Compiler Construction
Lent Term 2022

```
int main( int argc, char *argv[] )
{
  printf("hello world\n");
  return 0;
}
```

```
.LC0:
        .string   "hello world"
        .text
        .globl    main
        .type     main, @function
main:
.LFB0:
        .cfi_startproc
        pushq     %rbp
        .cfi_def_cfa_offset 16
        .cfi_offset 6, -16
        movq      %rsp, %rbp
        .cfi_def_cfa_register 6
        subq      $16, %rsp
        movl      %edi, -4(%rbp)
        movq      %rsi, -16(%rbp)
        movl      $.LC0, %edi
        call  puts
        movl      $0, %eax
        leave
        .cfi_def_cfa 7, 8
        ret
        .cfi_endproc
```


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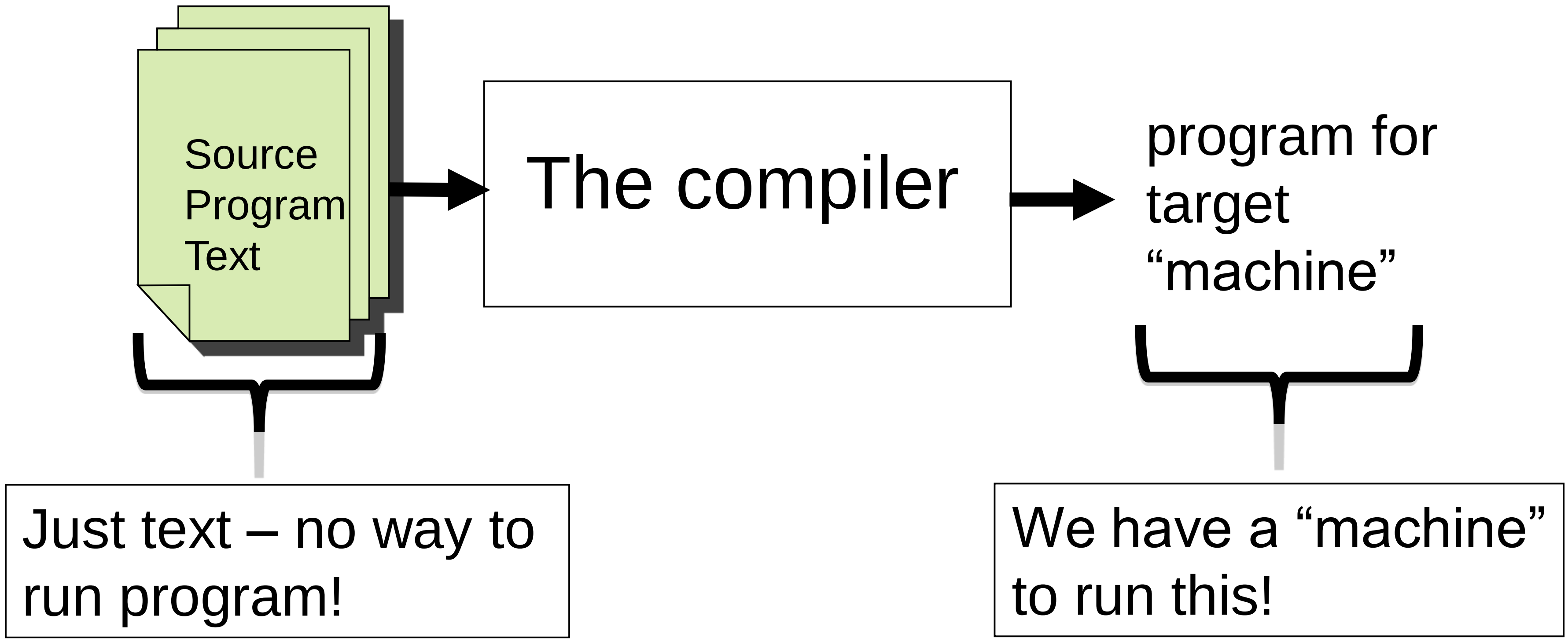
Why Study Compilers?

- **Although many of the basic ideas were developed over 60 years ago, compiler construction is still an evolving and active area of research and development.**
- **Compilers are intimately related to programming language design and evolution.**
- **Compilers are a Computer Science success story illustrating the hallmarks of our field --- higher-level abstractions implemented with lower-level abstractions.**
- **Every Computer Scientist should have a basic understanding of how compilers work.**

Compilation is a special kind of translation



A good compiler should ...

This course!

- <u>**be correct in the sense that meaning is preserved**</u>
- **produce usable error messages**

OptComp, Part II

- **generate efficient code**
- **itself be efficient**
- **be well-structured and maintainable**

Pick any 2?

Just 1?

Mind The Gap

High Level Language

- Machine independent
- Complex syntax
- Complex type system
- Variables
- Nested scope
- Procedures, functions
- Objects
- Modules
- …

Typical Target Language

- Machine specific
- Simple syntax
- Simple types
- memory, registers, words
- Single flat scope

Help!!! Where do we begin???

The Gap, illustrated

```
public class Fibonacci {
    public static long fib(int m) {
        if (m == 0) return 1;
        else if (m == 1) return 1;
             else return
                        fib(m - 1) + fib(m - 2);
    }
    public static void
        main(String[] args) {
        int m =
              Integer.parseInt(args[0]);
             System.out.println(
                  fib(m) + "\n");
    }
}
```

javac Fibonacci.java
javap –c Fibonacci.class

```
public class Fibonacci {
  public Fibonacci();
    Code:
       0: aload_0
       1: invokespecial #1
       4: return
  public static long fib(int);
    Code:
       0: iload_0
       1: ifne          6
       4: lconst_1
       5: lreturn
       6: iload_0
       7: iconst_1
       8: if_icmpne     13
      11: lconst_1
      12: lreturn
      13: iload_0
      14: iconst_1
      15: isub
      16: invokestatic  #2
      19: iload_0
      20: iconst_2
      21: isub
      22: invokestatic  #2
      25: ladd
      26: lreturn
```

```
public static void
   main(java.lang.String[]);
  Code:
     0: aload_0
     1: iconst_0
     2: aaload
     3: invokestatic  #3
     6: istore_1
     7: getstatic     #4
    10: new           #5
    13: dup
    14: invokespecial #6
    17: iload_1
    18: invokestatic  #2
    21: invokevirtual #7
    24: ldc           #8
    26: invokevirtual #9
    29: invokevirtual #10
    32: invokevirtual #11
    35: return
}
```

JVM bytecodes

The Gap, illustrated

fib.ml

```
(* fib : int -> int *)
let rec fib m =
    if m = 0
    then 1
    else if m = 1
         then 1
         else fib(m - 1) + fib (m - 2)
```

ocamlc –dinstr fib.ml

```
branch L2
L1:         acc 0
push
const 0
eqint
branchifnot L4
const 1
return 1
L4:         acc 0
push
const 1
eqint
branchifnot L3
const 1
return 1
L3:         acc 0
offsetint -2
push
offsetclosure 0
apply 1
push
acc 1
offsetint -1
push
offsetclosure 0
apply 1
addint
return 1
L2:         closurerec 1, 0
acc 0
makeblock 1, 0
pop 1
setglobal Fib!
```

OCaml VM bytecodes

The Gap, illustrated

fib.c

```
#include<stdio.h>

int Fibonacci(int);
int main()
{
   int n;
   scanf("%d",&n);
   printf("%d\n", Fibonacci(n));
   return 0;
}

int Fibonacci(int n)
{
   if ( n == 0 ) return 0;
   else if ( n == 1 ) return 1;
   else return ( Fibonacci(n-1) + Fibonacci(n-2) );
}
```

gcc –S fib.c

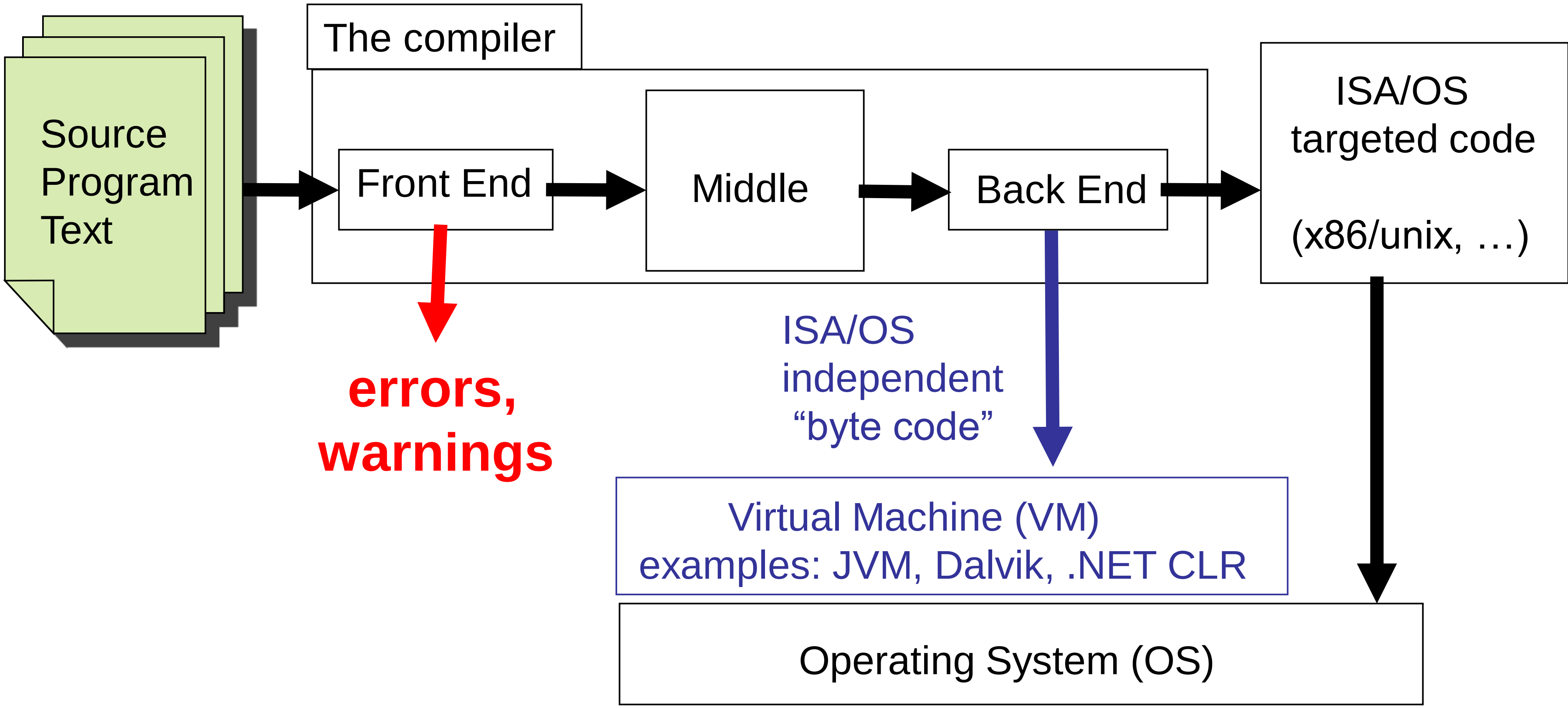
The Gap, illustrated

```
.section                __TEXT,__text,regular,pure_instructions
.globl                  _main
.align                  4, 0x90
_main:                          ## @main
.cfi_startproc
## BB#0:
pushq                   %rbp
Ltmp2:
.cfi_def_cfa_offset 16
Ltmp3:
.cfi_offset %rbp, -16
movq                    %rsp, %rbp
Ltmp4:
.cfi_def_cfa_register %rbp
subq                    $16, %rsp
leaq                    L_.str(%rip), %rdi
leaq                    -8(%rbp), %rsi
movl                    $0, -4(%rbp)
movb                    $0, %al
callq                   _scanf
movl                    -8(%rbp), %edi
movl                    %eax, -12(%rbp)         ## 4-byte Spill
callq                   _Fibonacci
leaq                    L_.str1(%rip), %rdi
movl                    %eax, %esi
movb                    $0, %al
callq                   _printf
movl                    $0, %esi
movl                    %eax, -16(%rbp)         ## 4-byte Spill
movl                    %esi, %eax
addq                    $16, %rsp
popq                    %rbp
ret
.cfi_endproc

.globl                  _Fibonacci
.align                  4, 0x90
_Fibonacci:                     ## @Fibonacci
.cfi_startproc
## BB#0:
pushq                   %rbp
Ltmp7:
.cfi_def_cfa_offset 16
Ltmp8:
.cfi_offset %rbp, -16
movq                    %rsp, %rbp
Ltmp9:
.cfi_def_cfa_register %rbp
subq                    $16, %rsp
movl                    %edi, -8(%rbp)
cmpl                    $0, -8(%rbp)
jne                     LBB1_2
## BB#1:
movl                    $0, -4(%rbp)
jmp                     LBB1_5
LBB1_2:
cmpl                    $1, -8(%rbp)
jne                     LBB1_4
## BB#3:
movl                    $1, -4(%rbp)
jmp                     LBB1_5
LBB1_4:
movl                    -8(%rbp), %eax
subl                    $1, %eax
movl                    %eax, %edi
callq                   _Fibonacci
movl                    -8(%rbp), %edi
subl                    $2, %edi
movl                    %eax, -12(%rbp)         ## 4-byte Spill
callq                   _Fibonacci
movl                    -12(%rbp), %edi         ## 4-byte Reload
addl                    %eax, %edi
movl                    %edi, -4(%rbp)
LBB1_5:
movl                    -4(%rbp), %eax
addq                    $16, %rsp
popq                    %rbp
ret
.cfi_endproc

.section                __TEXT,__cstring,cstring_literals
L_.str:                         ## @.str
.asciz                  "%d"

L_.str1:                        ## @.str1
.asciz                  "%d\n"


.subsections_via_symbols
```

x86/Mac OS

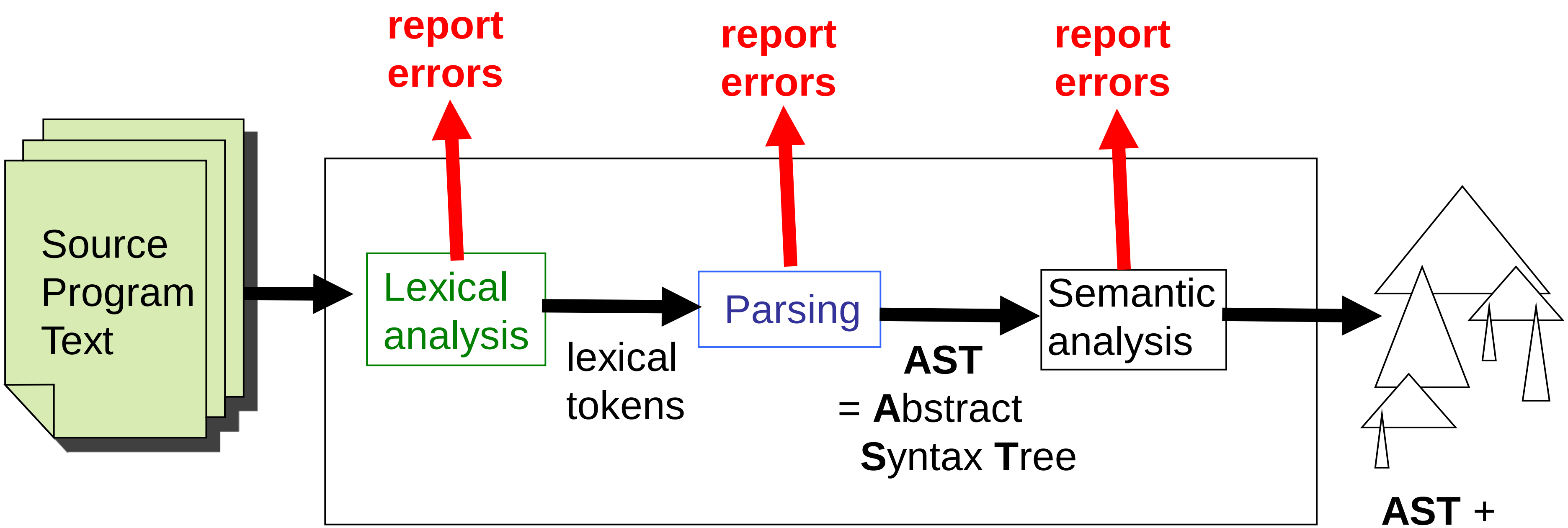
Conceptual view of a typical compiler

ISA = Instruction Set Architecture



Key to bridging **The Gap** : divide and conquer.
The gap is broken into small steps.
Each step broken into yet smaller steps …

The shape of a typical “front end”



AST +
other
info

Lexical theory based on finite automaton and regular expressions

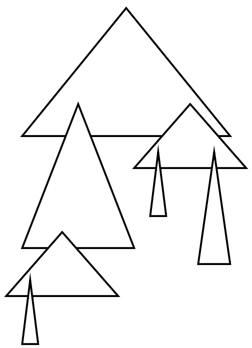
Parsing Theory based on push-down automaton and context-free grammars

Enforce “static sematics” of language: type checking, def/use rules, and so on (**SPL**!)

The AST output from the front-end should represent a legal program in the source language. (“Legal” of course does not mean “bug-free”!)

