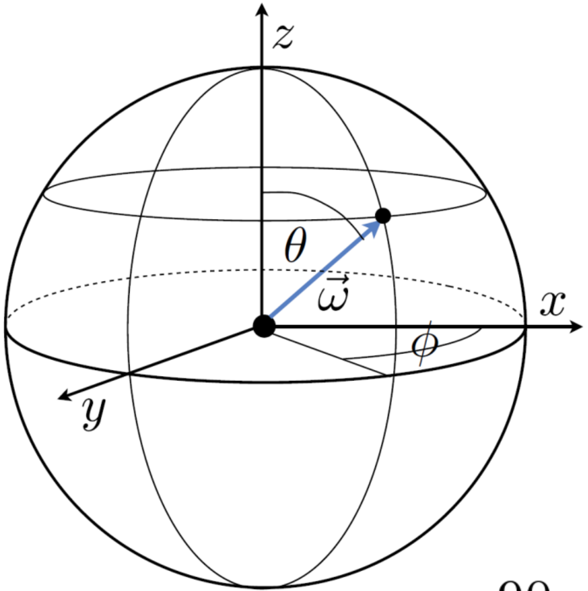


Basic Definitions

- Direction
  - point on the unit sphere
  - parameterized by two angles

$$\vec{\omega} = (\theta, \phi)$$

zenith (θ) azimuth (ϕ)



$$\vec{\omega}_x = \sin\theta\cos\phi$$

$$\vec{\omega}_y = \sin\theta\sin\phi$$

$$\vec{\omega}_z = \cos\theta$$

$$latitude = \frac{90}{\pi}(\pi - \theta) \qquad longitude = \frac{90}{\pi}\phi$$

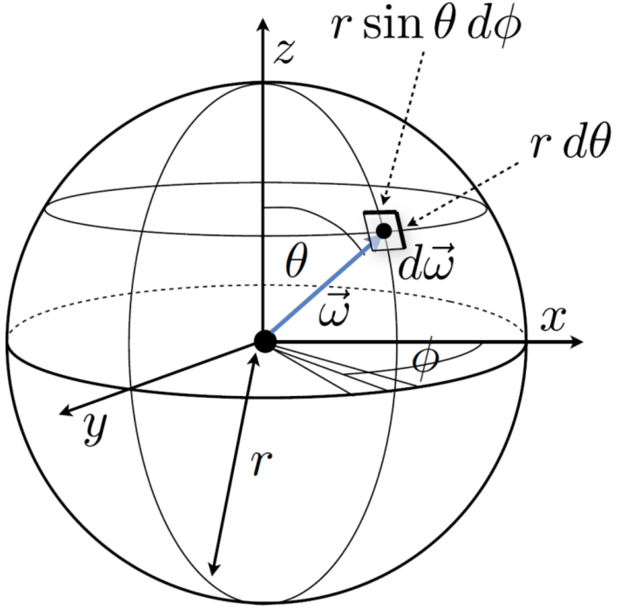
Basic Definitions

- Differential Solid Angle

$$dA = (r d\theta)(r \sin\theta d\phi)$$

$$d\vec{\omega} = \frac{dA}{r^2} = \sin\theta d\theta d\phi$$

$$\Omega = \int_{S^2} d\vec{\omega} = \int_0^{2\pi} \int_0^{\pi} \sin\theta d\theta d\phi = 4\pi$$



Basic Definitions

- Assume light consists of photons with
  - $\mathbf{x}$: Position
  - $\vec{\omega}$: Direction of motion
  - λ : Wavelength

- Each photon has an energy of: $\dfrac{hc}{\lambda}$
  - $h \approx 6.63 \cdot 10^{-34}\, m^2 \cdot kg/s$: Planck's constant
  - $c = 299{,}792{,}458\, m/s$: speed of light in vacuum
  - Unit of energy, Joule : $[J = kg \cdot m^2/s^2]$

