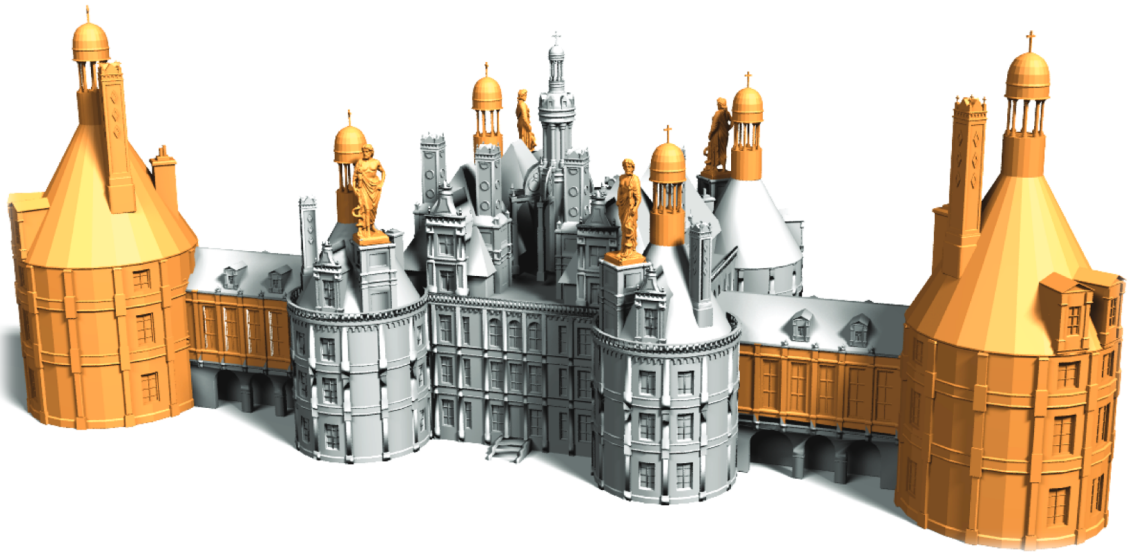
Geometry Representations

Dr Cengiz Öztireli



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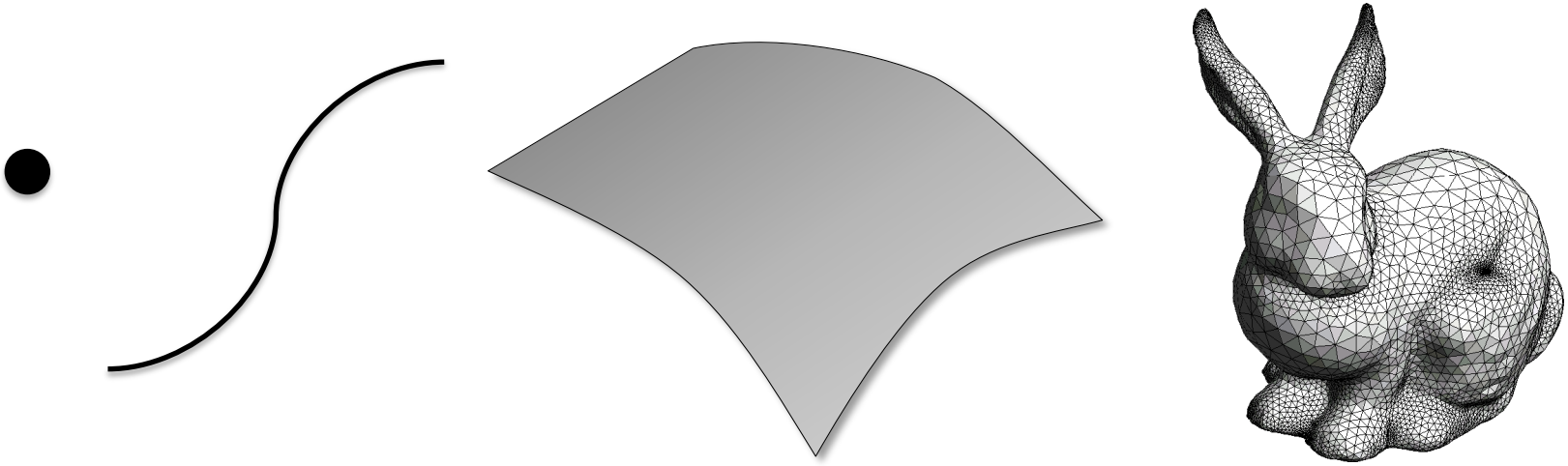
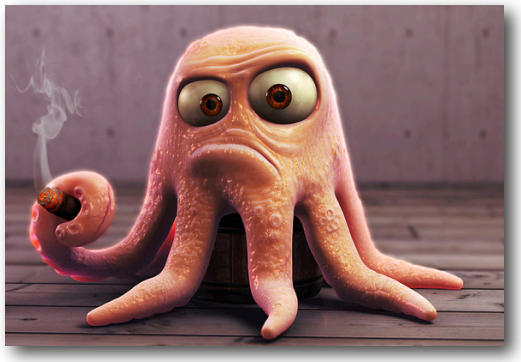
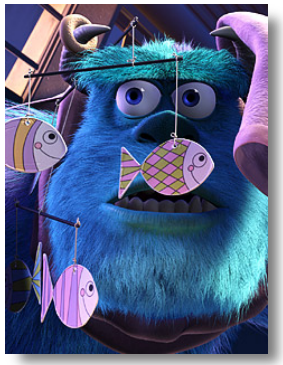
Geometry in Graphics