SPL = Semantics of Programming Languages, Part 1B

The middle



AST +
other
info

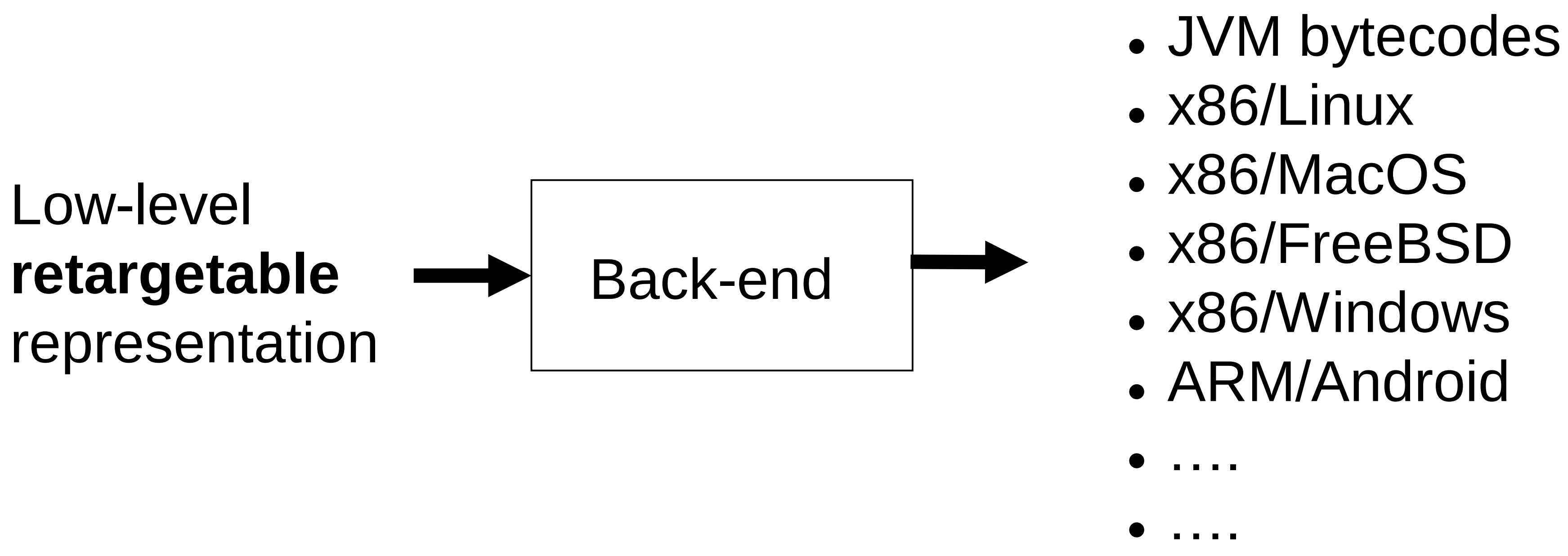
--High-level to
low-level

--Optimisations

Low-level
retargetable
representation

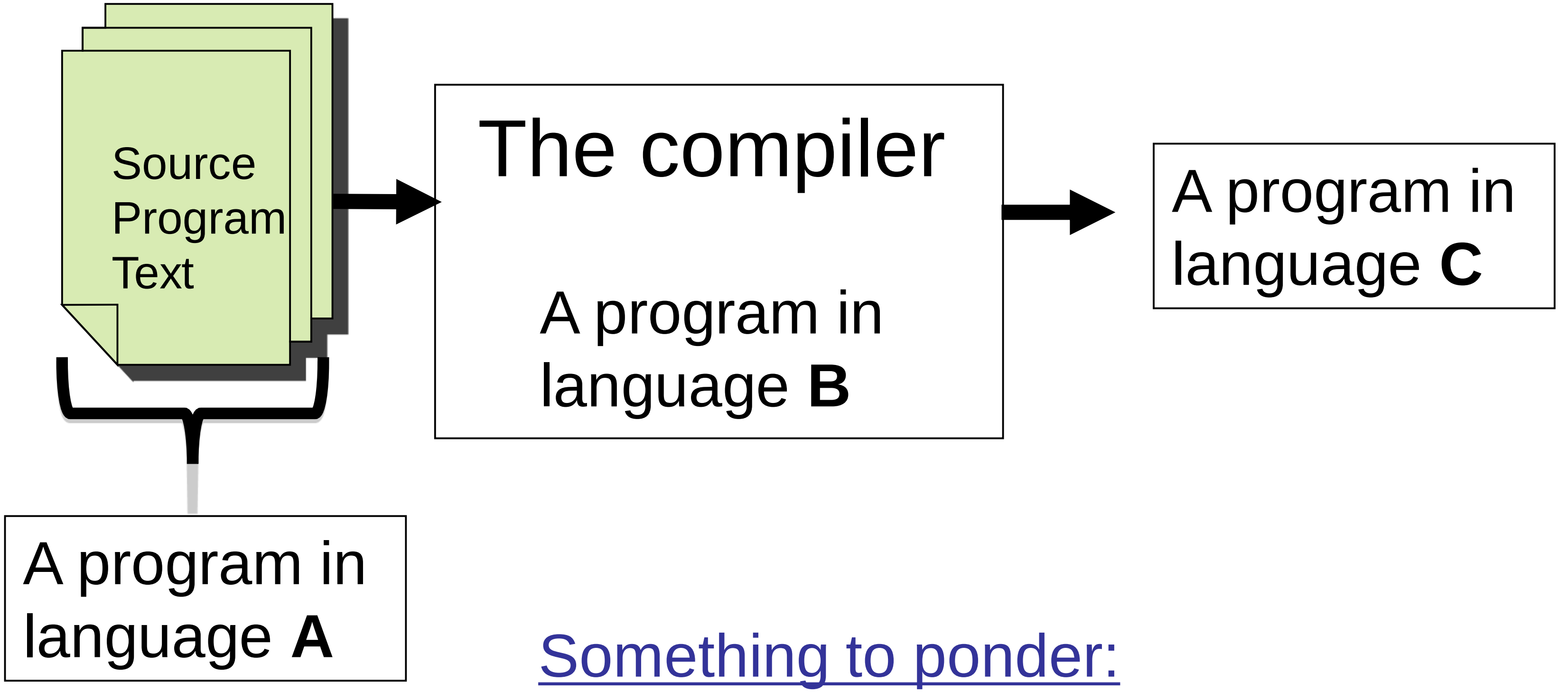
Trade-off: with more optimisations the generated code is (normally) **faster**, but the compiler is **slower**

The back-end



- Requires intimate knowledge of instruction set and details of target machine
- When generating assembler, need to understand details of OS interface
- Target-dependent optimisations happen here!

Compilers must be compiled



Something to ponder:
A compiler is just a program.
But how did it get compiled?
The OCaml compiler is written in OCaml.

How was the compiler compiled?

The Shape of this Course

- Part I (Lectures 2 – 6) :Lexical analysis and parsing
- Part II (Lectures 7 – 16) : Development of the SLANG (Simple LANGuage) compiler. SLANG is based on L3 from 1B Semantics.
- A compiler for SLANG, written in Ocaml, with link posted on the course web page.

Compiler Construction
Lent Term 2022
Lecture 2 : Lexical analysis

- Recall regular expressions
- Recall Finite Automata
- Recall NFA to DFA transformation
- What is the “lexing problem”?
- How DFAs are used to solve the lexing problem?

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What problem are we solving?

Translate a sequence of characters

```
if m = 0 then 1 else if m = 1 then 1 else fib (m - 1) +  fib (m -2)
```

into a sequence of **tokens**

```
IF, IDENT “m”, EQUAL, INT 0, THEN, INT 1, ELSE, IF,
IDENT “m”, EQUAL, INT 1, THEN, INT 1, ELSE, IDENT “fib”, LPAREN, IDENT “m”, SUB, INT 1,
RPAREN, ADD,
IDENT “fib”, LPAREN, IDENT “m”, SUB, INT 2, RPAREN
```

implemented with some data type

```
type token =
    | INT of int| IDENT of string | LPAREN | RPAREN
    | ADD | SUB | EQUAL | IF | THEN | ELSE
    | …
```

Regular expressions e over alphabet Σ

$$e \rightarrow \phi \mid \varepsilon \mid a \mid e+e \mid ee \mid e^{*} \qquad (a \in \Sigma)$$

$$M(e) \subseteq \Sigma^{*}$$

$$M(\phi) = \{\,\}$$

$$M(\varepsilon) = \{\varepsilon\}$$

$$M(a) = \{a\}$$

$$M(e_1 + e_2) = M(e_1) \cup M(e_2)$$

$$M(e_1 e_2) = \{w_1 w_2 \mid w_1 \in M(e_1),\ w_2 \in M(e_2)\}$$

$$M(e^0) = \{\varepsilon\}$$

$$M(e^{n+1}) = M(ee^n)$$

$$M(e^{*}) = \bigcup_{n \geq 0} M(e^n)$$

Regular Expression (RE) Examples

$$M((a+b)^* abb) =$$

$$\{abb,\ aabb,\ baabb,\ aaabb,\ ababb,$$

$$baabb,\ bbabb,\ aaaabb, \cdots\}$$

$$M((\Xi + \otimes)^* \Xi \otimes \otimes) =$$

$$\{\Xi \otimes \otimes,\ \Xi \Xi \otimes \otimes,\ \otimes \Xi \Xi \otimes \otimes,$$

$$\Xi \Xi \Xi \otimes \otimes,\ \Xi \otimes \Xi \otimes \otimes, \otimes \Xi \Xi \otimes \otimes,$$

$$\otimes \otimes \Xi \otimes \otimes,\ \Xi \Xi \Xi \Xi \otimes \otimes, \cdots\}$$

Review of Finite Automata (FA)

$$M = (Q, \Sigma, \delta, q_0, F)$$

Q : states $\qquad\qquad \Sigma$: alphabet

$q_0 \in \mathrm{Q}$: start state $\qquad F \subseteq \mathrm{Q}$: final states

$$\forall q \in Q, a \in \Sigma, \delta(\mathrm{q}, \mathrm{a}) \in \mathrm{Q}$$

for deterministic FA (DFA)

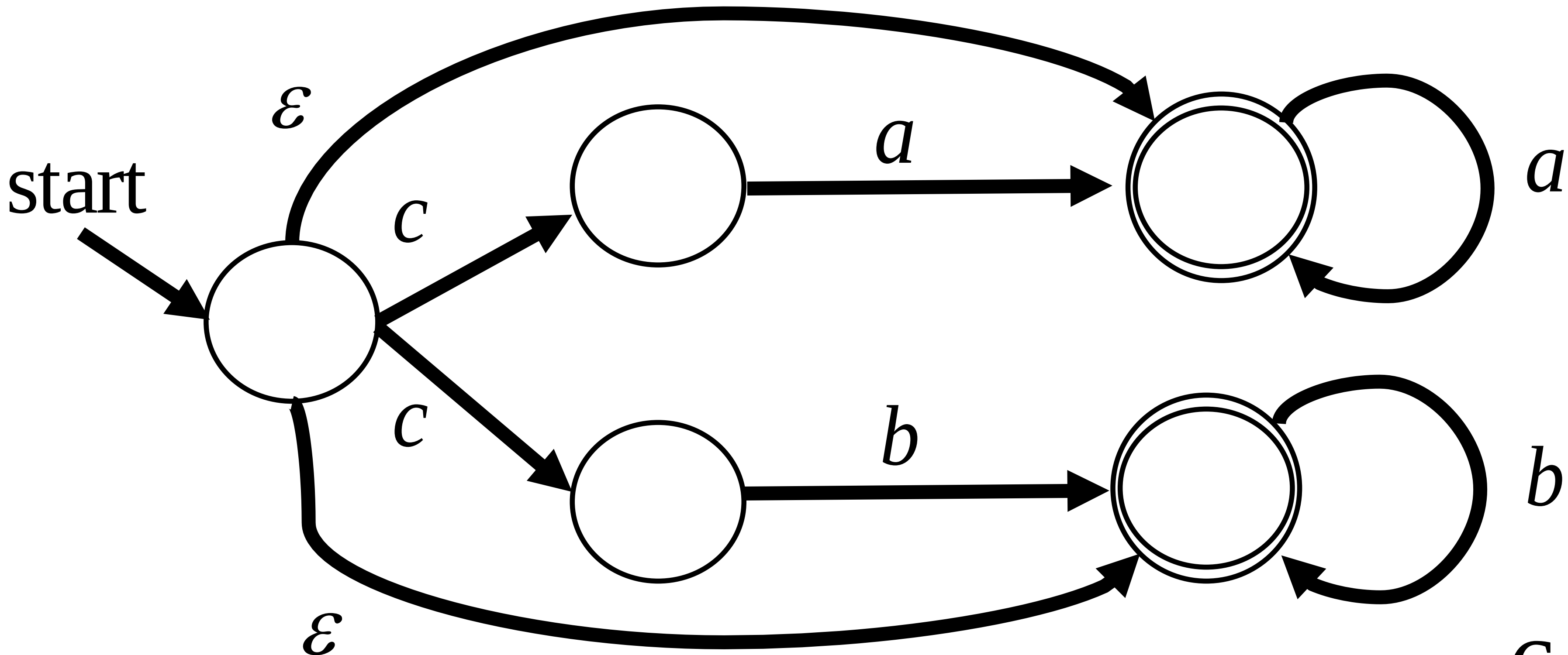
$$\forall q \in Q, a \in (\Sigma \cup \{\varepsilon\}), \delta(\mathrm{q}, \mathrm{a}) \subseteq \mathrm{Q}$$

for nondeterministic FA (NFA)

NFA Example

An NFA accepting

$$M(a^* + b^* + caa^* + cbb^*)$$



A bit of notation

For deterministic FA.

$$q \xrightarrow{\varepsilon} q$$

$$q_1 \xrightarrow{aw} q_3 \quad \text{if } \delta(q_1, a) = q_2 \text{ and } q_2 \xrightarrow{w} q_3$$

$$L(M) = \{w \mid \exists q \in F, q_0 \xrightarrow{w} q\}$$

For nondeterministic FA.

$$q \xrightarrow{\varepsilon} q$$

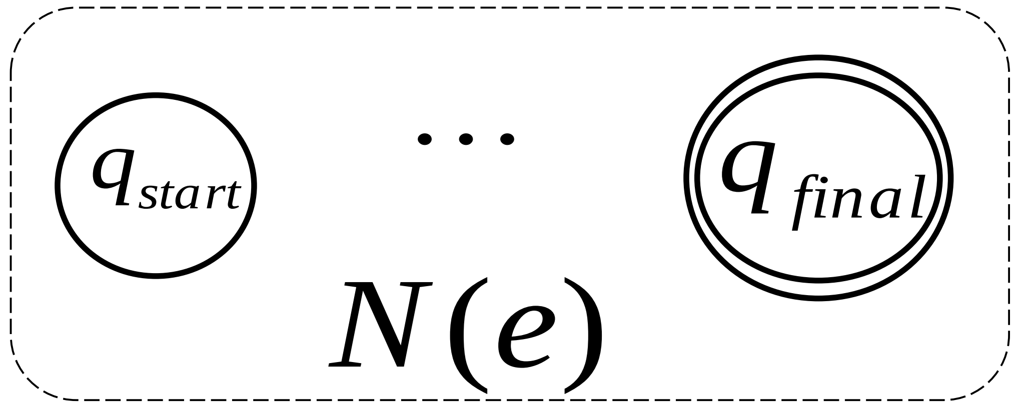
$$q_1 \xrightarrow{w} q_3 \quad \text{if } \ q_2 \in \delta(q_1, \varepsilon) \quad \text{and } q_2 \xrightarrow{w} q_3$$

$$q_1 \xrightarrow{aw} q_3 \quad \text{if } \mathrm{q}_2 \in \delta(q_1, a) \quad \text{and } q_2 \xrightarrow{w} q_3$$

$$L(M) = \{w \mid \exists q \in F, q_0 \xrightarrow{w} q\}$$

Review of RE -> NFA

$$e \quad \Rightarrow$$



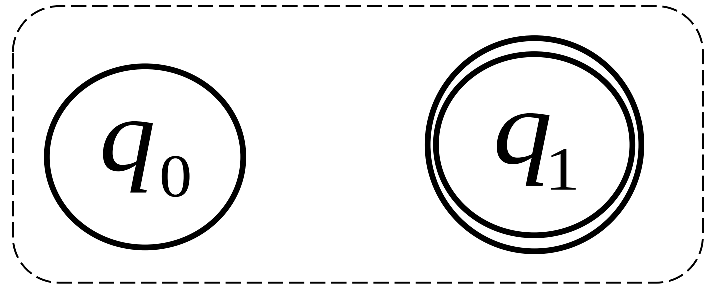
A regular expression.

A nondeterministic FA accepting *M(e) with a single final state.*

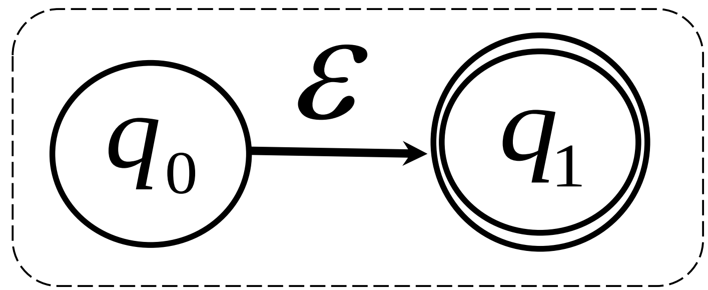
The construction is done by induction on the structure of *e*.