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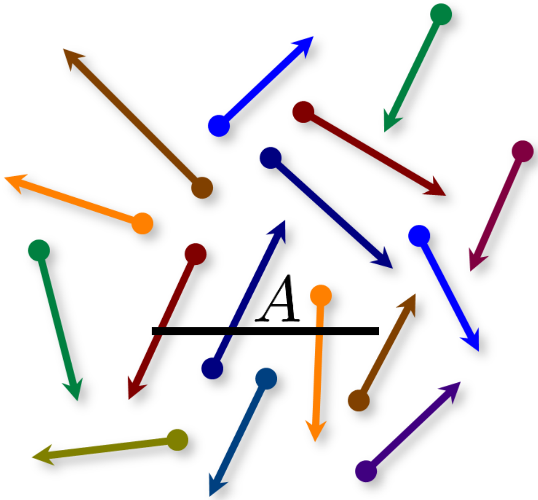
Radiometry

- Flux (radiant flux, power)
  - total amount of energy passing through surface or space per unit time

$$\Phi(A) \qquad \left[\frac{J}{s} = W\right]$$

  - examples:
    - number of photons hitting a wall per second
    - number of photons leaving a lightbulb per second

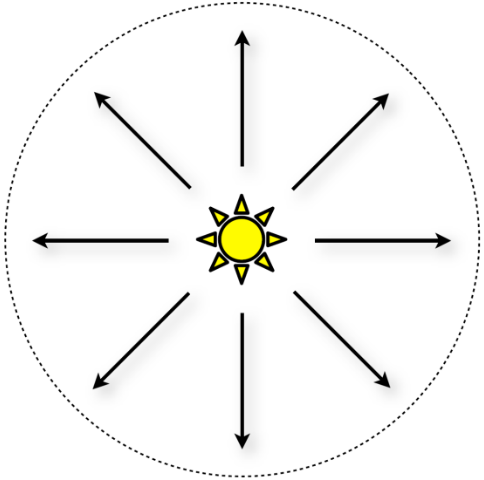


Radiometry

- Radiant intensity
  - Power (flux) per solid angle = directional density of flux

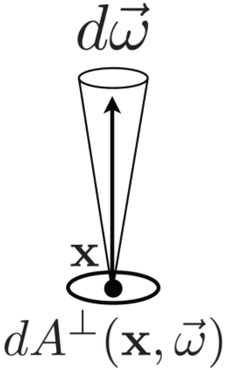
$$I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}} \qquad \left[\frac{W}{sr}\right] \qquad \Phi = \int_{S^2} I(\vec{\omega}) d\vec{\omega}$$

  - example:
    - power per unit solid angle emanating from a point source



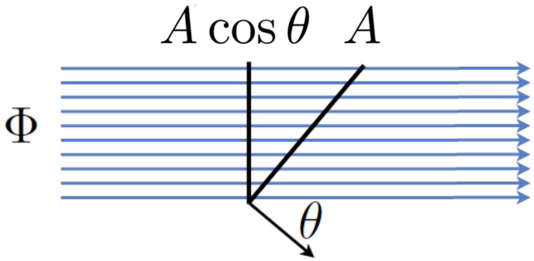
Radiometry

- Radiance
  - Radiant intensity per perpendicular unit area



$$L(\mathbf{x}, \vec{\omega}) = \frac{d^2\Phi(A)}{d\vec{\omega}\, dA^{\perp}(\mathbf{x}, \vec{\omega})}$$

$$= \frac{d^2\Phi(A)}{d\vec{\omega}\, dA(\mathbf{x}) \cos\theta} \quad \left[\frac{W}{m^2 sr}\right]$$



  - remains constant along a ray

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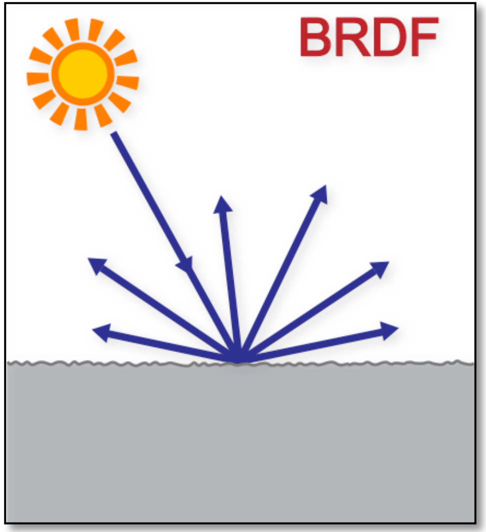
Reflection Models