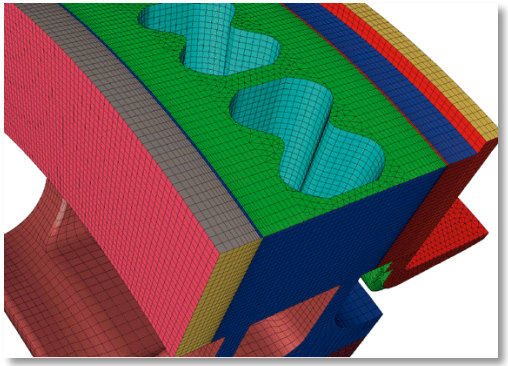


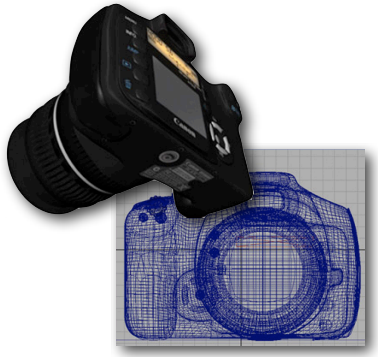
Applications





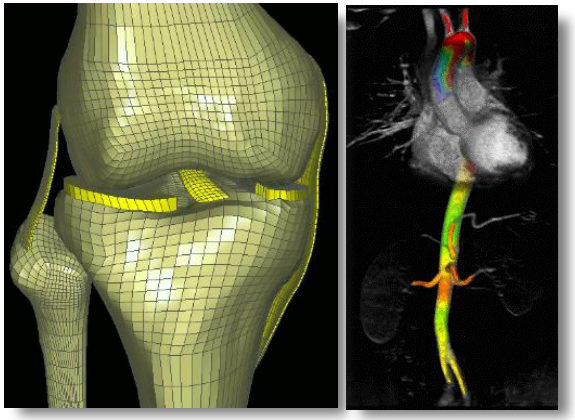
Games/Movies



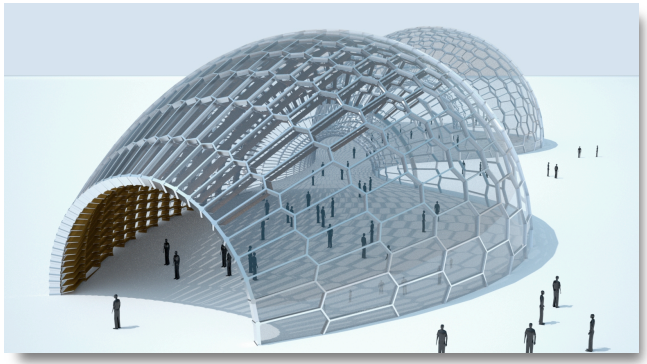


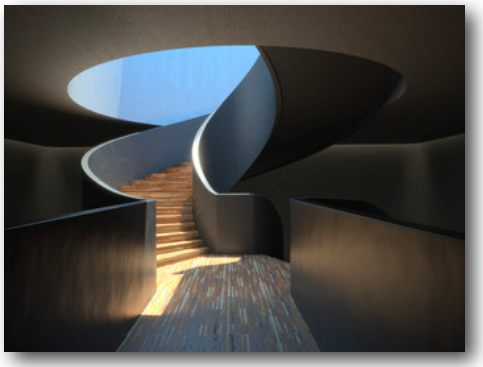
Engineering/Product design

Applications



Medicine/Biology





Architecture

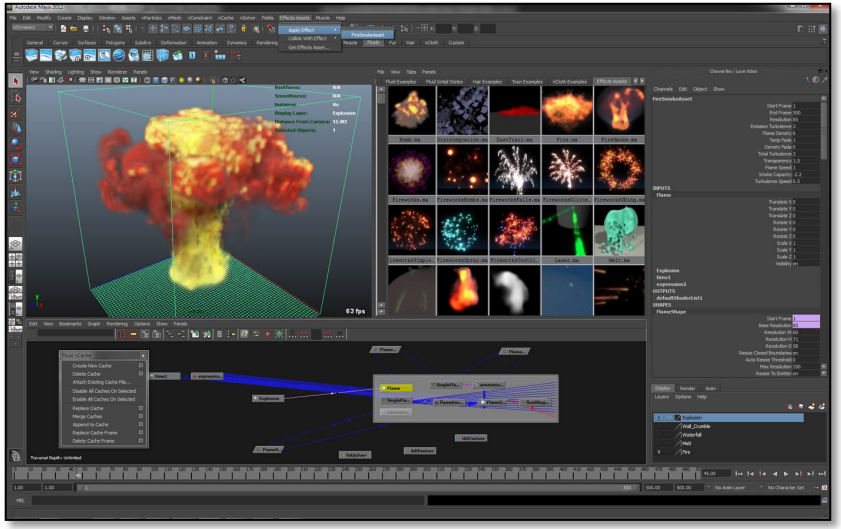
Sources of Geometry

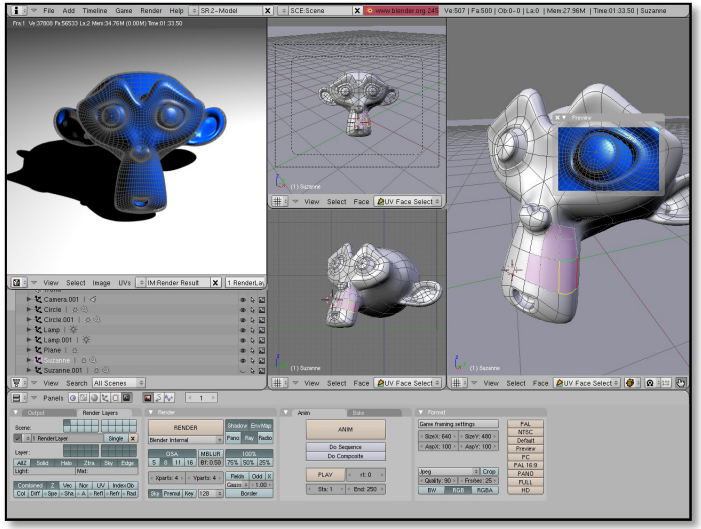
- Acquired real-world objects

3D Scanning

Sources of Geometry

- Digital 3D modeling



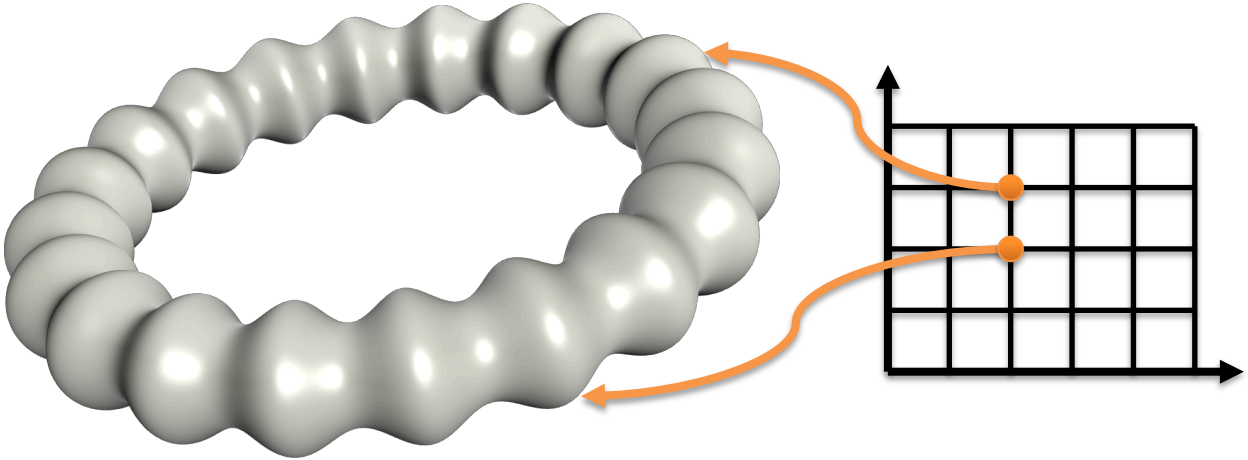


Geometry Representations

- Considerations
  - Storage
  - Acquisition of shapes