Review of RE -> NFA

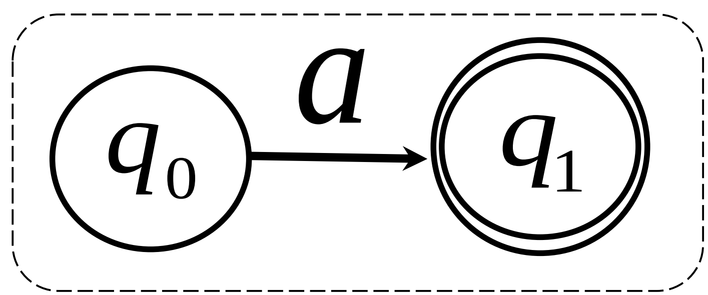
$$N(\phi) =$$



$$N(\varepsilon) =$$

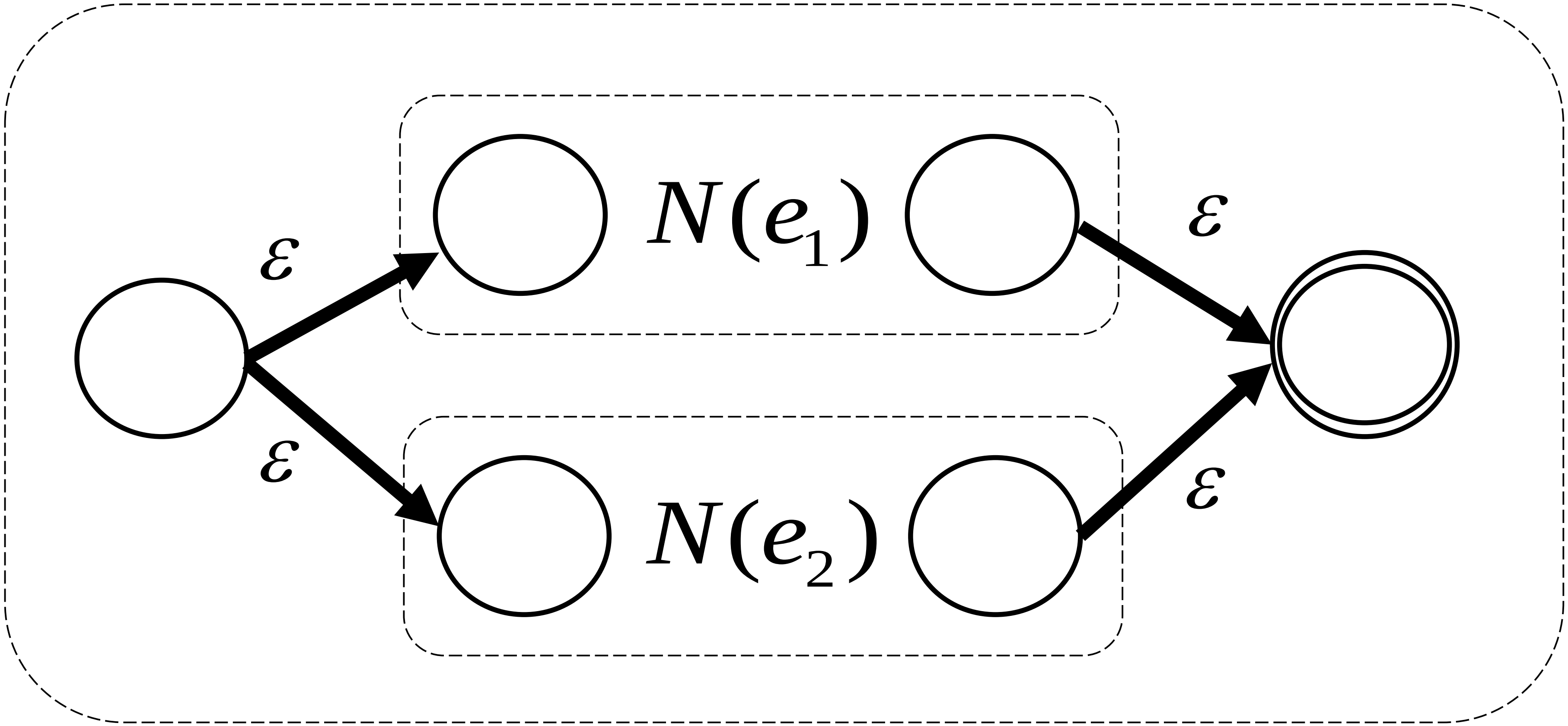


$$N(a) =$$



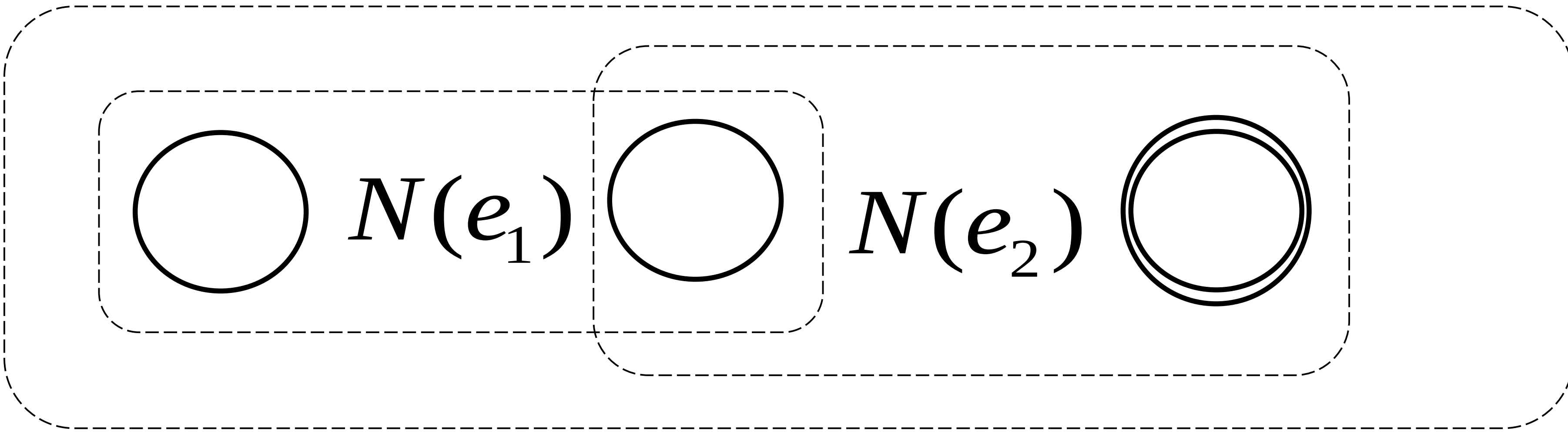
Review of RE -> NFA

$$N(e_1 + e_2) =$$



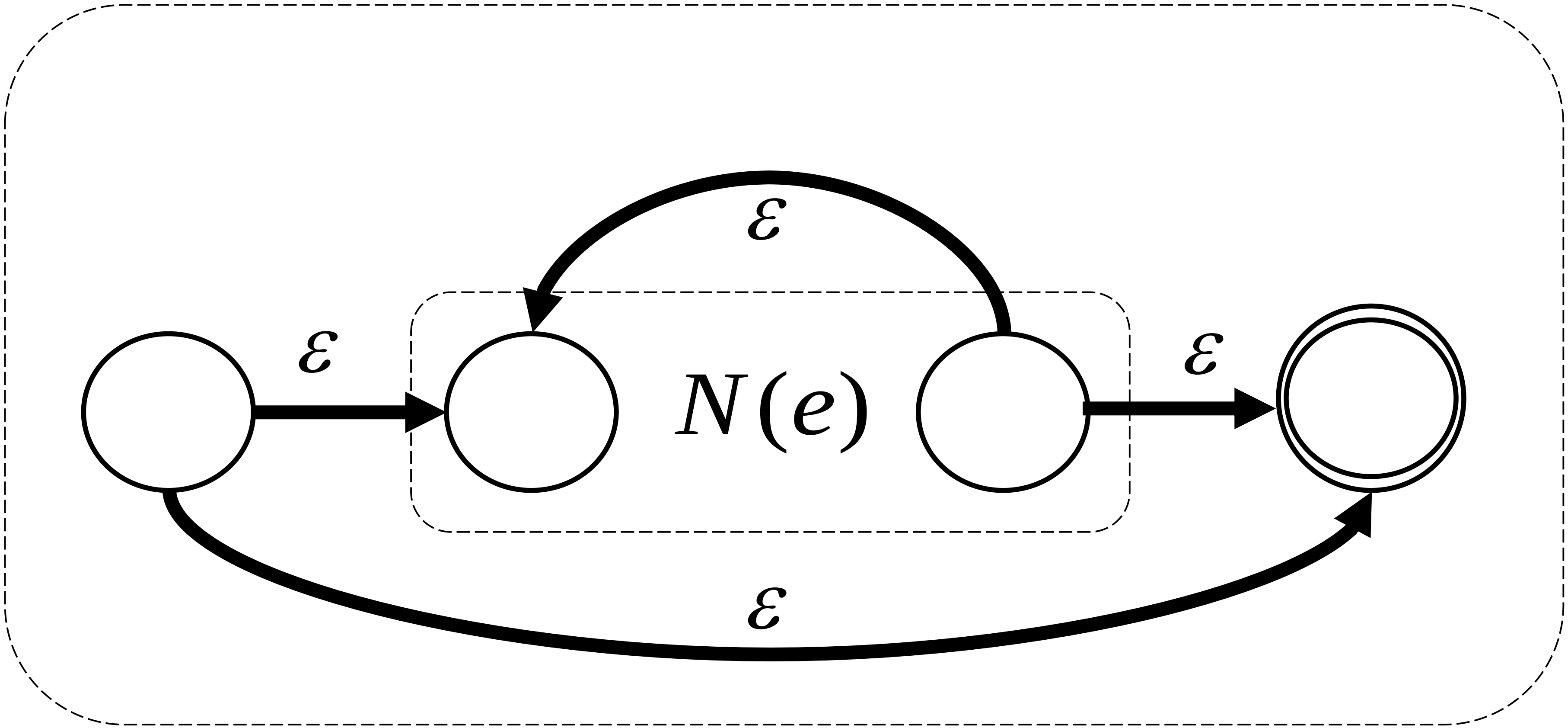
Review of RE -> NFA

$$N(e_1 e_2) =$$

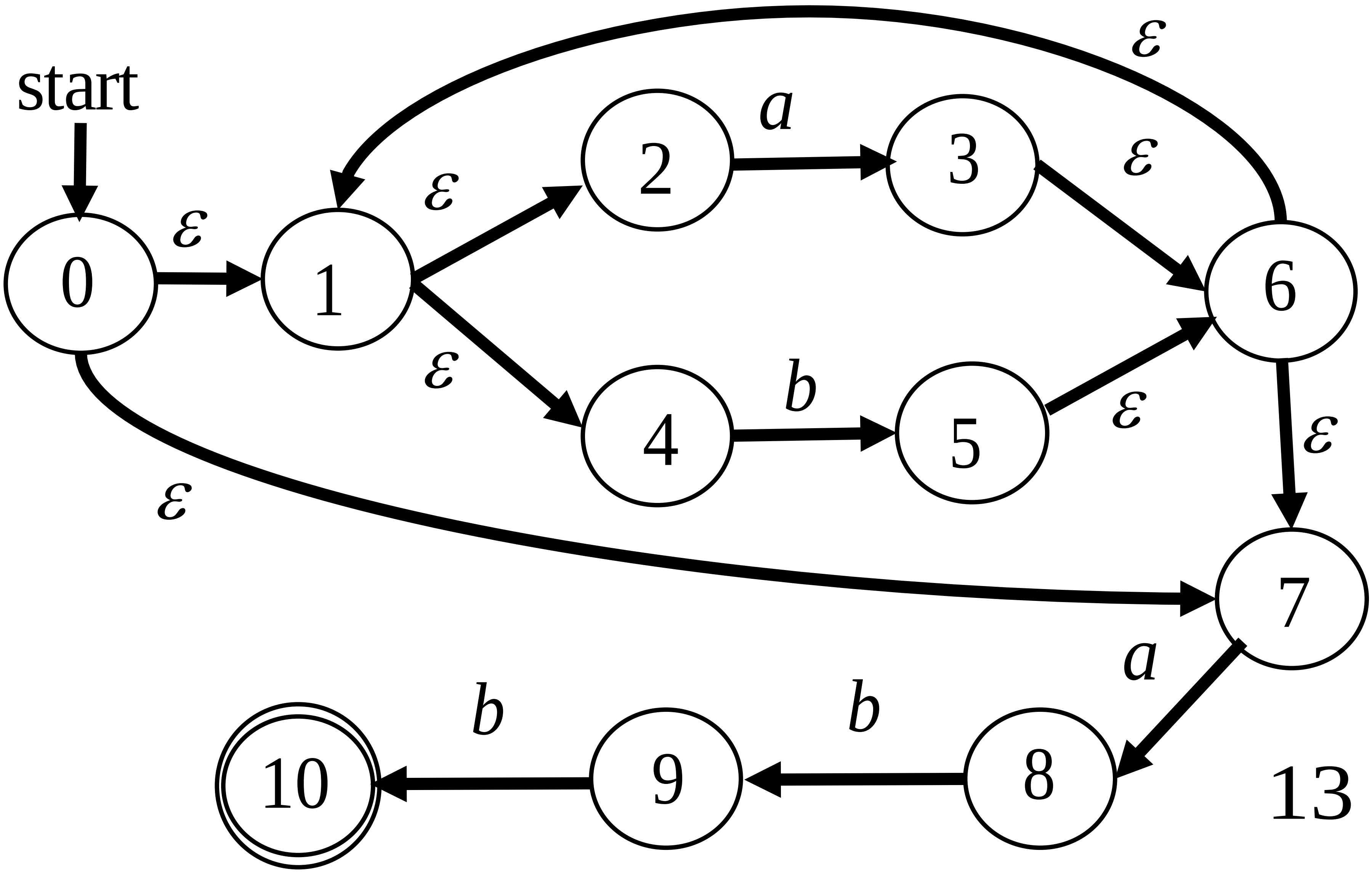


Review of RE -> NFA

$$N(e^*) =$$



$N((a+b)^{*}abb)$



Review of NFA -> DFA

$$M = (Q, \Sigma, \delta, q_0, F)$$

$$M^{'} = (Q^{'}, \Sigma, \delta^{'}, q_0^{'}, F^{'})$$

$$Q^{'} = \{S \mid S \subseteq Q\}$$

$$\varepsilon - closure(S) = \{q' \in Q \mid \exists q \in S, q \xrightarrow{\varepsilon} q'\}$$

$$\delta^{'}(S, a) = \varepsilon - closure(\{q' \in \delta(q, a) \mid q \in S\})$$

$$q_0^{'} = \varepsilon - closure\{q_0\}$$

$$F^{'} = \{S \subseteq Q \mid S \cap F \neq \phi\}$$

How do we compute $\varepsilon - closure(S)$?

$\varepsilon - closure(S)$:

push all elements of S onto a stack

result $:= S$

while stack not empty

pop q off the stack

for each $u \in \delta(q,\varepsilon)$

if $u \notin$ result

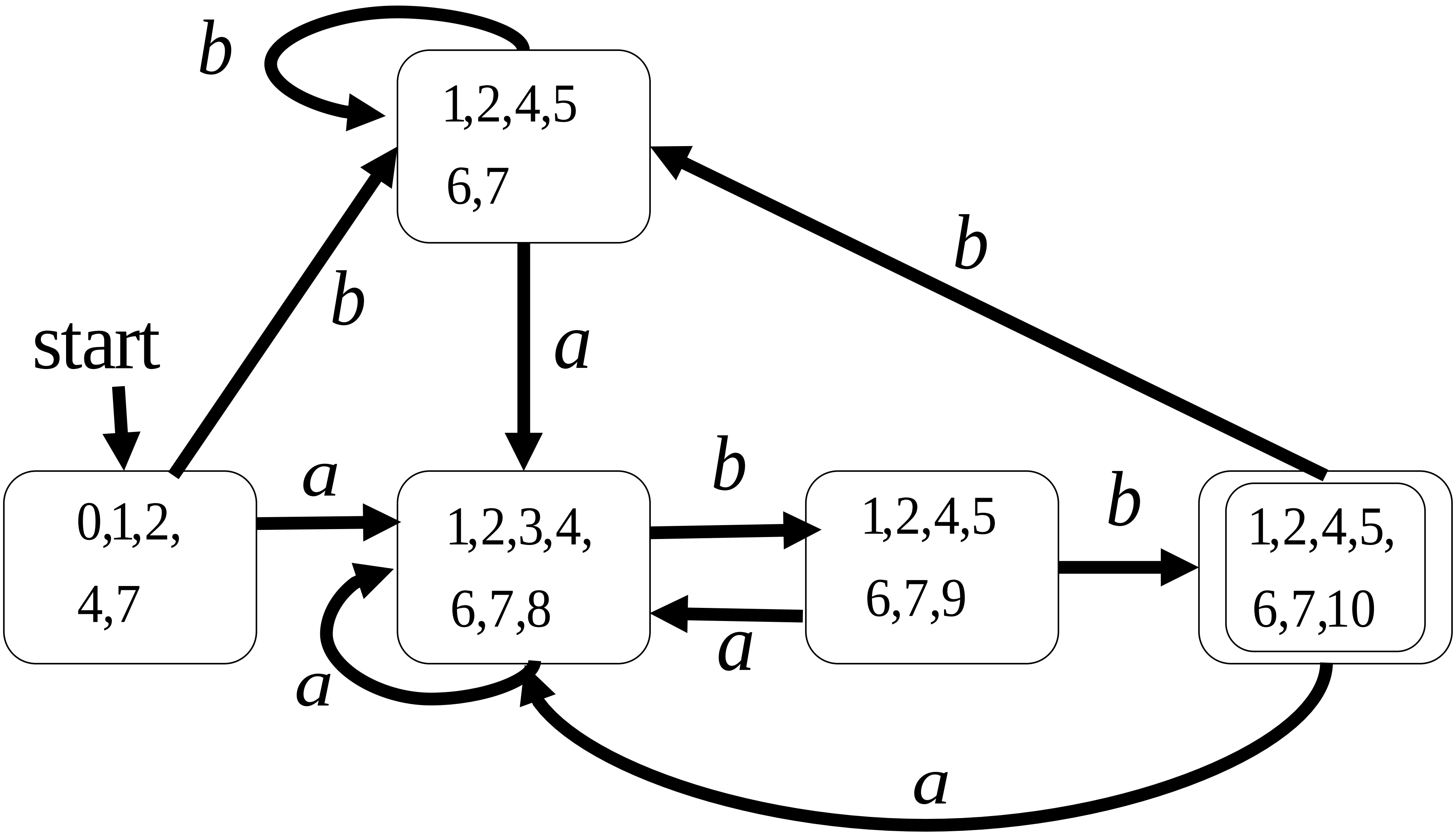
then result $:= \{u\} \cup$ result

push u on stack

return result

Look familiar?
It's just a version of transitive closure!

$DFA(N((a+b)^* abb))$



Traditional Regular Language Problem

$$\text{Given } e \text{ and } w, \text{ is } w \in L(e)?$$

Solution : construct NFA from e, then DFA, then run the DFA on w.

But is this a solution to the “lexing problem?

No!

Something closer to the "lexing problem"

Given $e_1, e_2 \cdots, e_k$ **and** W

find $(i_1, w_1), (i_2, w_2), \ldots (i_n, w_n)$ **so that**

$w = w_1 w_2 \ldots w_n$ **and** $\forall i \, \exists j \; w_i \in L(e_{i_j})$

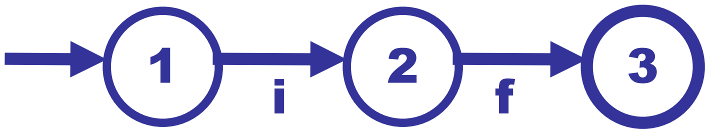
and what else?

The expressions are **ordered** by priority. Why? Is "if" a variable or a keyword? Need priority to resolve ambiguity (so "if" matched keyword RE before identifier RE.