- **B**idirectional **R**eflectance **D**istribution **F**unction (BRDF)



BRDF

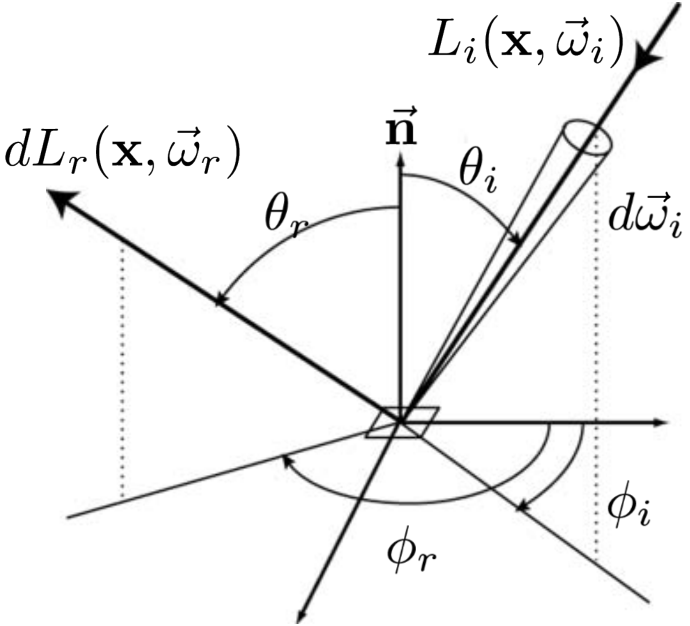
- **B**idirectional **R**eflectance **D**istribution **F**unction

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = \frac{dL_r(\mathbf{x}, \vec{\omega}_r)}{L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i d\vec{\omega}_i} \quad [1/sr]$$

BRDF

infinitesimal
reflected radiance

infinitesimal
solid angle



Reflection Equation

- The BRDF provides a relation between incident radiance and differential reflected radiance
- From this we can derive the **Reflection Equation**

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = \frac{dL_r(\mathbf{x}, \vec{\omega}_r)}{L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i}$$

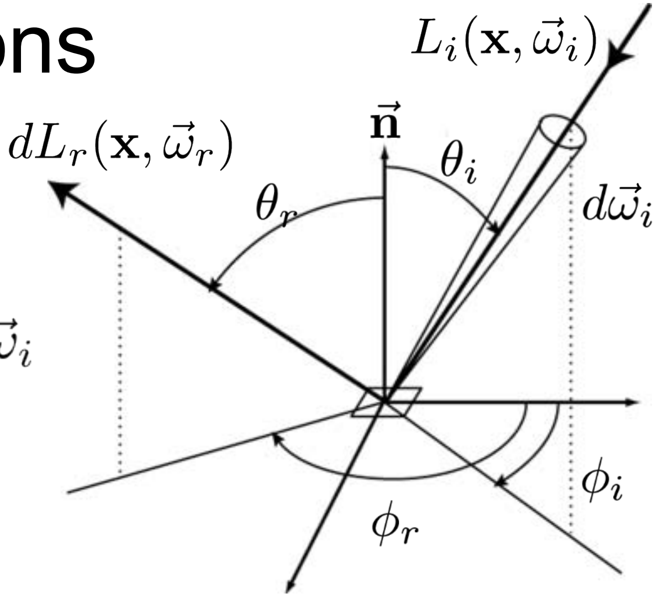
$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i = \frac{dL_r(\mathbf{x}, \vec{\omega}_r)}{d\vec{\omega}_i}$$

$$\int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i = L_r(\mathbf{x}, \vec{\omega}_r)$$