  - Creation of shapes
  - Editing shapes
  - Rendering shapes

Geometry Representations

- Parametric curves & surfaces



$$f : X \to Y, X \subseteq \mathbb{R}^m, Y \subseteq \mathbb{R}^n$$

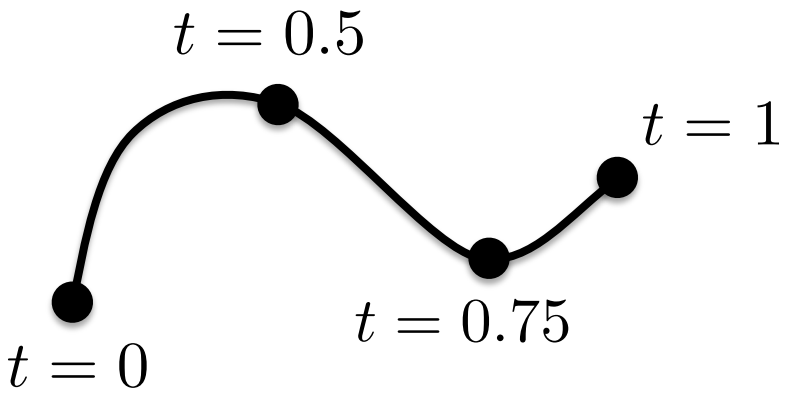
Geometry Representations

- Parametric curves & surfaces

Planar Curves

$$f : X \to Y, X \subseteq \mathbb{R}^m, Y \subseteq \mathbb{R}^n \qquad m = 1, n = 2$$

$$s(t) = (x(t), y(t))$$



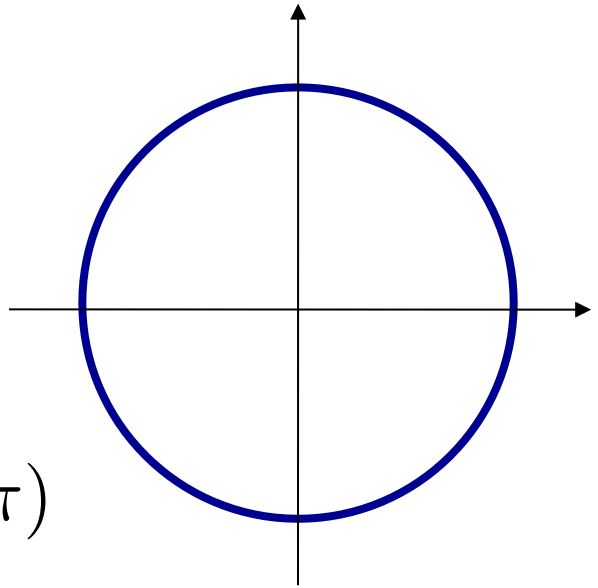
Geometry Representations

- Parametric curves & surfaces

Circle

$$\mathbf{p} : \mathbb{R} \to \mathbb{R}^2$$
$$t \mapsto \mathbf{p}(t) = (x(t), y(t))$$