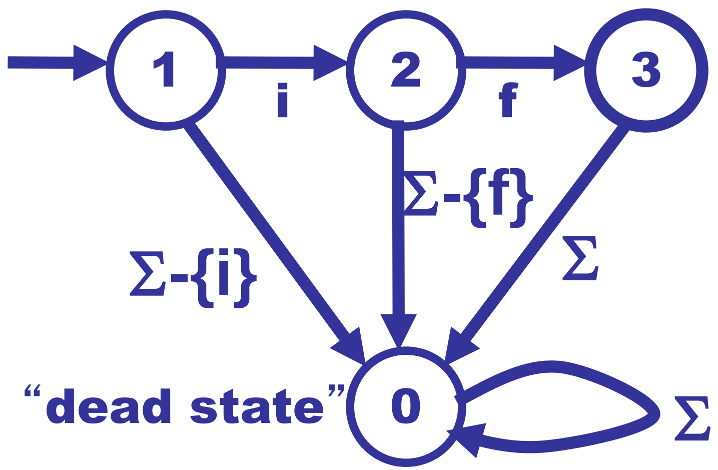
We need to do a **longest** match. Why? Is "ifif" a variable or two "if" keywords?

Define Tokens with Regular Expressions (Finite Automata)

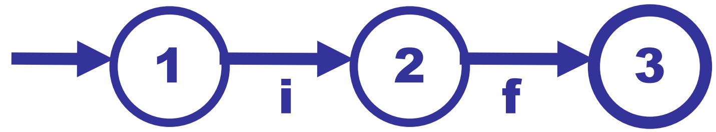
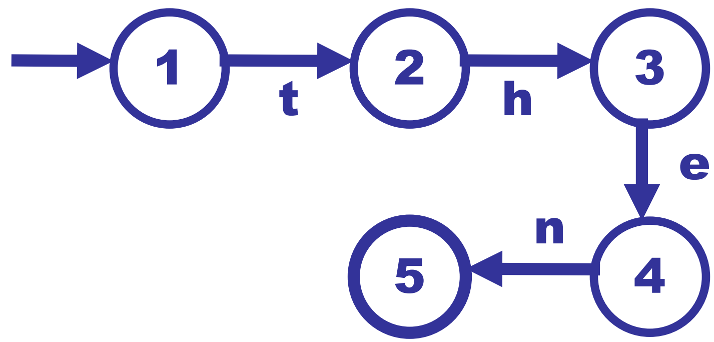
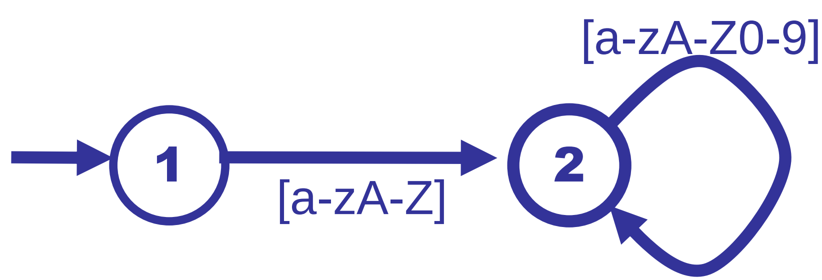
Keyword: if



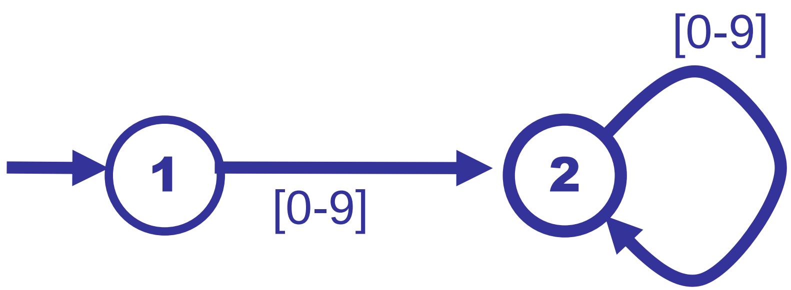
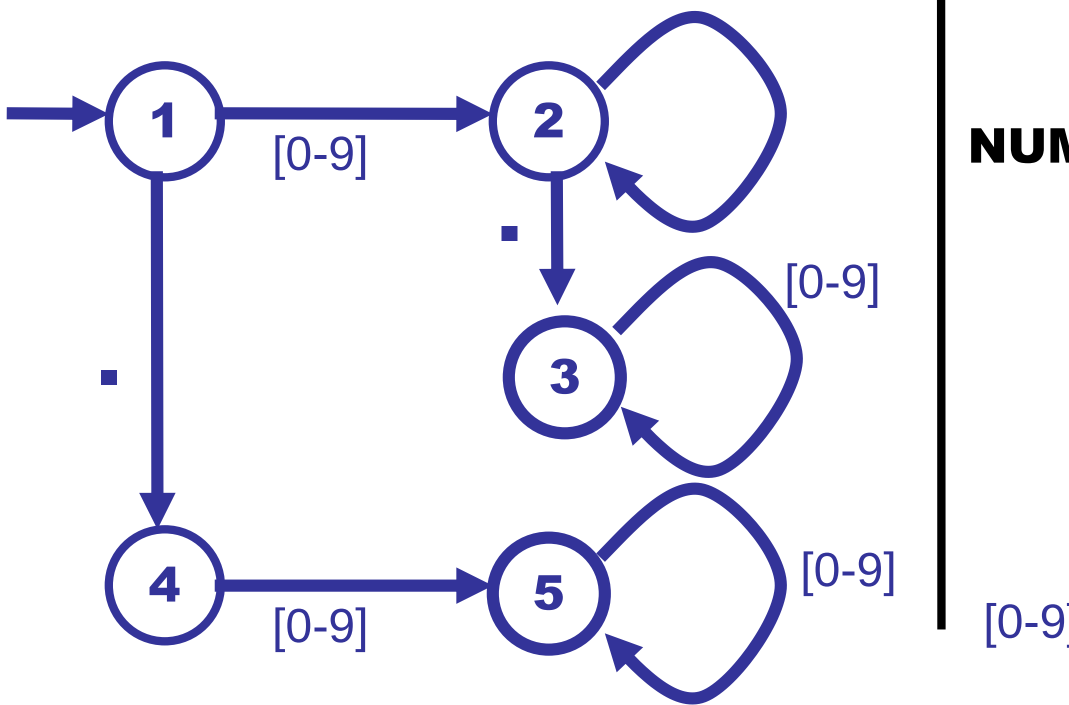
This FA is really shorthand for:



Define Tokens with Regular Expressions (Finite Automata)

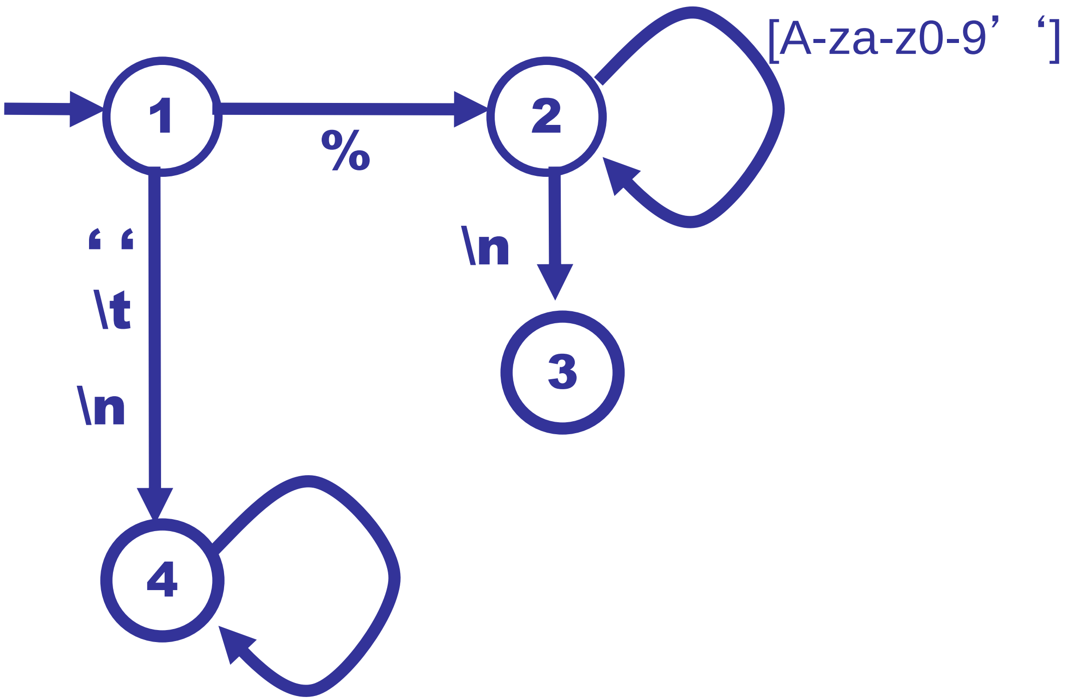
Regular Expression	Finite Automata	Token
Keyword: if		**KEY(IF)**
Keyword: then		**KEY(then)**
Identifier: [a-zA-Z][a-zA-Z0-9]*		**ID(s)**

Define Tokens with Regular Expressions (Finite Automata)

Regular Expression	Finite Automata	Token
number: **[0-9][0-9]***		**NUM(n)**
real: **([0-9]+ '.' [0-9]*)** **| ([0-9]* '.' [0-9]+)**		**NUM(n)**

No Tokens for “White-Space”

White-space with one line comments starting with %



Constructing a Lexer

INPUT: $e_1, e_2 \cdots, e_k$

an <u>ordered</u> list of regular expressions
Highest priority first, lowest last

NFA for $e = e_1 + e_2 + \cdots + e_k$

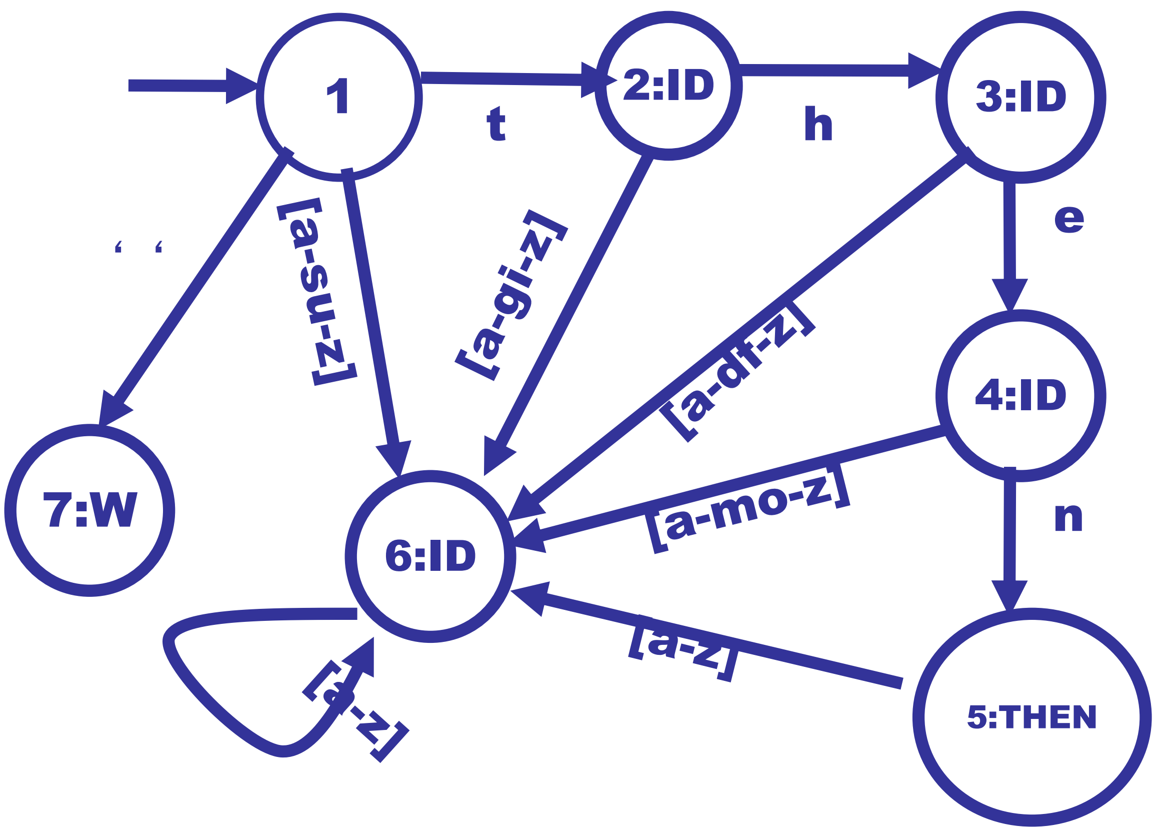
DFA with each final state associated with the e_i of highest priority.

Constructing a Lexer

(1) **Keyword : then**

(2) **Ident : [a-z][a-z]***

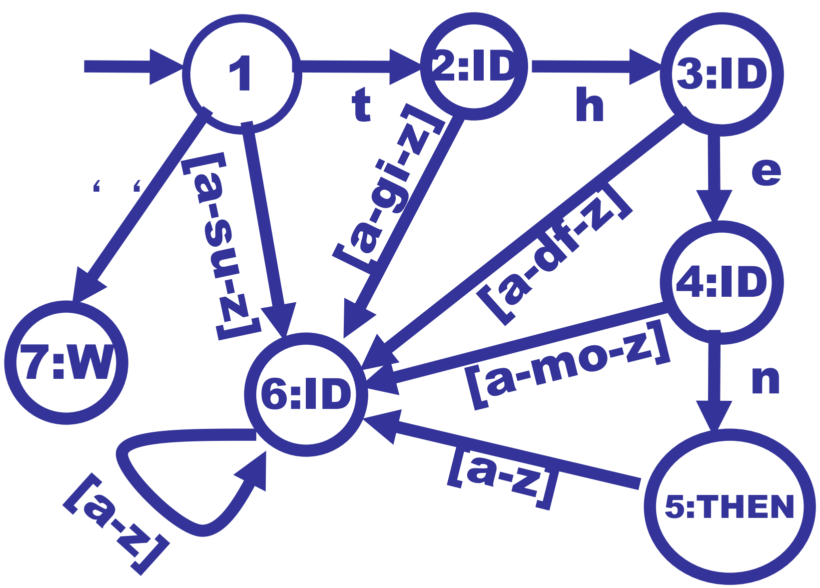
(2) **White-space: ‘ ‘**



State 5 could accept either an ID or the keyword “then”. The priority rules eliminates this ambiguity and associates state 5 with the keyword.

What about longest match?

Start in initial state,
Repeat:
(1) read input until dead state is reached. Emit token associated with last accepting state.
(2) reset state to start state



| = current position, $ = EOF

Input	current state	last accepting state
|then thenx$	1	0
t|hen thenx$	2	2
th|en thenx$	3	3
the|n thenx$	4	4
then| thenx$	5	5
then |thenx$	0	5 EMIT KEY(THEN)
then| thenx$	1	0 RESET
then |thenx$	7	7
then t|henx$	0	7 EMIT WHITE(' ')
then |thenx$	1	0 RESET
then t|henx$	2	2
then th|enx$	3	3
then the|nx$	4	4
then then|x$	5	5
then thenx|$	6	6
then thenx$|	0	6 EMIT ID(thenx)

Compiler Construction
Lent Term 2022
Lecture 3: Context-Free Grammars

- Context-Free Grammars (CFGs)
- Each CFG generates a Context-Free Language (CFL)
- Push-down automata (PDAs)
- PDAs recognize CFLs
- Ambiguity is the central problem

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Programming Language Syntax

6.7 Declarations

Syntax

declaration:
  declaration-specifiers init-declarator-list$_{opt}$ **;**
  static_assert-declaration

declaration-specifiers:
  storage-class-specifier declaration-specifiers$_{opt}$
  type-specifier declaration-specifiers$_{opt}$
  type-qualifier declaration-specifiers$_{opt}$
  function-specifier declaration-specifiers$_{opt}$
  alignment-specifier declaration-specifiers$_{opt}$

init-declarator-list:
  init-declarator
  init-declarator-list **,** *init-declarator*

init-declarator:
  declarator
  declarator **=** *initializer*

A small fragment of the C standard. How can we turn this specification into a parser that reads a text file and produces a syntax tree?