Reflection Equation

- The reflected radiance due to incident illumination from all directions

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$



The Rendering Equation

- The outgoing light is the sum of emitted and incoming

$$L_o(\mathbf{x}, \vec{\omega}_o) = L_e(\mathbf{x}, \vec{\omega}_o) + L_r(\mathbf{x}, \vec{\omega}_o)$$

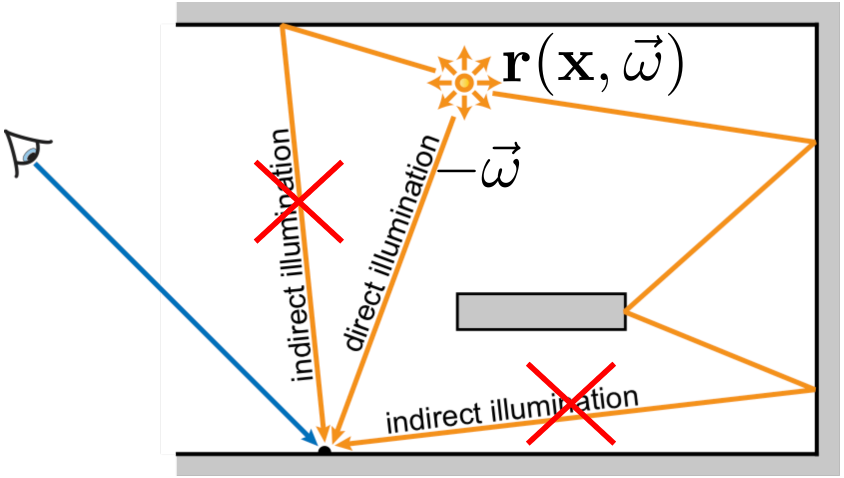
$$\underbrace{L_o(\mathbf{x}, \vec{\omega}_o)}_{\text{outgoing light}} = \underbrace{L_e(\mathbf{x}, \vec{\omega}_o)}_{\text{emitted light}} + \underbrace{\int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_o) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i}_{\text{reflected light}}$$

Energy is conserved!

Direct Illumination

- All light comes directly from emitters, i.e. light sources

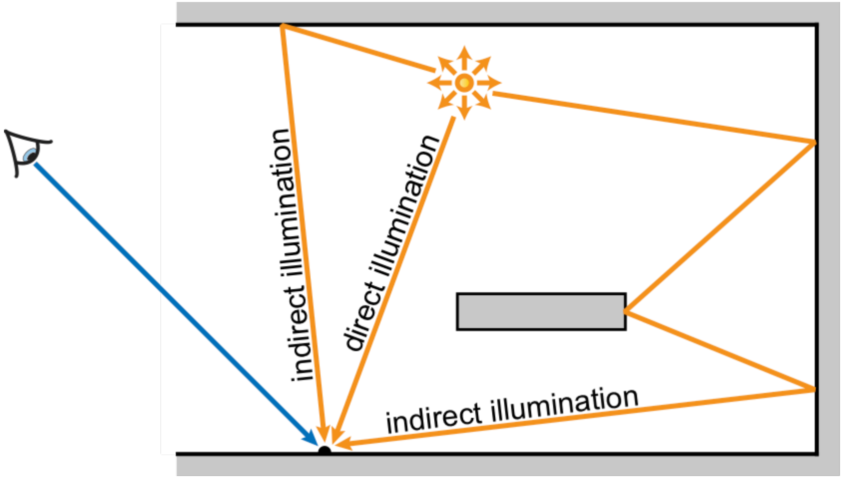
$$L_o(\mathbf{x}, \vec{\omega}_o) = L_e(\mathbf{x}, \vec{\omega}_o) + \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_o) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