$$\mathbf{p}(t) = r\,(\cos(t), \sin(t)) \quad t \in [0, 2\pi)$$



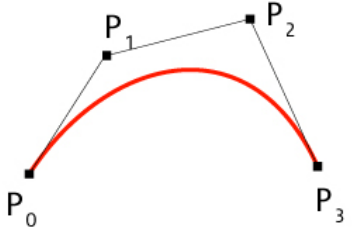
Geometry Representations

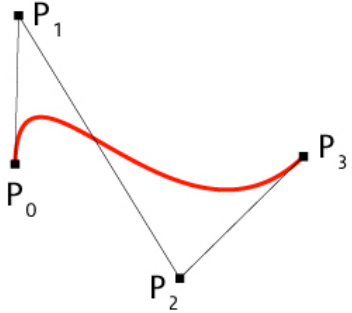
- Parametric curves & surfaces

Bezier Curves

$$s(t) = \sum_{i=0}^{n} \mathbf{p}_i B_i^n(t)$$

$$B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i}$$





Geometry Representations

- Parametric curves & surfaces

Space Curves in 3D

$$f : X \to Y, X \subseteq \mathbb{R}^m, Y \subseteq \mathbb{R}^n$$

$$m = 1, n = 3$$

$$s(t) = (x(t), y(t), z(t))$$

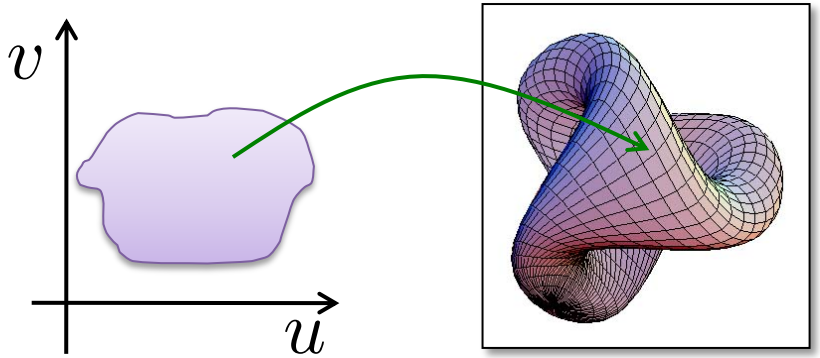
Geometry Representations

- Parametric curves & surfaces

Surfaces

$$f : X \to Y, X \subseteq \mathbb{R}^m, Y \subseteq \mathbb{R}^n \qquad m = 2, n = 3$$