Context-Free Grammars (CFGs)

$$G = (N, T, P, S)$$

N : set of nonterminals

T : set of terminals

$P \subseteq N \times (N \cup T)^*$: a set of productions

$S \in \mathrm{N}$: start symbol

Each $(A, \alpha) \in P$ is written as $A \rightarrow \alpha$

Example CFG

$$G_1 = (N_1, T_1, P_1, E)$$

$N_1 = \{\mathrm{E}\}$ $\qquad$ $T_1 = \{+, *, (,), \mathrm{id}\}$

P_1 :

$$\mathrm{E} \rightarrow \mathrm{E} + \mathrm{E} \mid \mathrm{E} * \mathrm{E} \mid (\mathrm{E}) \mid \mathrm{id}$$

This is shorthand for

$$P_1 = \{(\mathrm{E}, \mathrm{E} + \mathrm{E}), (\mathrm{E}, \mathrm{E} * \mathrm{E}), (\mathrm{E}, (\mathrm{E})), (\mathrm{E}, \mathrm{id})\}$$

Derivations

Notation conventions:

$$\alpha, \beta, \gamma, \cdots \in (N \cup T)^*$$

$$A, B, C, \cdots \in N$$

Given: $\alpha A \beta$ and a production $A \to \gamma$

a derivation step is written as

$$\alpha A \beta \Rightarrow \alpha \gamma \beta$$

$\Rightarrow^+$ means one or more derivation steps and

$\Rightarrow^*$ means zero or more derivation steps

Example derivations

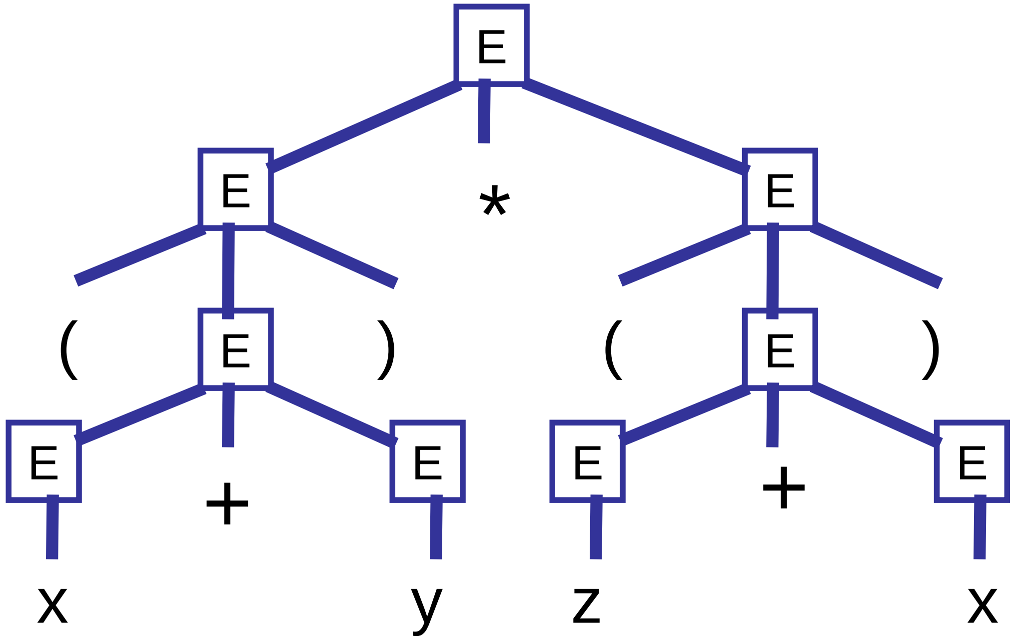
$$
\begin{aligned}
\mathrm{E} &\Rightarrow \mathrm{E} * \mathrm{E} \\
&\Rightarrow (E) * E \\
&\Rightarrow (E + E) * E \\
&\Rightarrow (x + E) * E \\
&\Rightarrow (x + y) * E \\
&\Rightarrow (x + y) * (E) \\
&\Rightarrow (x + y) * (E + E) \\
&\Rightarrow (x + y) * (z + E) \\
&\Rightarrow (x + y) * (z + x)
\end{aligned}
$$

A leftmost derivation

$$
\begin{aligned}
\mathrm{E} &\Rightarrow \mathrm{E} * \mathrm{E} \\
&\Rightarrow E * (E) \\
&\Rightarrow E * (E + E) \\
&\Rightarrow E * (E + x) \\
&\Rightarrow E * (z + x) \\
&\Rightarrow (E) * (z + x) \\
&\Rightarrow (E + E) * (z + x) \\
&\Rightarrow (E + y) * (z + x) \\
&\Rightarrow (x + y) * (z + x)
\end{aligned}
$$

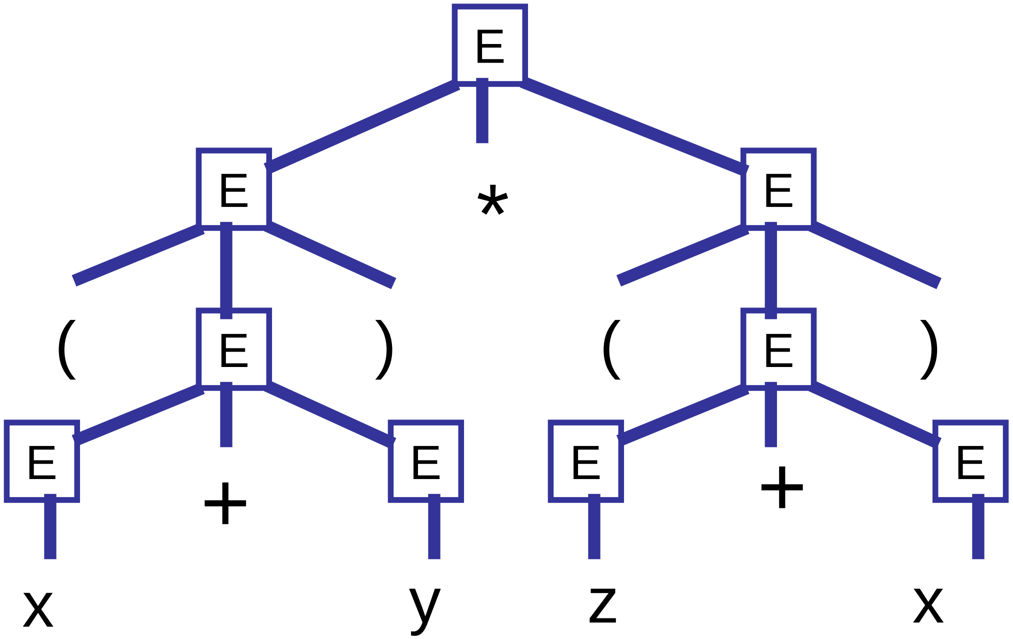
A rightmost derivation

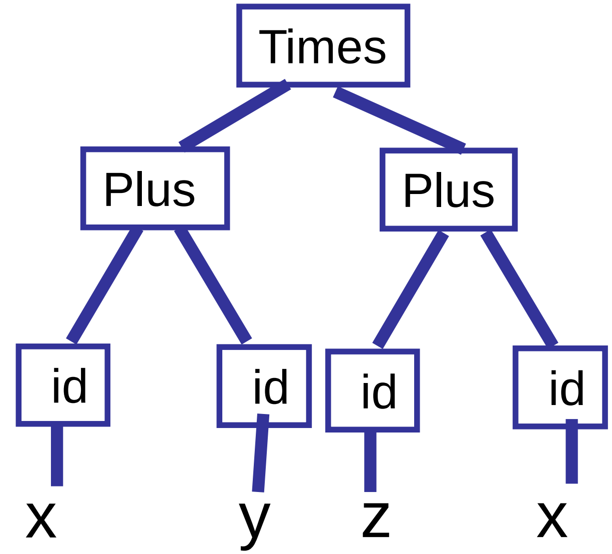
Derivation Trees



The derivation tree for (x + y) * (z + x). All derivations of this expression will produce the same derivation tree.

Concrete vs. Abstract Syntax Trees





**parse tree =
derivation tree =
concrete syntax tree**

An AST contains only the information needed to generate an intermediate representation

L(G) = The Language Generated by Grammar G

$$L(G) = \left\{ w \in T^* \mid S \Rightarrow^+ w \right\}$$

For example, if G has productions

$$\mathrm{S} \rightarrow \mathrm{aSb} \mid \varepsilon$$

then

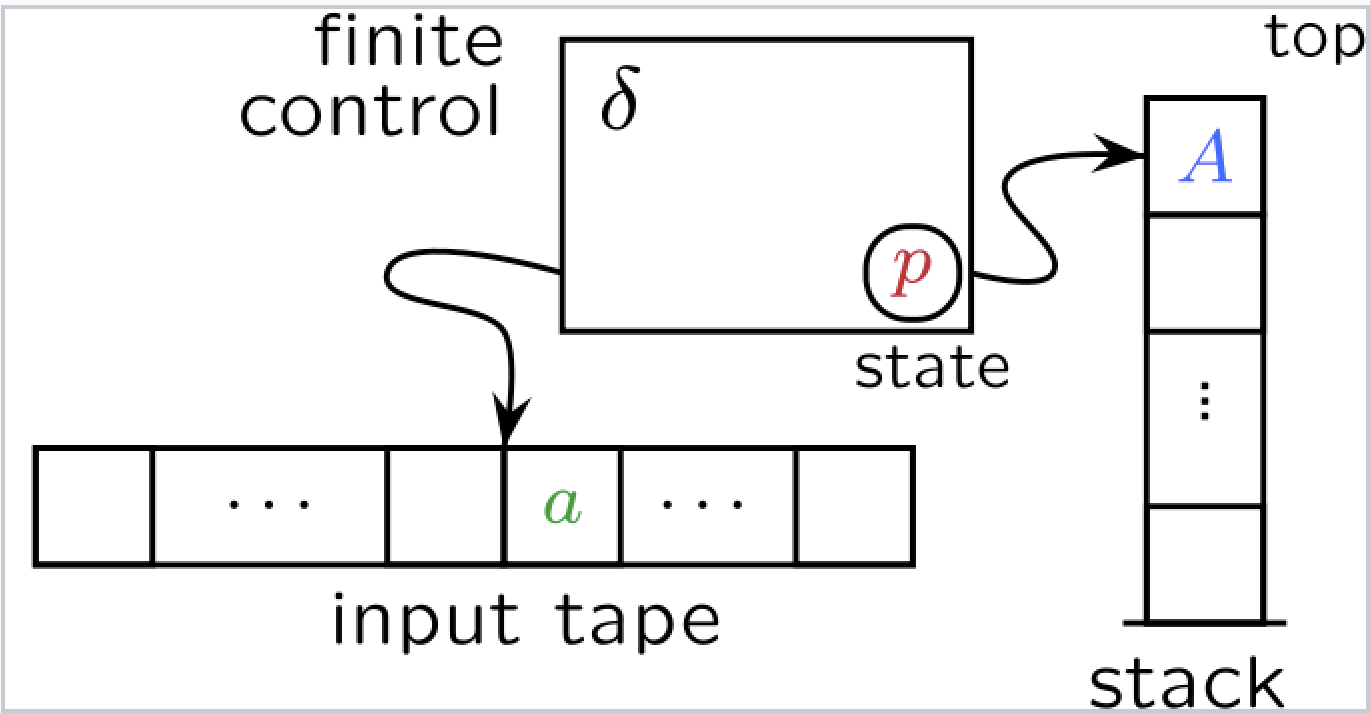
$$L(G) = \left\{ a^n b^n \mid n \geq 0 \right\}.$$

So CFGs can capture more than regular languages!

Pushdown Automata (PDAs)

Regular languages are accepted by Finite Automata.
Context-free languages are accepted by Pushdown Automata, a finite automata augmented with a stack.

Illustration from https://en.wikipedea.org/wiki/Pushdown_automaton



A diagram of a pushdown automaton

Pushdown Automata (PDAs)

$$M = (Q, \Sigma, \Gamma, \delta, q_0, Z)$$

Q : states Σ : alphabet Γ : stack symbols

$q_0 \in \mathrm{Q}$: start state

$\mathrm{Z} \in \Gamma$: initial stack symbol

$$\delta : \forall q \in Q, a \in (\Sigma \cup \{\varepsilon\}), X \in \Gamma,$$

$$\delta(\mathrm{q}, \mathrm{a}, \mathrm{X}) \subseteq \mathrm{Q} \times \Gamma^{*}$$

Pushdown Automata (PDAs)

$(q', \beta) \in \delta(\mathrm{q}, \mathrm{a}, \mathrm{X})$ means that when the machine is in state q reading a with X on top of the stack, it can move to state q' and replace X with β. That is, it "pops" X and "pushes" β (leftmost symbol is top of stack).

Pushdown Automata (PDAs)

For $q \in Q, w \in \Sigma^*, \alpha \in \Gamma^*$

$$(q, w, \alpha)$$

is called an instantaneous description (ID). It denotes the PDA in state q looking at the first symbol of w, with α on the stack (top at left).

Language accepted by a PDA

For $(q', \beta) \in \delta(q, a, X)$, $a \in \Sigma$ define

the relation $\rightarrow$ on IDs as

$$(q, aw, X\alpha) \rightarrow (q', w, \beta\alpha)$$

and for $(q', \beta) \in \delta(q, \varepsilon, X)$ as

$$(q, w, X\alpha) \rightarrow (q', w, \beta\alpha)$$

$$L(M) =$$

$$\{w \in \Sigma^* \mid \exists q \in Q, (q_0, w, Z) \rightarrow^+ (q, \varepsilon, \varepsilon)\}$$

Exercise : work out the details of this PDA

$$(q_{0,} aaabbb, Z)$$

$$\to (q_a, aabbb, A)$$

$$\to (q_a, abbb, AA)$$

$$\to (q_a, bbb, AAA)$$

$$\to (q_b, bb, AA)$$

$$\to (q_b, b, A)$$

$$\to (q_b, \varepsilon, \varepsilon)$$

$$L(M) = \left\{ a^n b^n \mid n \geq 0 \right\}$$

PDAs and CFGs Facts
(we will not prove them)

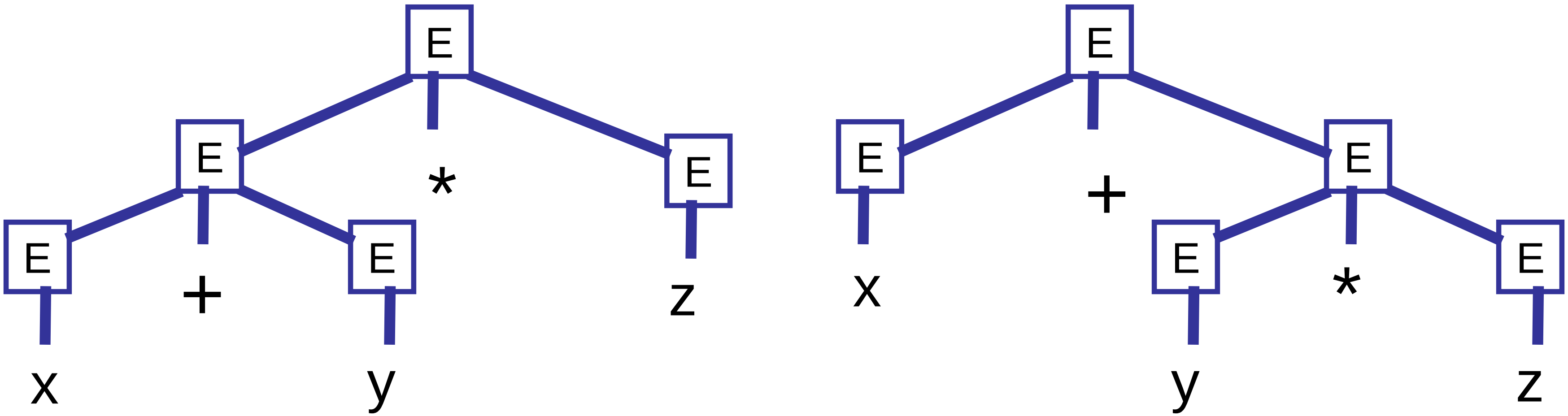
1) For every CFG G there is a PDA M such that $L(G) = L(M)$.

2) For every PDA M there is a CFG G such that $L(G) = L(M)$.

Parsing problem solved? Given a CFG G just construct the PDA M? Not so fast!

For programming languages we want M to be deterministic!

Origins of nondeterminism? Ambiguity!



Both derivation trees correspond "x + y * z".
But (x+y) * z is not the same as x + (y * z).

This type of ambiguity will cause problems when we try to go from program texts to derivation trees! Semantic ambiguity!

We can often modify the grammar in order to eliminate ambiguity

$$G_2 = (N_2, T_1, P_2, E)$$

$N_2 = \{\mathrm{E}, \mathrm{T}, \mathrm{F}\}$ $\qquad$ $T_1 = \{+, *, (,), \mathrm{id}\}$

$P_2:$

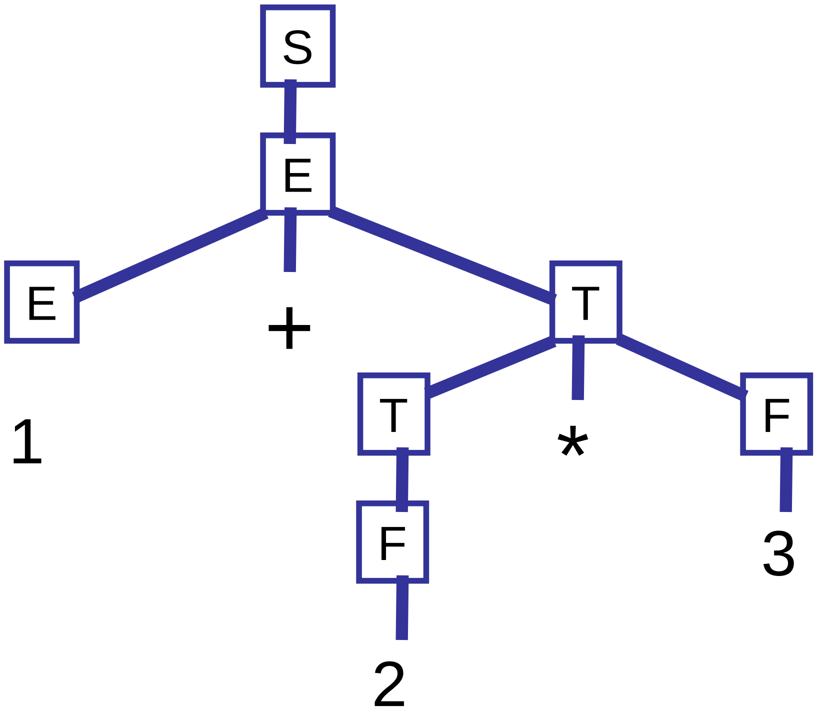
$\mathrm{E} \to \mathrm{E} + \mathrm{T} \mid \mathrm{T}$ $\qquad$ (expressions)

$\mathrm{T} \to \mathrm{T} * \mathrm{F} \mid \mathrm{F}$ $\qquad$ (terms)

$\mathrm{F} \to (\mathrm{E}) \mid \mathrm{id}$ $\qquad$ (factors)

Can you prove that $L(G_1) = L(G_2)$?

The modified grammar eliminates ambiguity



This is now the *unique* derivation tree for
x + y * z

Fun Fun Facts

(1) Some context-free languages are *inherently ambiguous* --- every context-free grammar for them will be ambiguous. For example:

$$L = \left\{ a^n b^n c^m d^m \mid m \geq 1, n \geq 1 \right\}$$
$$\cup \left\{ a^n b^m c^m d^n \mid m \geq 1, n \geq 1 \right\}$$

(2) Checking for ambiguity in an arbitrary context-free grammar is not decidable! Ouch!

(3) Given two grammars G1 and G2, checking L(G1) = L(G2) is not decidable! Ouch!

See Hopcroft and Ullman, "Introduction to Automata Theory, Languages, and Computation"

Two approaches to building stack-based parsing machines: top-down and bottom-up

- **Top Down : attempts a <u>left-most derivation</u>. We will look at two techniques:**
  - **Recursive decent (hand coded)**
  - **Predictive parsing (table driven)**
- **Bottom-up : attempts a <u>right-most derivation backwards</u>. We will look at two techniques:**
  - **SLR(1) : Simple LR(1)**
  - **LR(1)**

Bottom-up techniques are strictly more powerful. That is, they can parse more grammars.