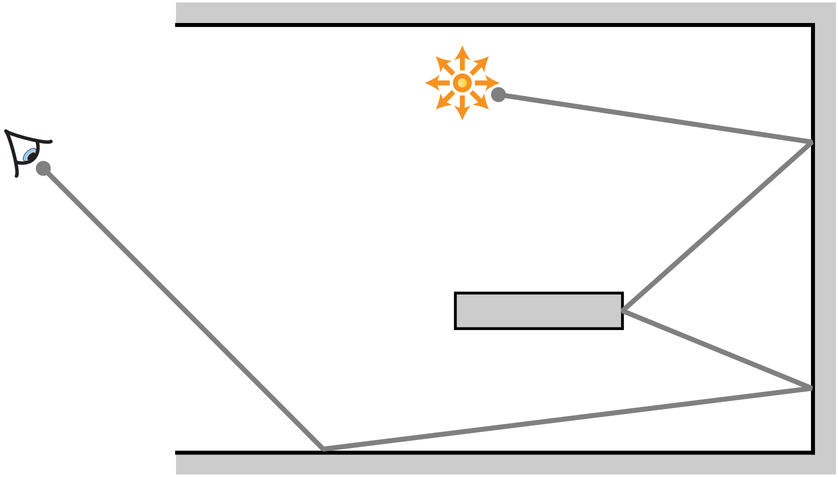


$$L_i(\mathbf{x}, \vec{\omega}) = L_e(\mathbf{r}(\mathbf{x}, \vec{\omega}), -\vec{\omega})$$

Global Illumination

- Consider all light – including bounces





x

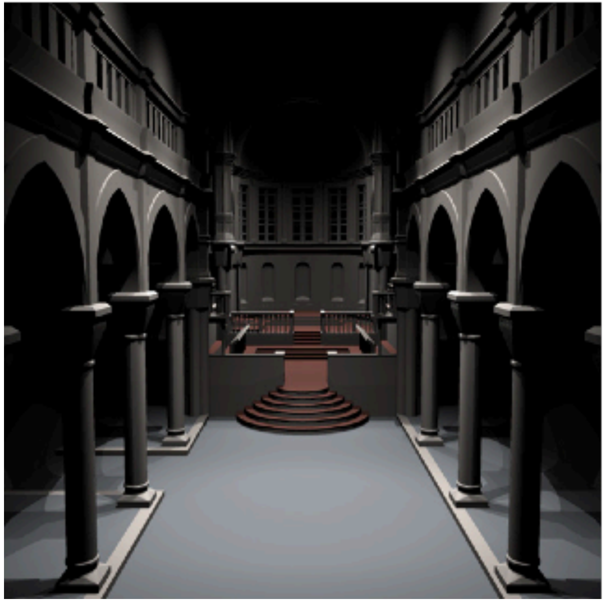
light path

Global Illumination

- Connects a **light source** to a **sensor**
- Constructed by tracing from:
  - light source… *light tracing*
  - from sensor… *path tracing*
  - or from both… *bidirectional path tracing*
- Length of light path:
  - 2 segments… *direct illumination, direct lighting*
  - 2 segments… *indirect illumination, indirect lighting*