$$s(u, v) = (x(u, v), y(u, v), z(u, v))$$



Geometry Representations

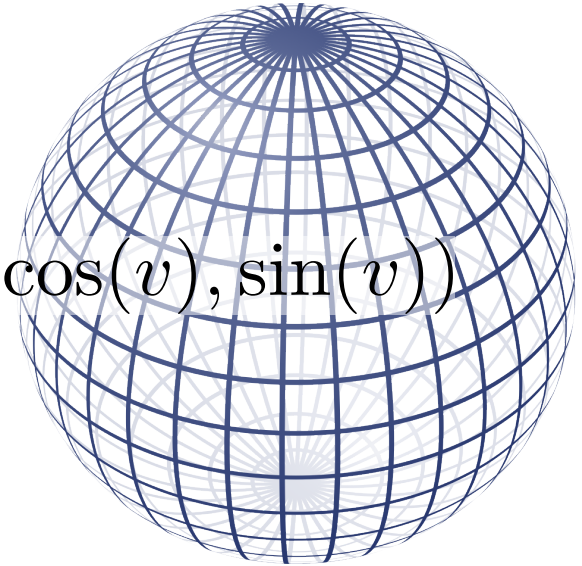
- Parametric curves & surfaces

Sphere

$$s : \mathbb{R}^2 \rightarrow \mathbb{R}^3$$

$$s(u, v) = r\,(\cos(u)\cos(v), \sin(u)\cos(v), \sin(v))$$

$$(u, v) \in [0, 2\pi) \times [-\pi/2, \pi/2]$$

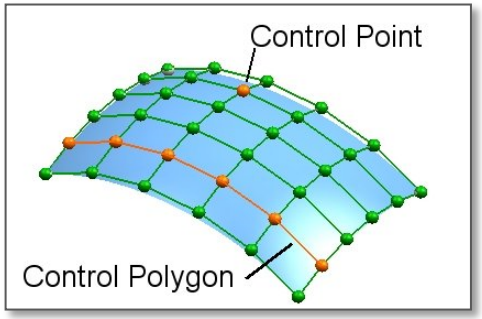


Geometry Representations

- Parametric curves & surfaces

Bezier Surfaces

$$s(u,v) = \sum_{i=0}^{m} \sum_{j=0}^{n} \mathbf{p}_{i,j} B_i^m(u) B_j^n(v)$$



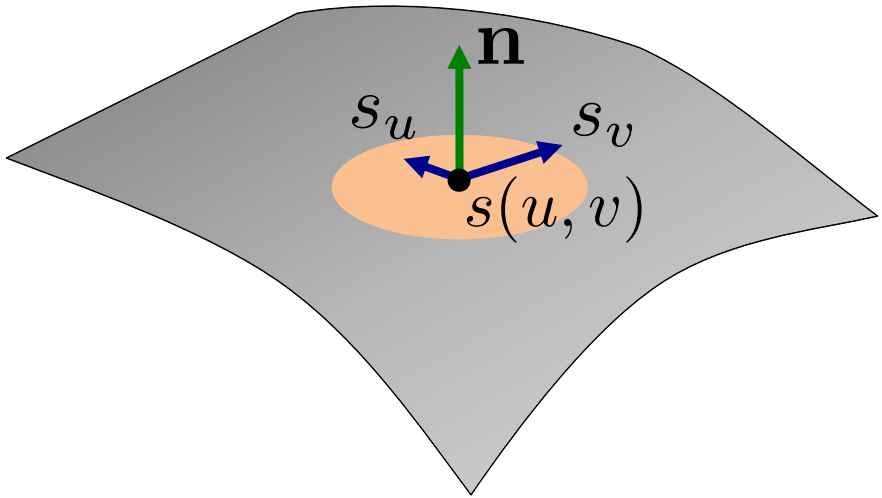
Geometry Representations

- Parametric curves & surfaces

Normal and Tangent plane

$$s_u = \frac{\partial s(u,v)}{\partial u} \qquad s_v = \frac{\partial s(u,v)}{\partial v}$$

$$\mathbf{n} = \frac{s_u \times s_v}{\|s_u \times s_v\|}$$



Geometry Representations

- Parametric curves & surfaces

Volumetric Representations

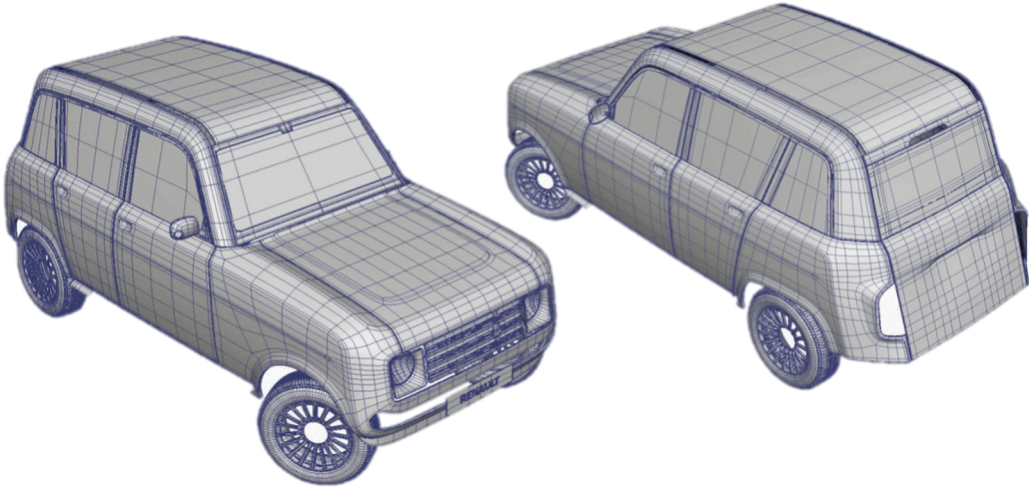
$$f : X \to Y, X \subseteq \mathbb{R}^m, Y \subseteq \mathbb{R}^n \quad m = 3, n = 1$$

Geometry Representations

- Parametric curves & surfaces
  - + Easy to generate points on a curve/surface
  - + Easy point-wise differential properties
  - + Easy to control by hand
  - − Hard to determine inside/outside
  - − Hard to determine if a point is on a curve/surface
  - − Hard to generate by reverse engineering