Recursive Descent Parsing

(G5)

S :: = if E then S else S
      | begin S L
      | print E

E ::= NUM = NUM

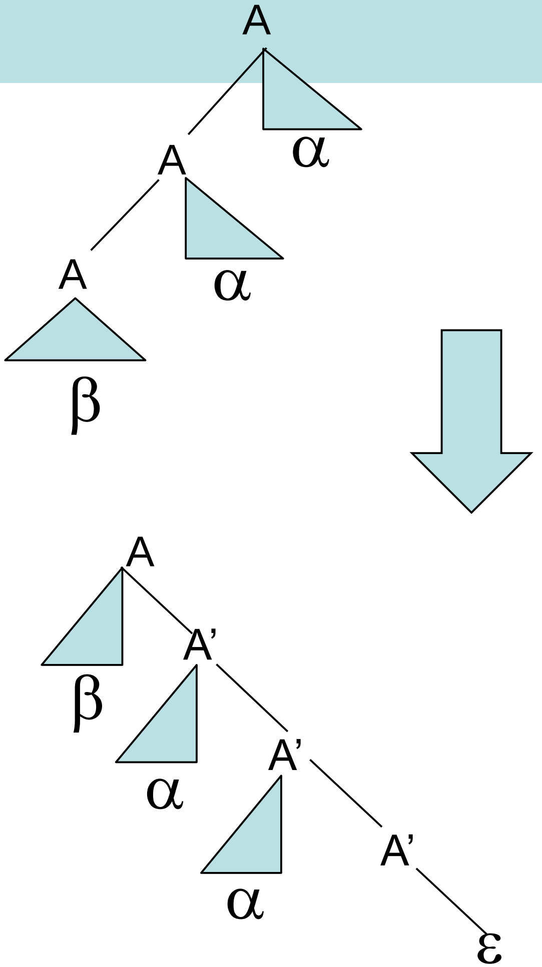
L ::= end
    | ; S L

```
int tok = getToken();

void advance() {tok = getToken();}
void eat (int t) {if (tok == t) advance(); else
error();}

void S() {switch(tok) {
      case IF:    eat(IF); E(); eat(THEN);
                  S(); eat(ELSE); S(); break;
      case BEGIN: eat(BEGIN); S(); L(); break;
      case PRINT: eat(PRINT); E(); break;
      default: error();
     }}

void L() {switch(tok) {
      case END:  eat(END); break;
      case SEMI: eat(SEMI); S(); L(); break;
      default: error();
     }}

void E() {eat(NUM) ; eat(EQ); eat(NUM); }
```

Parse corresponds to a left-most derivation constructed in a “top-down” manner

Example From Andrew Appel, “Modern Compiler Implementation in Java” page 46

But "left recursion" $E \to E + T$ in G_2 will lead to an infinite loop!



Eliminate left recursion!

A -> Aα1 | Aα2 | . . . | Aαk |
β1 | β2 | . . . | βn

A -> β1 A' | β2 A' | . . . | βn A'

A' -> α1 A' | α2 A'| . . . | αk A' | ε

For eliminating left-recursion in general, see Aho and Ullman.

Eliminate left recursion

$$G_3 = (N_3, T_1, P_3, E)$$

$N_2 = \{\mathrm{E}, \mathrm{E'}, \mathrm{T}, \mathrm{T'}, \mathrm{F}\}$ $\qquad T_1 = \{+, *, (,), \mathrm{id}\}$

P_2 :

$$\mathrm{E} \to \mathrm{T\ E'}$$

$$E' \to +T\ E' \mid \varepsilon$$

$$\mathrm{T} \to \mathrm{F\ T'}$$

$$T' \to *F\ T' \mid \varepsilon$$

$$\mathrm{F} \to (\mathrm{E}) \mid \mathrm{id}$$

Can you prove that $L(G_2) = L(G_3)$?

Recursive descent pseudocode

$$\mathrm{getE}() = \mathrm{getT}(); \mathrm{getE}'()$$

$$\mathrm{get}E'() = \text{if } \ \mathrm{token}() = "+" \text{ then eat}("+"); \mathrm{get}T(); \mathrm{get}E'()$$

$$\mathrm{getT}() = \mathrm{getF}(); \mathrm{getT}'()$$

$$\mathrm{get}T'() = \text{if } \ \mathrm{token}() = "*" \text{ then eat}("*"); \mathrm{get}F(); \mathrm{get}T'()$$

$$\mathrm{getF}() = \text{if } \ \mathrm{token}() = id$$

$$\text{then eat}(id)$$

$$\text{else eat}("("); \mathrm{get}E(); \text{eat}(")")$$

Where’s the stack machine?
It’s implicit in the call stack!

Parsing (x+y)*(z+x) using a call to getE()

			eat(“(“)	getE()	
		getF()	getF()	getF()	
	getT()	getT()	getT()	getT()	…
getE()	getE()	getE()	getE()	getE()	

call stack over time …

Compiler Construction
Lent Term 2022
Lecture 4: Table-driven top-down (LL) parsing

1. LL(k) vs LR(k) parsing
2. Automating left-most derivations?
3. FIRST, FOLLOW, and the LL(1) parsing table.
4. LL(1) table-based parsing
5. Computing FIRST and FOLLOW

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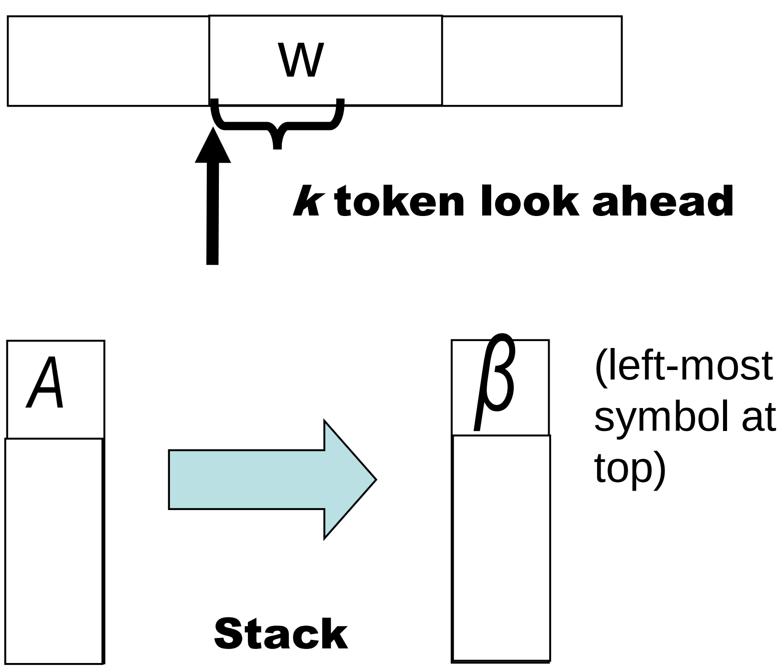
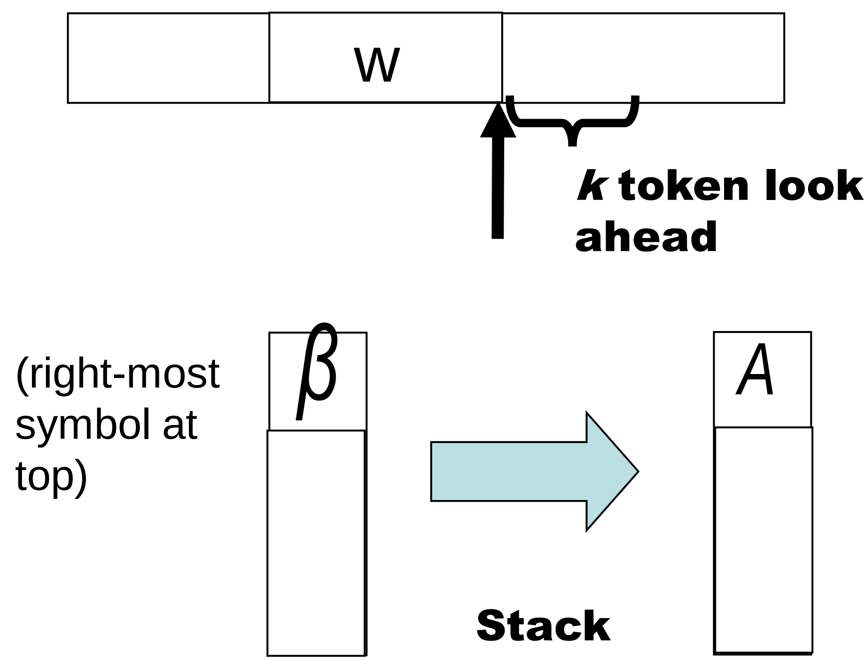
LL(k) and LR(k)

- **LL(k)** : (**L**)eft-to-right parse, (**L**)eft-most derivation, k-symbol lookahead. Based on looking at the next k tokens, an LL(k) parser must *predict* the next production. We have been looking at LL(1).
- **LR(k)** : (**L**)eft-to-right parse, (**R**)ight-most derivation, k-symbol lookahead. Postpone production selection until *the entire* right-hand-side has been seen (and as many as k symbols beyond). LR parsers perform a rightmost derivation <u>backwards</u>!

LL(k) vs. LR(k) reductions (SLR(1) as well)

$$A \to \beta \Rightarrow^{+} w \qquad \beta \in (T \cup N)^{*} \qquad w \in T^{*}$$

$LL(k)$	$LR(k)$
	

For LL(1), augment Grammar with end-of-input

$$G_3' = (N_3', T_3, P_3', S)$$

$$N_3' = \{\mathrm{E}, \mathrm{E'}, \mathrm{T}, \mathrm{T'}, \mathrm{F}, \mathrm{S}\} \qquad T_3 = \{+, *, (,), \mathrm{id}, \$\}$$

P_3' :

$S \to E\$$ ($ is end of input marker)

$$\mathrm{E} \to \mathrm{T\ E'}$$

$$E' \to +T\ E' \mid \varepsilon$$

$$\mathrm{T} \to \mathrm{F\ T'}$$

$$T' \to *F\ T' \mid \varepsilon$$

$$\mathrm{F} \to (\mathrm{E}) \mid \mathrm{id}$$

Leftmost derivations

$$w \in T^* \qquad \alpha, \beta \in (N \cup T)^*$$

Given : $wA\beta$ and a production $A \to \gamma$

a leftmost derivation step is written as

$$wA\beta \Rightarrow_{lm} w\gamma\beta$$

A left-most derivation of *(x+y)*

$$
\begin{aligned}
S &\Rightarrow_{lm} E\$ \\
&\Rightarrow_{lm} TE'\$ \\
&\Rightarrow_{lm} FT'E'\$ \\
&\Rightarrow_{lm} (E)T'E'\$ \\
&\Rightarrow_{lm} (TE')T'E'\$ \\
&\Rightarrow_{lm} (FT'E')T'E'\$ \\
&\Rightarrow_{lm} (xT'E')T'E'\$ \\
&\Rightarrow_{lm} (xE')T'E'\$ \\
&\Rightarrow_{lm} (x+TE')T'E'\$ \\
&\Rightarrow_{lm} (x+FT'E')T'E'\$ \\
&\Rightarrow_{lm} (x+yT'E')T'E'\$ \\
&\Rightarrow_{lm} (x+yE')T'E'\$ \\
&\Rightarrow_{lm} (x+y)T'E'\$ \\
&\Rightarrow_{lm} (x+y)E'\$ \\
&\Rightarrow_{lm} (x+y)\$
\end{aligned}
$$

Idea: Can we turn left-most derivations into a stack machine (a PDA)? Perhaps this will work: If $S \Rightarrow_{lm}^{+} w\alpha\$$ then w has been read from the input and α is on on the stack.

This looks promising. But can we make it work?

input	stack	via production
$(x+y)\$$	S	$S \to E\$$
$(x+y)\$$	$E\$$	$E \to TE'$
$(x+y)\$$	$TE'\$$	$T \to FT'$
$(x+y)\$$	$FT'E'\$$	$F \to (E)$
$(x+y)\$$	$(E)T'E'\$$	match
$x+y)\$$	$E)T'E'\$$	$E \to TE'$
$x+y)\$$	$TE')T'E'\$$	$T \to FT'$
$x+y)\$$	$FT'E')T'E'\$$	$F \to id$
$x+y)\$$	$idT'E')T'E'\$$	match
$+y)\$$	$T'E')T'E'\$$	$T' \to \varepsilon$

But how do we automate selection of the production to use at each step?

input	stack	via production
$+y)\$$	$E')T'E'\$$	$E' \to +TE'$
$+y)\$$	$+TE')T'E'\$$	match
$y)\$$	$TE')T'E'\$$	$T \to FT'$
$y)\$$	$FT'E')T'E'\$$	$F \to id$
$y)\$$	$idT'E')T'E'\$$	match
$)\$$	$T'E')T'E'\$$	$T' \to \varepsilon$
$)\$$	$E')T'E'\$$	$E' \to \varepsilon$
$)\$$	$)T'E'\$$	match
$\$$	$T'E'\$$	$T' \to \varepsilon$
$\$$	$E'\$$	$E' \to \varepsilon$
$\$$	$\$$	accept!

FIRST (we will see how to compute later)

$$\text{FIRST}(\alpha) = \left\{ a \in T \mid \exists \beta \in (N \cup T)^{*}, \alpha \Rightarrow^{*} a\beta \right\}$$

$S \to E\$$	$\text{FIRST}(S) = \{ (, id \}$
$\text{E} \to \text{T E'}$	$\text{FIRST}(E) = \{ (, id \}$
$E' \to +T\ E' \mid \varepsilon$	$\text{FIRST}(E') = \{ +, \varepsilon \}$
$\text{T} \to \text{F T'}$	$\text{FIRST}(T) = \{ (, id \}$
$T' \to\ *F\ T' \mid \varepsilon$	$\text{FIRST}(T') = \{ *, \varepsilon \}$
$\text{F} \to (\text{E}) \mid \text{id}$	$\text{FIRST}(T) = \{ (, id \}$

FOLLOW (we will see how to compute later)

$$\text{FOLLOW}(A) = \left\{ a \mid \exists \alpha\ \beta, S \Rightarrow^{+} \alpha A a \beta \right\}$$

$S \to E\$$	
$\mathrm{E} \to \mathrm{T\ E'}$	$\text{FOLLOW}(E) = \{\ ),\$\}$
$E' \to +T\ E' \mid \varepsilon$	$\text{FOLLOW}(E') = \{\ ),\$\}$
$\mathrm{T} \to \mathrm{F\ T'}$	$\text{FOLLOW}(T) = \{+,),\$\}$
$T' \to\ *F\ T' \mid \varepsilon$	$\text{FOLLOW}(T') = \{+,),\$\}$
$\mathrm{F} \to (\mathrm{E}) \mid \mathrm{id}$	$\text{FOLLOW}(F) = \{+,*,),\$\}$

$$")" \in \text{FOLLOW}(E)?$$

$$S \Rightarrow E\$ \Rightarrow TE'\$ \Rightarrow FT'E'\$ \Rightarrow (E)T'E'\$$$

The LL(1) Parsing table *M*

for all $A \in N$, $a \in T$, $M[A, a] = \{\}$

for each $A \in N$

for each production $A \to \alpha$

if $a \in \mathrm{FIRST}(\alpha)$ and $a \neq \varepsilon$

then $\mathrm{M}[A, a] = M[A, a] \cup \{A \to \alpha\}$

else if $\varepsilon \in \mathrm{FIRST}(\alpha)$

then for each $b \in \mathrm{FOLLOW}(A)$

$\mathrm{M}[A, b] = M[A, b] \cup \{A \to \alpha\}$

Table M for grammar G_3'

	id	$+$	$*$	$($	$)$	$\$$
E	$E \to TE'$			$E \to TE'$		
E'		$E' \to +TE'$			$E' \to \varepsilon$	$E' \to \varepsilon$
T	$T \to FT'$			$T \to FT'$		
T'		$T' \to \varepsilon$	$T' \to *FT'$		$T' \to \varepsilon$	$T' \to \varepsilon$
F	$F \to id$			$F \to (E)$		

The LL(1) Parsing Algorithm

$a := LexNextToken()$

$X := TopOfStack()$

while $(X \neq \$)$

  if $X = a$ (* a match *)

  then pop; $a := LexNextToken()$

  else if $M[X,a] = \{X \rightarrow \alpha\}$

    then pop; push α (leftmost symbol on top)

  $X :=$ TopOfStack()

Now use M to parse $(x+y)$ ...

input	stack	action
$(x+y)\$$	S	$M[S,(\,]=\{S \to E\$\}$
$(x+y)\$$	$E\$$	$M[E,(\,]=\{E \to TE'\}$
$(x+y)\$$	$TE'\$$	$M[T,(\,]=\{T \to FT'\}$
$(x+y)\$$	$FT'E'\$$	$M[F,(\,]=\{F \to (E)\}$
$(x+y)\$$	$(E)T'E'\$$	$match$
$x+y)\$$	$E)T'E'\$$	$M[E,id]=\{E \to TE'\}$
$x+y)\$$	$TE')T'E'\$$	$M[T,id]=\{T \to FT'\}$
$x+y)\$$	$FT'E')T'E'\$$	$M[F,id]=\{F \to id\}$
$x+y)\$$	$idT'E')T'E'\$$	$match$
$+y)\$$	$T'E')T'E'\$$	$M[T',+]=\{T' \to \varepsilon\}$

… kachunk, kachunk, kachunk …

input	stack	action
$+y)\$$	$E')T'E'\$$	$M[E',+]=\{E'\to +TE'\}$
$+y)\$$	$+TE')T'E'\$$	$match$
$y)\$$	$TE')T'E'\$$	$M[T,id]=\{T\to FT'\}$
$y)\$$	$FT'E')T'E'\$$	$M[F,id]=\{F\to id\}$
$y)\$$	$idT'E')T'E'\$$	$match$
$)\$$	$T'E')T'E'\$$	$M[T',)]=\{T'\to \varepsilon\}$
$)\$$	$E')T'E'\$$	$M[E',)]=\{E'\to \varepsilon\}$
$)\$$	$)T'E'\$$	$match$
$\$$	$T'E'\$$	$M[T',\$]=\{T'\to \varepsilon\}$
$\$$	$E'\$$	$M[E',\$]=\{E'\to \varepsilon\}$
$\$$	$\$$	$accept$