Global Illumination

Direct illumination

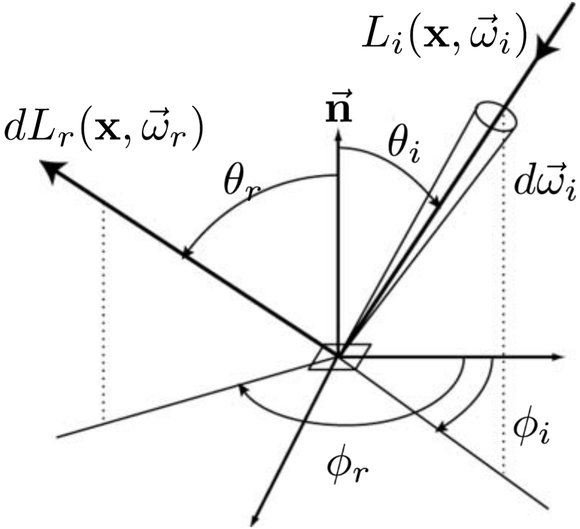


Indirect illumination

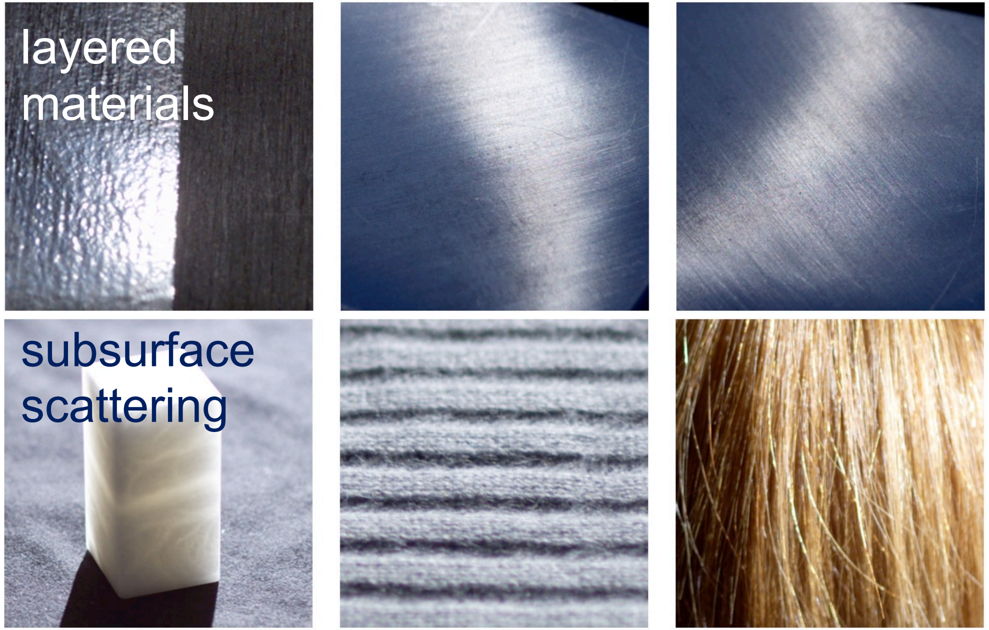
Direct + Indirect

BRDFs - Complex Reflections

$$\int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

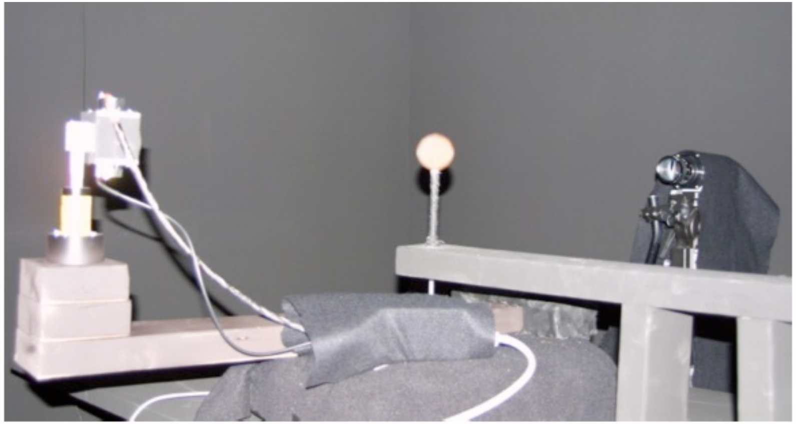


anisotropic reflections

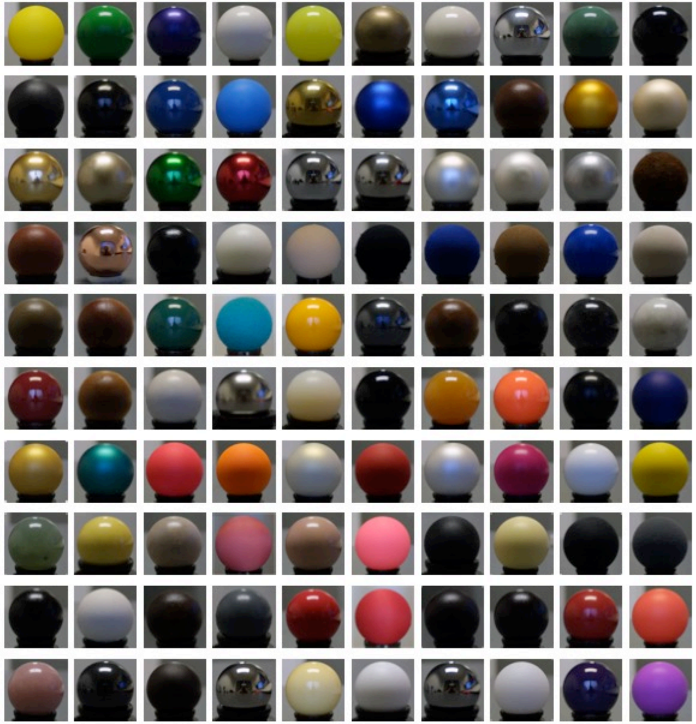


volumetric structures

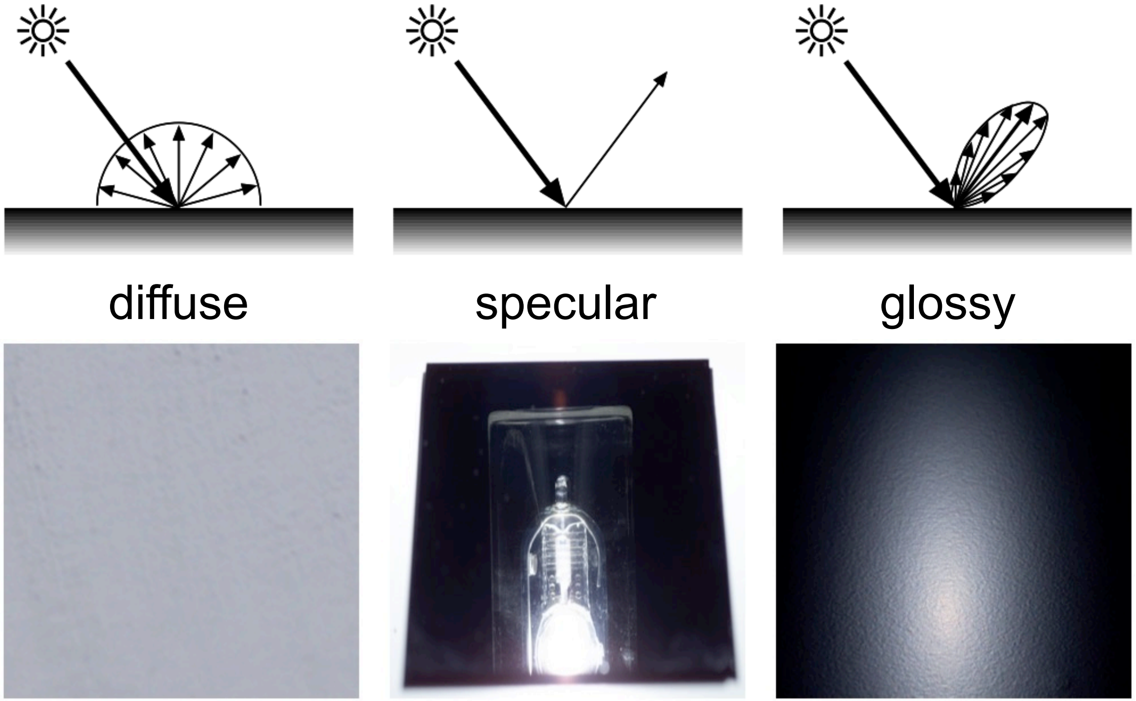
Measuring BRDFs



Matusik et al.: Efficient Isotropic BRDF Measurement, Eurographics Symposium on Rendering 2003



Simpler Reflections



Hendrik Lensch, Efficient Image-Based Appearance Acquisition of Real-World Objects, Ph.D. thesis, 2004

Diffuse Reflection

- For diffuse reflection, the BRDF is a constant:

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

$$L_r(\mathbf{x}) = f_r \int_{H^2} L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

$$L_r(\mathbf{x}) = f_r \, E_i(\mathbf{x})$$