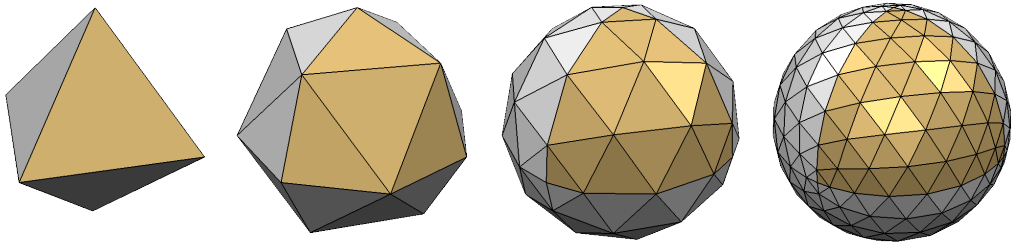
Geometry Representations

- Polygonal Meshes



Geometry Representations

- Polygonal Meshes



Piecewise linear approximation

Geometry Representations

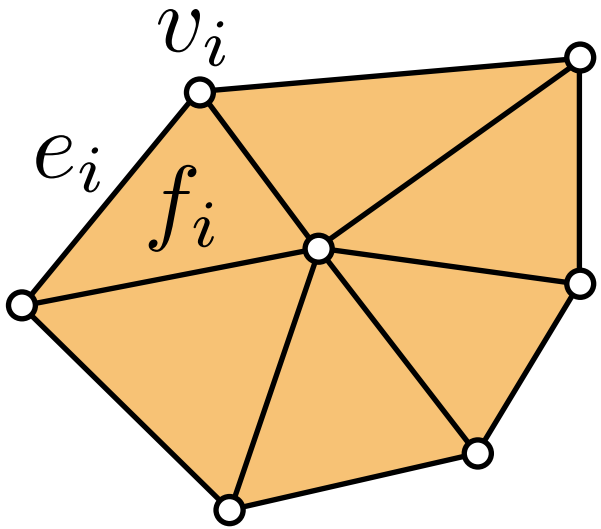
- Triangle Meshes

$$V = \{v_1, \ldots, v_n\}$$

$$E = \{e_1, \ldots, e_k\}, \quad e_i \in V \times V$$

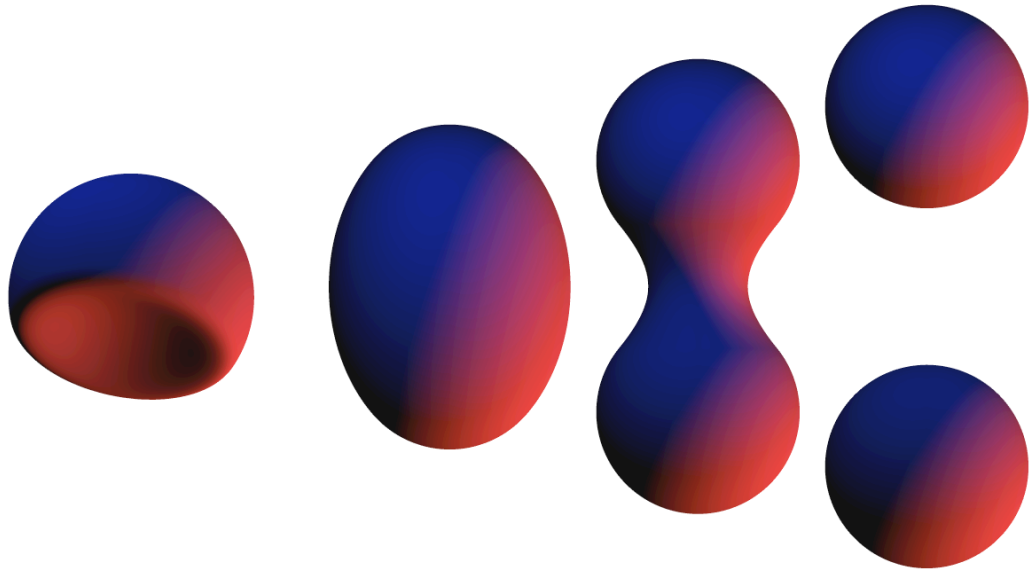
$$F = \{f_1, \ldots, f_m\}, \quad f_i \in V \times V \times V$$

$$P = \{\mathbf{p}_1, \ldots, \mathbf{p}_n\}, \quad \mathbf{p}_i \in \mathbb{R}^3$$