NULLABLE

$$\text{NULLABLE}(\alpha) = \text{true}$$

$$\text{if and only if } \alpha \Rightarrow^{*} \varepsilon.$$

$$\text{NULLABLE}(\varepsilon) = \mathit{true}$$

$$\text{NULLABLE}(c) = \mathit{false} \qquad (c \in T)$$

$$\text{NULLABLE}(A) = \qquad (A \in N)$$

$$\bigvee_{A \to \alpha} \text{NULLABLE}(\alpha)$$

$$\text{NULLABLE}(X\beta) = \qquad (X \in T \cup N)$$

$$\text{NULLABLE}(X) \wedge \text{NULLABLE}(\beta)$$

Computing FIRST

$\text{for all } a \in T, \text{FIRST(a)} := \{a\}$

$\text{for all } A \in N, \text{FIRST(A)} := \{\,\}$

$\text{while FIRST changes}$

$\quad \text{if } A \to \varepsilon \text{ is a production}$

$\quad \text{then FIRST(A)} := \text{FIRST(A)} \cup \{\varepsilon\}$

$\quad \text{if } A \to \mathrm{X}_1\mathrm{X}_2 \cdots \mathrm{X}_\mathrm{k} \text{ is a production}$

$\quad \text{then } \mathrm{j} = 1; \text{done} := \text{false}$

$\qquad \text{while not done and } \mathrm{j} \le \mathrm{k}$

$\qquad\quad \text{FIRST(A)} := \text{FIRST(A)} \cup (\text{FIRST}(\mathrm{X}_\mathrm{j}) - \{\varepsilon\})$

$\qquad\quad \text{if NULLABLE}(\mathrm{X}_\mathrm{j})$

$\qquad\quad \text{then } \mathrm{j} := \mathrm{j} + 1$

$\qquad\quad \text{else done} := \textit{true}$

$\qquad \text{if } \mathrm{j} = \mathrm{k} + 1 \text{ then FIRST(A)} := \text{FIRST(A)} \cup \{\varepsilon\}$

Computing FOLLOW

for all $A \in N$, FOLLOW$(A) := \{\}$

FOLLOW(S) $:= \{\$\}$ (S is the start symbol)

while FOLLOW changes

if $A \rightarrow \alpha B \beta$ is a production ($B \in N, \beta \neq \varepsilon$)

then FOLLOW(B) $:=$ FOLLOW(B) $\cup$ (FIRST(β) - $\{\varepsilon\}$)

if $A \rightarrow \alpha B \beta$ is a production and $\varepsilon \in$ FIRST(β)

then FOLLOW(B) $:=$ FOLLOW(B) $\cup$ FOLLOW(A)

if $A \rightarrow \alpha B$ is a production ($B \in N$)

then FOLLOW(B) $:=$ FOLLOW(B) $\cup$ FOLLOW(A)

Many grammars cannot be parsed LL(1)

$$S \to d \mid XYS$$
$$Y \to c \mid \varepsilon$$
$$X \to Y \mid a$$

	FIRST	FOLLOW
S	$\{a,c,d\}$	$\{\}$
Y	$\{c\}$	$\{a,c,d\}$
X	$\{a,c\}$	$\{a,c,d\}$

$$M[S,d] = \{\, S \to d,\ S \to XYS \,\}$$

This is ambiguity!

Grammar is not LL(1)!

Bottom-up (LR) parsing to the rescue!

$$G_2 = (N_2, T_1, P_2, E)$$

$N_2 = \{\mathrm{E}, \mathrm{T}, \mathrm{F}\}$ $\qquad$ $T_1 = \{+, *, (,), \mathrm{id}\}$

$\mathrm{E} \to \mathrm{E} + \mathrm{T} \mid \mathrm{T}$ $\qquad$ $\mathrm{T} \to \mathrm{T} * \mathrm{F} \mid \mathrm{F}$ $\qquad$ $\mathrm{F} \to (\mathrm{E}) \mid \mathrm{id}$

With LR parsing we no longer have to eliminate left recursion from the grammar!

Compiler Construction
Lent Term 2022
Lecture 5 : Theoretical foundations of Bottom-up (LR) parsing

1. This lecture develops a general theory for non-deterministic bottom-up parsing
2. Next lecture will present two techniques for imposing determinism --- SLR(1) parsing and LR(1) parsing.

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This grammar will be our running example

$$G_2 = (N_2, T_1, P_2, E')$$

$N_2 = \{\mathrm{E'}, \mathrm{E}, \mathrm{T}, \mathrm{F}\}$ $\qquad T_1 = \{+, *, (,), \mathrm{id}\}$

$$
\begin{aligned}
P_2 : \mathrm{E'} &\to \mathrm{E} \\
\mathrm{E} &\to \mathrm{E} + \mathrm{T} \mid \mathrm{T} && \text{(expressions)} \\
\mathrm{T} &\to \mathrm{T} * \mathrm{F} \mid \mathrm{F} && \text{(terms)} \\
\mathrm{F} &\to (\mathrm{E}) \mid \mathrm{id} && \text{(factors)}
\end{aligned}
$$

Note: E’ was added for convenience to ensure that there is a single starting production.

Rightmost derivations

$$w \in T^{*} \qquad \alpha, \beta \in (N \cup T)^{*}$$

Given : $\alpha A w$ and a production $A \rightarrow \beta$

a rightmost derivation step is written as

$$\alpha A w \Rightarrow_{rm} \alpha \beta w$$

A rightmost derivation of *(x+y)*

$$
\begin{aligned}
E' &\Rightarrow_{rm} E \\
&\Rightarrow_{rm} T \\
&\Rightarrow_{rm} F \\
&\Rightarrow_{rm} (E) \\
&\Rightarrow_{rm} (E+T) \\
&\Rightarrow_{rm} (E+F) \\
&\Rightarrow_{rm} (E+y) \\
&\Rightarrow_{rm} (T+y) \\
&\Rightarrow_{rm} (F+y) \\
&\Rightarrow_{rm} (x+y)
\end{aligned}
$$

Top-down (LL) parsing is based on
<u>left-most</u> derivations.

Bottom-up (LR) parsing is based on
<u>right-most</u> derivations.

But Bottom-up parsers perform the derivation in reverse!

$$
\begin{aligned}
S &\Rightarrow_{rm} E \\
&\Rightarrow_{rm} T \\
&\Rightarrow_{rm} F \\
&\Rightarrow_{rm} (E) \\
&\Rightarrow_{rm} (E+T) \\
&\Rightarrow_{rm} (E+F) \\
&\Rightarrow_{rm} (E+y) \\
&\Rightarrow_{rm} (T+y) \\
&\Rightarrow_{rm} (F+y) \\
&\Rightarrow_{rm} (x+y)
\end{aligned}
$$

FLIP!

Start parse

$$
\begin{aligned}
(x+y) &\Leftarrow \\
(F+y) &\Leftarrow \\
(T+y) &\Leftarrow \\
(E+y) &\Leftarrow \\
(E+F) &\Leftarrow \\
(E+T) &\Leftarrow \\
(E) &\Leftarrow \\
F &\Leftarrow \\
T &\Leftarrow \\
E &\Leftarrow E'
\end{aligned}
$$

Finish

Can we transform a backwards derivation into an execution of a stack machine?

$$
\begin{aligned}
(x+y) &\Leftarrow \\
(F+y) &\Leftarrow \\
(T+y) &\Leftarrow \\
(E+y) &\Leftarrow \\
(E+F) &\Leftarrow \\
(E+T) &\Leftarrow \\
(E) &\Leftarrow \\
F &\Leftarrow \\
T &\Leftarrow \\
E &\Leftarrow E'
\end{aligned}
$$

View the reversed derivation as a stack machine (use $ as stack bottom and end-of-input).

Can we make this work?

stack	input
$\$$	$(x+y)\$$
$\$(F$	$+y)\$$
$\$(T$	$+y)\$$
$\$(E$	$+y)\$$
$\$(E+F$	$)\$$
$\$(E+T$	$)\$$
$\$(E)$	$\$$
$\$F$	$\$$
$\$T$	$\$$
$\$E$	$\$$
$\$E'$	$\$$

Let's try to formalize such a parser

An LR parser configuration has the form

$$\$\alpha,\ x\$$$

$$(\alpha \text{ is the stack, } x \text{ the remaining input})$$

The configuration is valid when there exists a right-most derivation of the form

$$S \Rightarrow^{*}_{rm} \alpha\, x$$

Let's try to formalize our (non-deterministic) parser

Suppose

$$\alpha A x \Rightarrow_{rm} \alpha\beta B z x$$

Our "backwards" parser MIGHT move from one configuration to another like so:

$$\$\alpha\beta Bz,\ x\$ \xrightarrow{\quad reduce \quad} \$\alpha A,\ x\$$$

This action is called a reduction using production $A \rightarrow \beta B z$

Are reduction actions sufficient?

Suppose we have the derivation

$$\alpha A x \Rightarrow_{rm} \alpha\beta B z x \Rightarrow_{rm} \alpha\beta\gamma z x$$

using $A \rightarrow \beta B z$ and then $B \rightarrow \gamma$.

Simulating this in reverse, our parser gets stuck:

$$\$\alpha\beta\gamma\,,\ zx\$$$

$$\xrightarrow{\quad reduce \quad} \$\alpha\beta B,\ zx\$$$

$$\xrightarrow{\quad ??? \quad} ???$$

We want $\beta B z$ on top of the stack!

We need an action that <u>shifts</u> a terminal onto the stack!

$$\boxed{\alpha A x \Rightarrow_{rm} \alpha\beta B z x \Rightarrow_{rm} \alpha\beta\gamma z x}$$

$$\$\alpha\beta\gamma\,,\ zx\$$$

$$\xrightarrow{reduce} \$\alpha\beta B,\ zx\$$$

$$\xrightarrow{shift(s)} \$\alpha\beta B z,\ x\$$$

$$\xrightarrow{reduce} \$\alpha A,\ x\$$$

How do we know when to stop shifting? Here we don't want to gobble up *x*!

Sanity check.

Let's make sure that this can work when B does not appear in the right - hand side of A's production,

$$\alpha BxAz \Rightarrow_{rm} \alpha Bxyz \Rightarrow_{rm} \alpha\gamma xyz$$

using production $A \rightarrow y$, then $B \rightarrow \gamma$.

Our parser's possible actions :

$$\$\alpha\gamma\,,\ xyz\$$$

$$\xrightarrow{\quad reduce \quad} \$\alpha B,\ xyz\$$$

$$\xrightarrow{\quad shift(s) \quad} \$\alpha Bxy,\ z\$$$

$$\xrightarrow{\quad reduce \quad} \$\alpha BxA,\ z\$$$

All good! But again, how do we know when to reduce and when to stop shifting?

Shift and reduce are sufficient.

The previous two slides demonstrate that if we have a derivation

$$S \Rightarrow^{*}_{\mathrm{rm}} w$$

Then we can always "replay it" in reverse using shift/reduce actions

$$\$, w\$ \rightarrow^{*} \$S, \$$$

This tells us that shift and reduce are sufficient.

However, when we are parsing a w we won't have access to a derivation to replay! So our parser will be non-deterministic and GUESS what the future holds!

Replay parsing of *(x+y)* using shift/reduce actions.
X=top-of-stack, a = next input token

stack	input	action[X, a]
$\$$	$(x+y)\$$	shift
$\$($	$x+y)\$$	shift
$\$(x$	$+y)\$$	reduce $F \rightarrow id$
$\$(F$	$+y)\$$	reduce $T \rightarrow F$
$\$(T$	$+y)\$$	reduce $E \rightarrow T$
$\$(E$	$+y)\$$	shift
$\$(E+$	$y)\$$	shift

... informal shift/reduce parse continued

stack	input	action[X, a]
$\$(E+y$	$)\$$	reduce $F \rightarrow id$
$\$(E+F$	$)\$$	reduce $T \rightarrow F$
$\$(E+T$	$)\$$	reduce $E \rightarrow E+T$
$\$(E$	$)\$$	shift
$\$(E)$	$\$$	reduce $F \rightarrow (E)$
$\$F$	$\$$	reduce $T \rightarrow F$
$\$T$	$\$$	reduce $F \rightarrow E$
$\$E$	$\$$	reduce $\mathrm{S} \rightarrow \mathrm{E}$
$\$E'$	$\$$	accept!

How do we decide when to shift and when to reduce?

Suppose $A \to \beta\gamma$ is a production. When our parser is in the configuration

$$\$\alpha\beta\gamma, x\$$$

we MIGHT want to reduce with $A \to \beta\gamma$.

However, if we have

$$\$\alpha\beta, x\$$$

we MIGHT want to continue parsing with the hope of eventually getting $\beta\gamma$ on top of the stack so that we can then reduce to A.

LR(0) items record how much of a production's right-hand side we have already parsed

For every grammar production

$$\mathrm{A} \rightarrow \beta\gamma \qquad (\beta, \gamma \in (N \cup T)^*)$$

produce the LR(0) item

$$\mathrm{A} \rightarrow \beta \bullet \gamma$$

Interpretation of $A \rightarrow \beta \bullet \gamma$: we have already parsed some input x derivable from β $(\beta \Rightarrow^*_{rm} x)$ and we MIGHT next see some input derivable from γ.

$LR(0)$ items for grammar G_2

$E' \to \bullet E$

$E \to \bullet E + T$

$E \to E \bullet + T$

$E \to E + \bullet T$

$E \to E + T \bullet$

$E \to \bullet T$

$E \to T \bullet$

$E' \to E \bullet$

$T \to \bullet T * T$

$T \to T \bullet * F$

$T \to T * \bullet F$

$T \to T * F \bullet$

$T \to \bullet F$

$T \to F \bullet$

$F \to \bullet (E)$

$F \to (\bullet E)$

$F \to (E \bullet)$

$F \to (E) \bullet$

$F \to \bullet id$

$F \to id \bullet$

Valid LR(0) items

Definition. Item $A \to \beta \bullet \gamma$ is valid for $\phi\beta$ if there exists a derivation

$$S \Rightarrow^*_{rm} \phi A x \Rightarrow_{rm} \phi\beta\gamma x$$

If item $A \to \beta \bullet \gamma$ is valid for $\phi\beta$ then our parser could use the item as a guide when in configuration

$$\$\phi\beta,\ z\$.$$

Suppose $A \to \beta B \gamma$ and $B \to \alpha_1 \mid \alpha_2 \mid \cdots \mid \alpha_k$. Consider the ways in which items for these productions might be used as parsing guides.

Derivation	Parse	Possible guides
S	$\$S, \$^*$	
$\Rightarrow^*_{rm} \phi A x$	$^*\leftarrow \$\phi A, x\$$	
$\Rightarrow_{rm} \phi\beta B\gamma x$	$\leftarrow \$\phi\beta B\gamma, x\$$	$A \to \beta B\gamma \bullet$
$\Rightarrow^*_{rm} \phi\beta B z x$	$^*\leftarrow \$\phi\beta B, zx\$$	$A \to \beta B \bullet \gamma$
$\Rightarrow_{rm} \phi\beta\alpha_i z x$	$\leftarrow \$\phi\beta\alpha_i, zx\$$	$B \to \alpha_i \bullet$
$\Rightarrow^*_{rm} \phi\beta u z x$	$^*\leftarrow \$\phi\beta, uzx\$$	$A \to \beta \bullet B\gamma,\ B \to \bullet\alpha_i$

Using items as parsing guides

Suppose our parser is in the config

$$\$\phi\beta,\ cz\$$$

and $A \to \beta \bullet c\gamma$ is valid for $\phi\beta$.

Then we MIGHT shift c onto the stack :

$$\$\phi\beta,\ cz\$ \xrightarrow{shift} \$\phi\beta c,\ z\$$$

Suppose our parser is in the config

$$\$\phi\beta,\ z\$$$

and $A \to \beta \bullet$ is valid for $\phi\beta$.

Then we MIGHT perform a reduction :

$$\$\phi\beta,\ z\$ \xrightarrow{reduce} \$\phi A,\ z\$$$

Using items as parsing guides

Suppose our parser is in the config

$$\$\phi\beta,\ z\$$$

which we will assume is valid, so $S \Rightarrow^*_{rm} \phi\beta z$.

Suppose $A \to \beta \bullet \gamma$ is valid for $\phi\beta$.

Then γ MIGHT capture the future of our parse (the past of that derivation). That is, it MIGHT be that

$$S \Rightarrow^*_{rm} \phi A x \Rightarrow_{rm} \phi\beta\gamma x \Rightarrow^*_{rm} \phi\beta y x = \phi\beta z$$

If so, our parser MIGHT proceed like so:

$$\$\phi\beta,\ z\$ = \$\phi\beta,\ yx\$ \to^* \$\phi\beta\gamma,\ x\$ \xrightarrow{reduce} \$\phi A,\ x\$.$$

That is, our parser could guess that γ will derive a prefix of the remaining input z.

The KEY idea in LR parsing

Augment our shift/reduce parser in such a way that in every configuration it can derive the set of all items valid for the the contents of the current stack.

Then at each step the parser can (non-deterministically) select an item from this set to use as a guide.

Defined a NFA with LR(0) items as states!

$$\boxed{A \to \beta \bullet c\gamma} \xrightarrow{\quad c \quad} \boxed{A \to \beta c \bullet \gamma}$$

$$\boxed{A \to \beta \bullet B\gamma} \xrightarrow{\quad B \quad} \boxed{A \to \beta B \bullet \gamma}$$

$$\boxed{A \to \beta \bullet B\gamma} \xrightarrow{\quad \varepsilon \quad} \boxed{B \to \bullet \alpha_i}$$

The initial state q_0 is this item constructed

from the unique starting production

$E' \to \bullet E$ (for example)

and every item (state) is a final state.

Let δ_G be the transition function of this NFA.