Geometry Representations

- Implicit surfaces



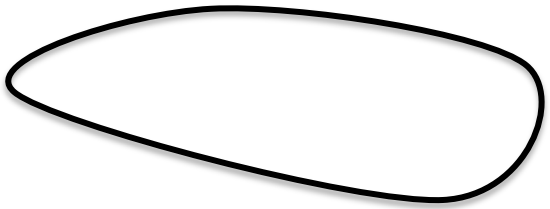
Geometry Representations

- Implicit curves & surfaces

$$f : \mathbb{R}^m \rightarrow \mathbb{R}$$

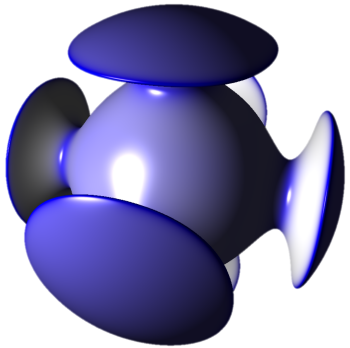
Planar Curves

$$S = \{x \in \mathbb{R}^2 | f(x) = 0\}$$



Surfaces in 3D

$$S = \{x \in \mathbb{R}^3 | f(x) = 0\}$$



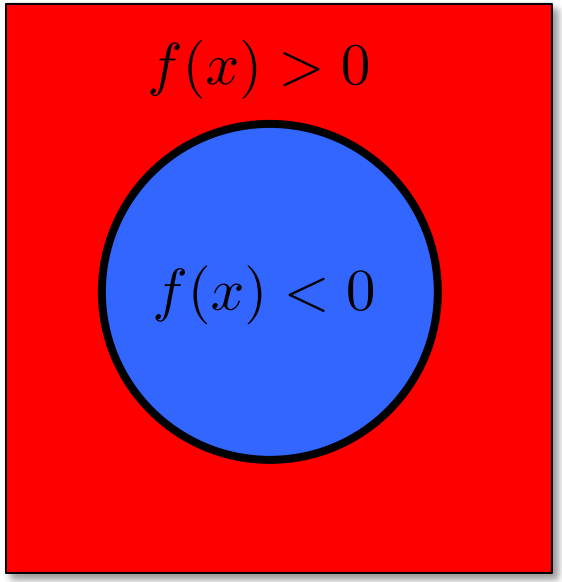
Geometry Representations

- Implicit curves & surfaces

$$\{x \in \mathbb{R}^m | f(x) > 0\}$$ Outside

$$\{x \in \mathbb{R}^m | f(x) = 0\}$$ Curve/Surface

$$\{x \in \mathbb{R}^m | f(x) < 0\}$$ Inside

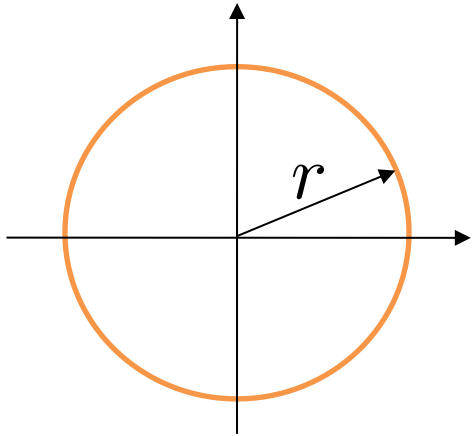


Geometry Representations

- Implicit curves & surfaces

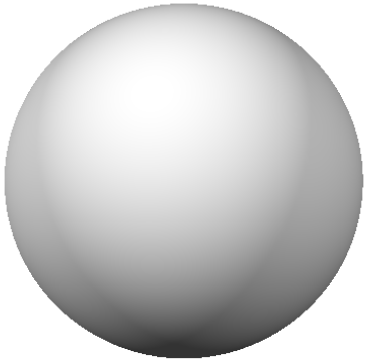
Circle

$$f(x, y) = x^2 + y^2 - r^2$$



Sphere

$$f(x, y, z) = x^2 + y^2 + z^2 - r^2$$



Geometry Representations

- Implicit curves & surfaces

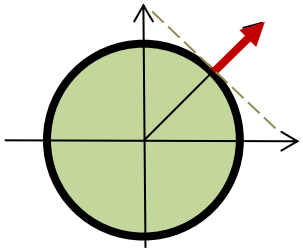
Surface Normal

$$\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)^T$$

Sphere

$$f(x,y,z) = x^2 + y^2 + z^2 - r^2$$

$$\nabla f(x,y,z) = (2x, 2y, 2z)^T$$

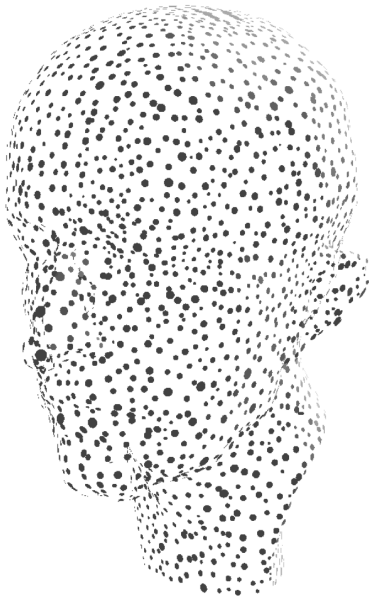
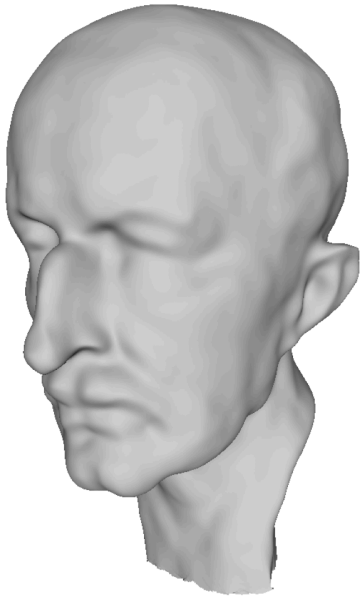


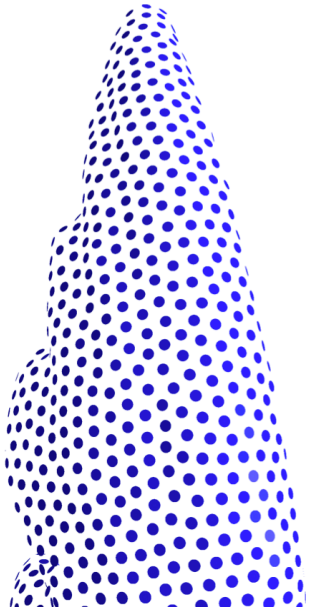
Geometry Representations

- Implicit curves & surfaces
  - + Easy to determine inside/outside
  - + Easy to determine if a point is on a curve/surface
  - + Easy to combine
  - − Hard to generate points on a curve/surface
  - − Limited set of surfaces
  - − Does not lend itself to (real-time) rendering

Geometry Representations

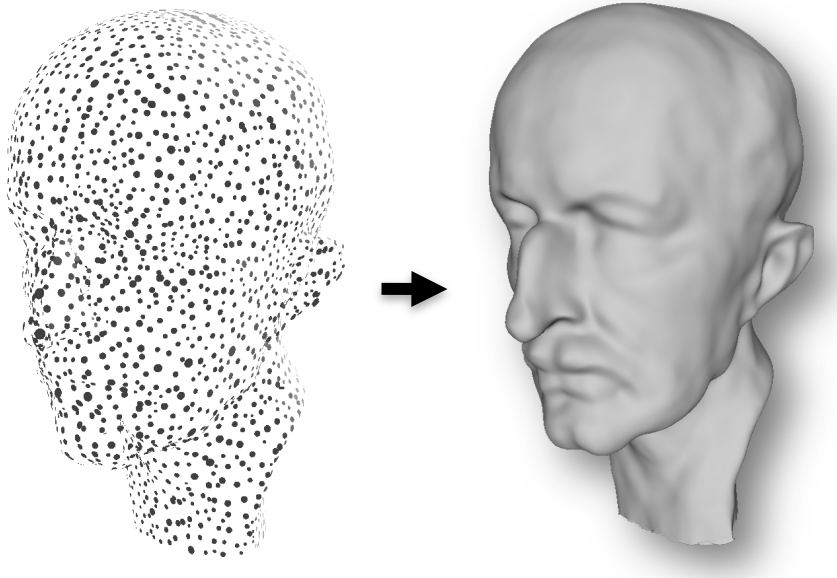
- Point Set Surfaces



Geometry Representations

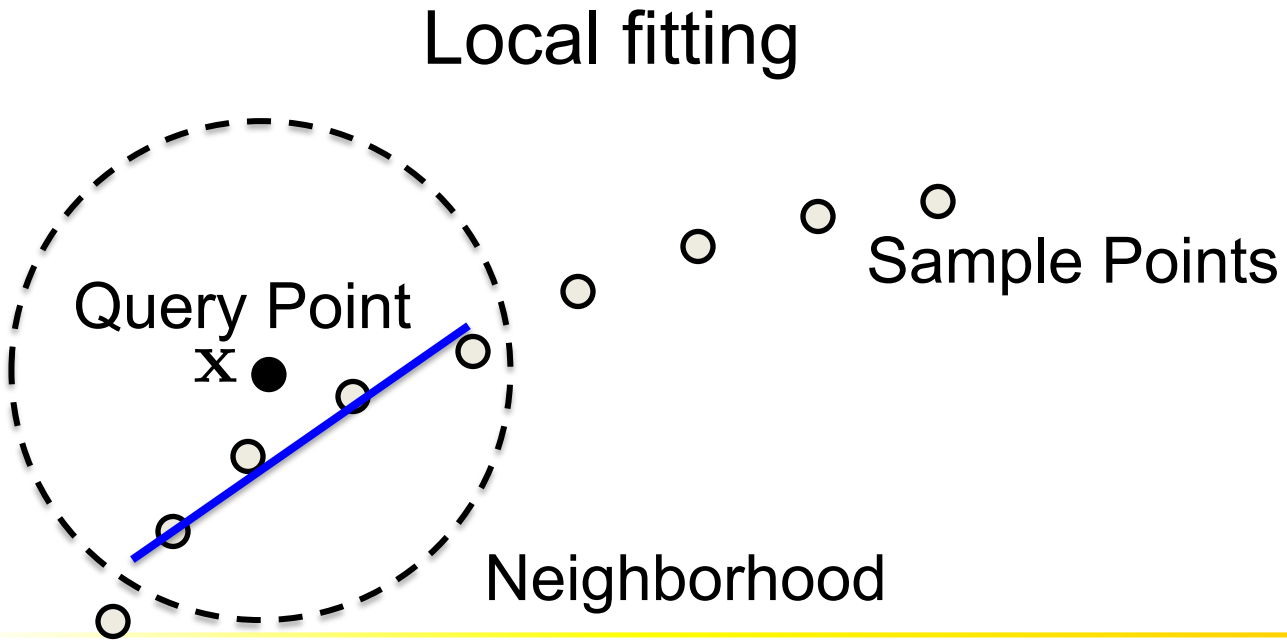
- Point Set Surfaces



Only point-wise attributes
Approximation methods
Smooth surfaces
Works on acquired data

Geometry Representations

- Point Set Surfaces

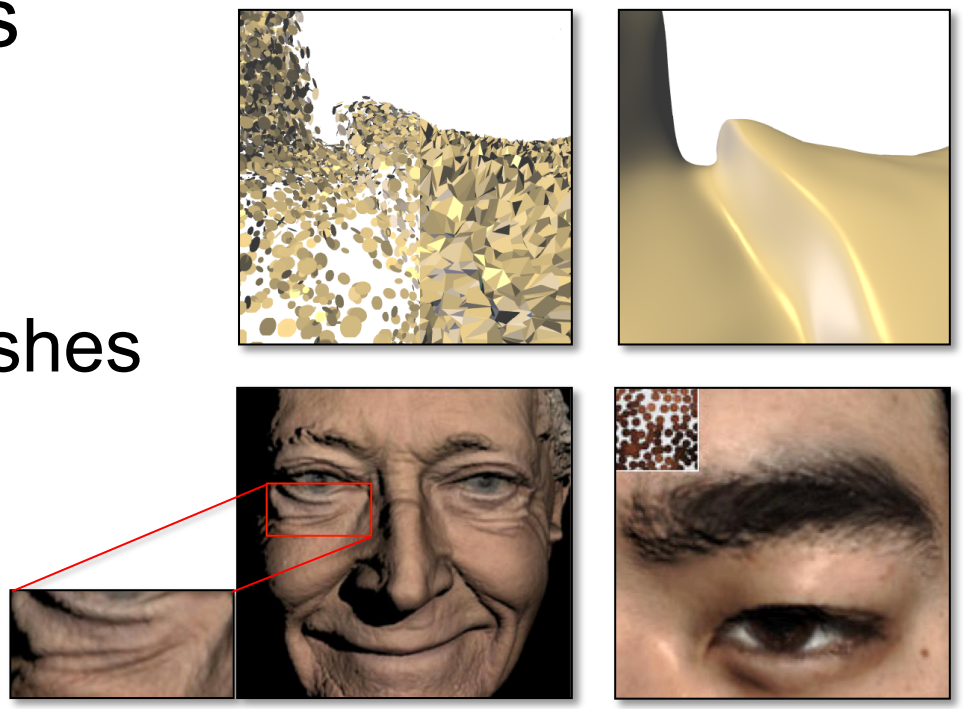


Geometry Representations

- Point Set Surfaces
  - Implicit representation & fast projection

Geometry Representations

- Point Set Surfaces
  - Robust to noise
  - Direct rendering
  - Conversion to meshes



Geometry Representations

- Point Set Surfaces
  - + Easy to determine inside/outside
  - + Easy to determine if a point is on the curve/surface
  - + Easy to generate points on the curve/surface
  - + Suitable for reconstruction from general data
  - + Direct real-time rendering
  - − Not efficient to use in some modeling tasks