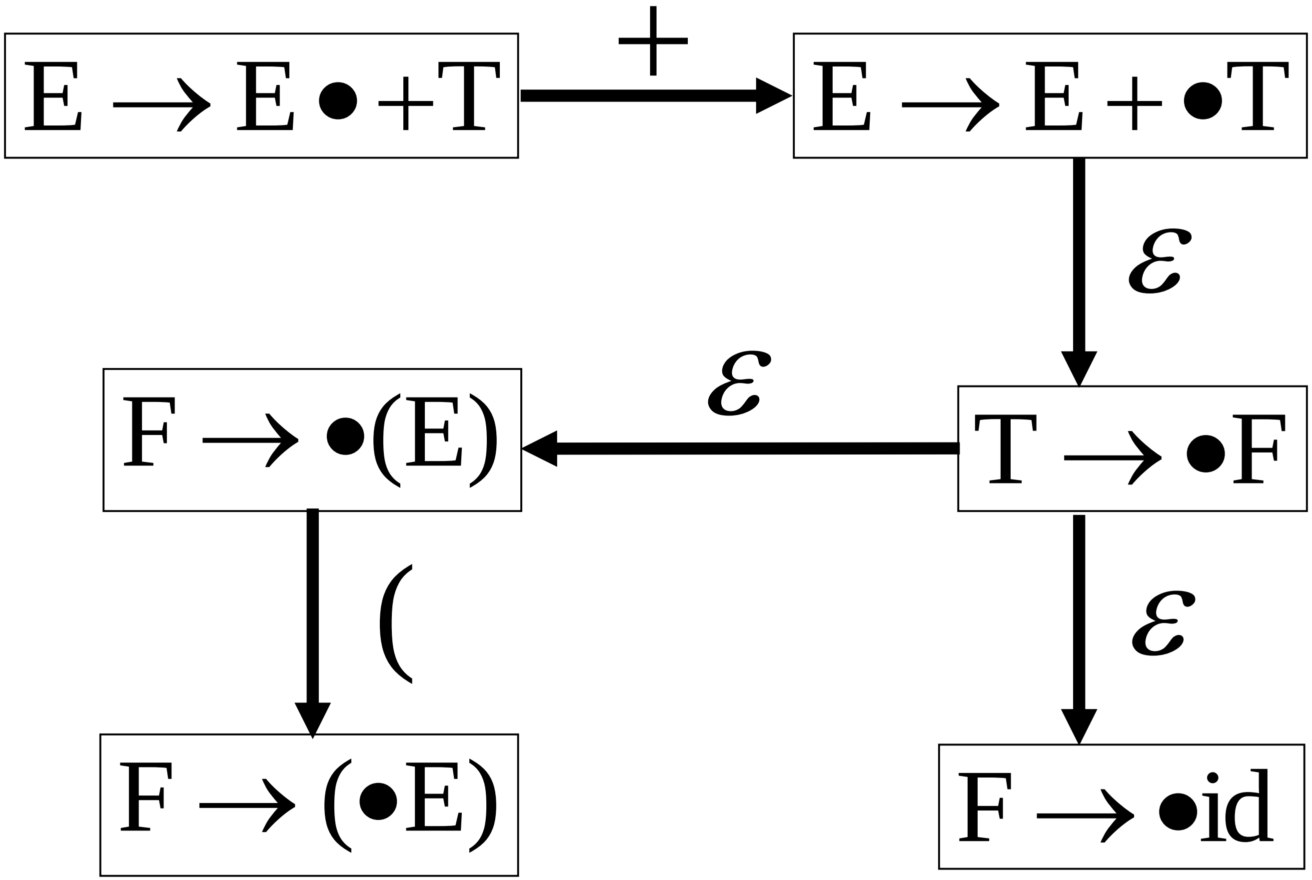
Main LR parsing theorem

Theorem. $A \to \beta \bullet \gamma \in \delta_G(q_0, \phi\beta)$ if and only if $A \to \beta \bullet \gamma$ is valid for $\phi\beta$.

Amazing fact: the language of the stack is regular!

See proof (not examinable) in Introduction to Automata Theory, Languages, and Computation. Hopcroft and Ullman.

A few NFA transitions for grammar G_2



A non-deterministic LR parsing algorithm

c := first symbol of input w$

while(true)

$\alpha :=$ the stack

if $A \to \beta \bullet c\gamma \in \delta_G(q_0, \alpha)$

then shift c onto the stack

c := next input token;

if A $\to \beta \bullet \in \delta_G(q_0, \alpha)$

then reduce : pop β off the stack

and then push A onto the stack;

if S $\to \beta \bullet \in \delta_G(q_0, \alpha)$

then accept and exit if no more input;

if none of the above then ERROR

This is non-deterministic since multiple conditions can be true and multiple items can match any condition.

How can we make the algorithm deterministic?

1. **The easy part: convert the NFA to a DFA**
2. **When there are shift/reduce or reduce/reduce conflicts, find some way of making a deterministic choice.**
3. **For (2), peek into the input buffer.**
4. **For (3), use FIRST and/or FOLLOW!**

Note : no matter how we do this there will be non-ambiguous grammars for which our deterministic parser will fail.

Next lecture : we will look at two popular approaches, SLR(1) and LR(1).

Compiler Construction
Lent Term 2022
Lecture 6: Deterministic SLR(1) and LR(1) parsing

1. SLR(1) parsing
2. LR(1) parsing.

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Our goal: impose deterministic choices on this non-deterministic LR parsing algorithm

c := first symbol of input w$

while(true)

$\alpha :=$ the stack

if $A \to \beta \bullet c\gamma \in \delta_G(q_0, \alpha)$

then shift c onto the stack

c := next input token;

if $A \to \beta \bullet \in \delta_G(q_0, \alpha)$

then reduce : pop β off the stack

and then push A onto the stack;

if $S \to \beta \bullet \in \delta_G(q_0, \alpha)$

then accept and exit if no more input;

if none of the above then ERROR

This is non-deterministic since multiple conditions can be true and multiple items can match any condition.

The easy part: NFA → DFA

In general, add new production $\text{S}' \to \text{S}$, where

S is the original start symbol. For the simple term

grammar G_2, add production

$$\text{E}' \to \text{E}$$

which produces the NFA start state

$$q_0 = \text{E}' \to \bullet\text{E}$$

The DFA start state is then

$$\varepsilon - \text{closure}(\{\text{E}' \to \bullet\text{E}\}) = \boxed{\begin{array}{l} \text{E}' \to \bullet\text{E} \\ \text{E} \to \bullet\text{E} + \text{T} \\ \text{E} \to \bullet\text{T} \\ \text{T} \to \bullet\text{T} * \text{F} \\ \text{T} \to \bullet\text{F} \\ \text{F} \to \bullet(E) \\ \text{F} \to \bullet\text{id} \end{array}}$$

The DFA transition function δ

For this DFA

$$\delta(\mathrm{I}, \mathrm{X}) = \varepsilon\text{-closure}(\{\mathrm{A} \to \alpha \mathrm{X} \bullet \beta \mid \mathrm{A} \to \alpha \bullet \mathrm{X}\beta \in I\})$$

Many books calls this GOTO(I, X).

and repeat the construction of DFA

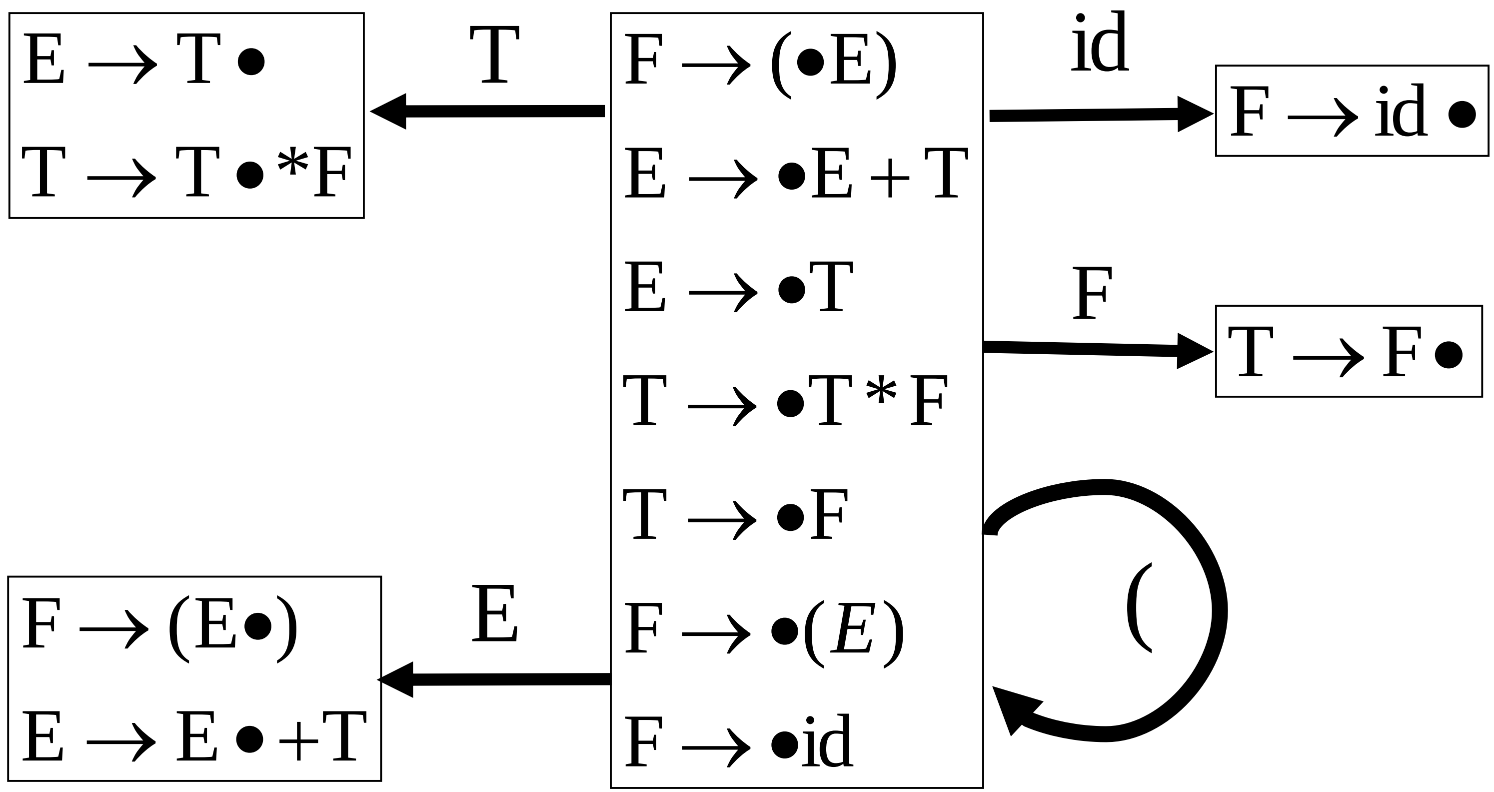
specialise d to LR(0) items (using

function called CLOSURE). I see no reason to do

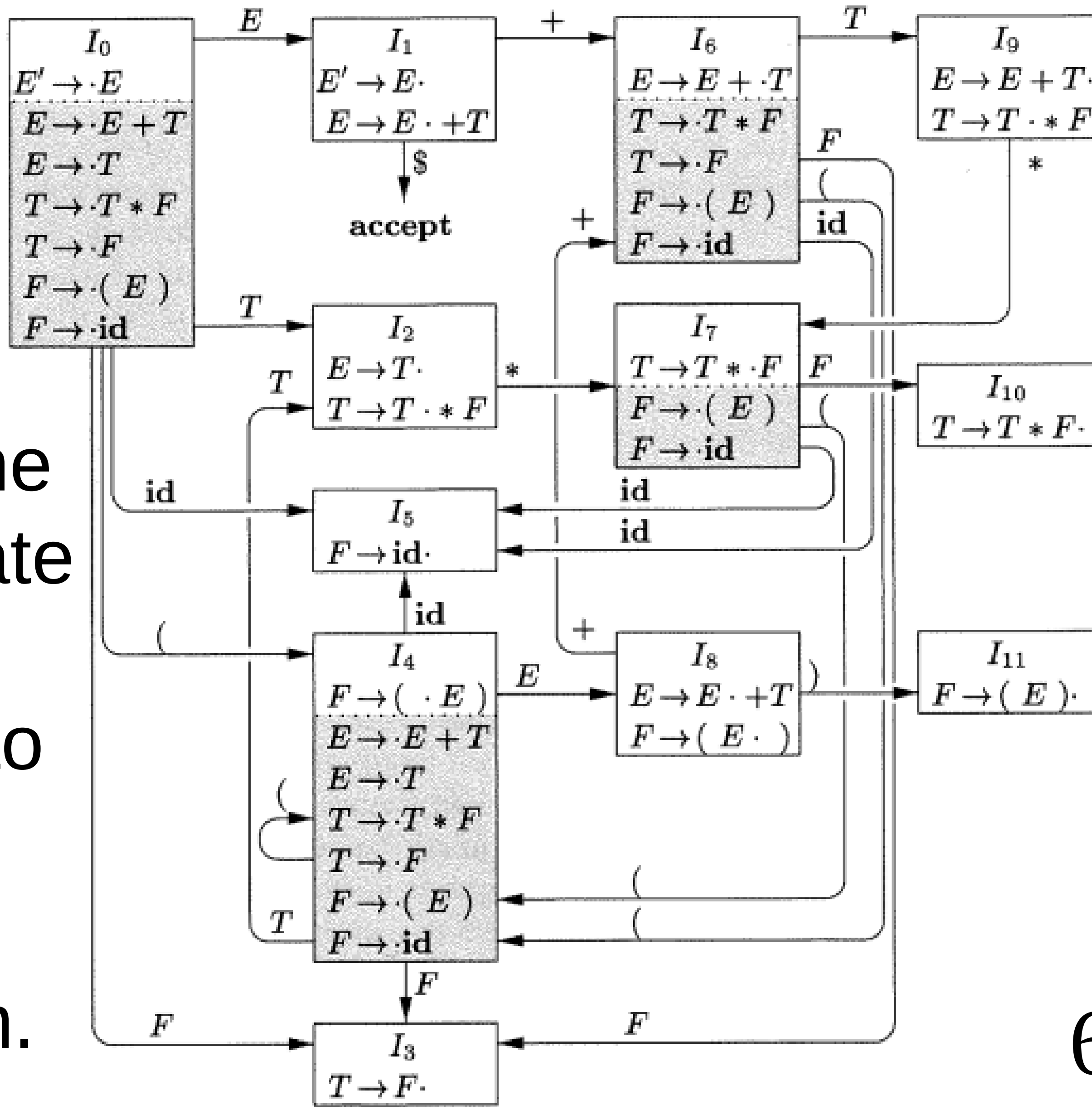
this since we already know how to build a DFA

from an NFA (see Lexing lecture).

A few DFA transitions for grammar G_2



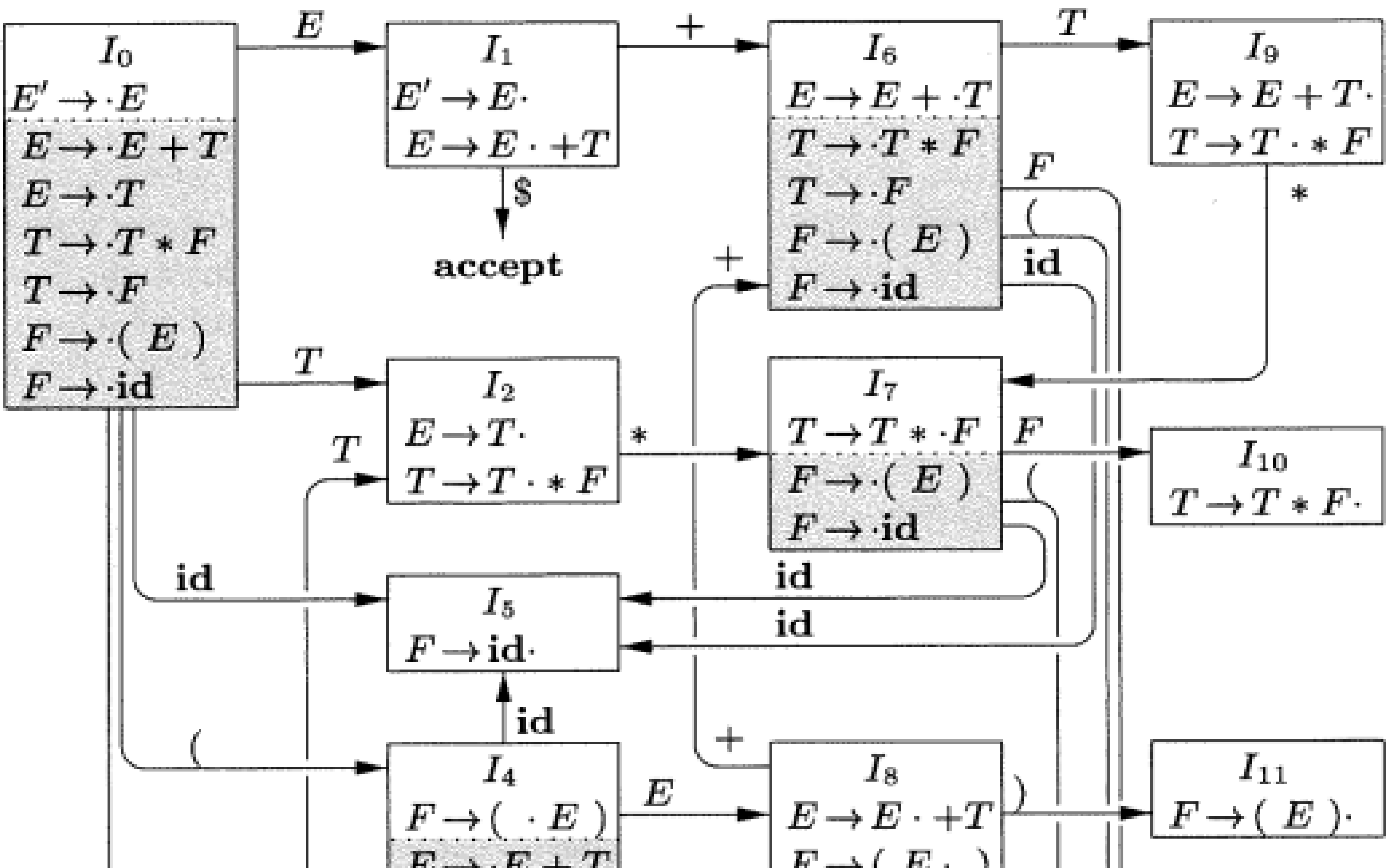
Full DFA for the stack language of G_2



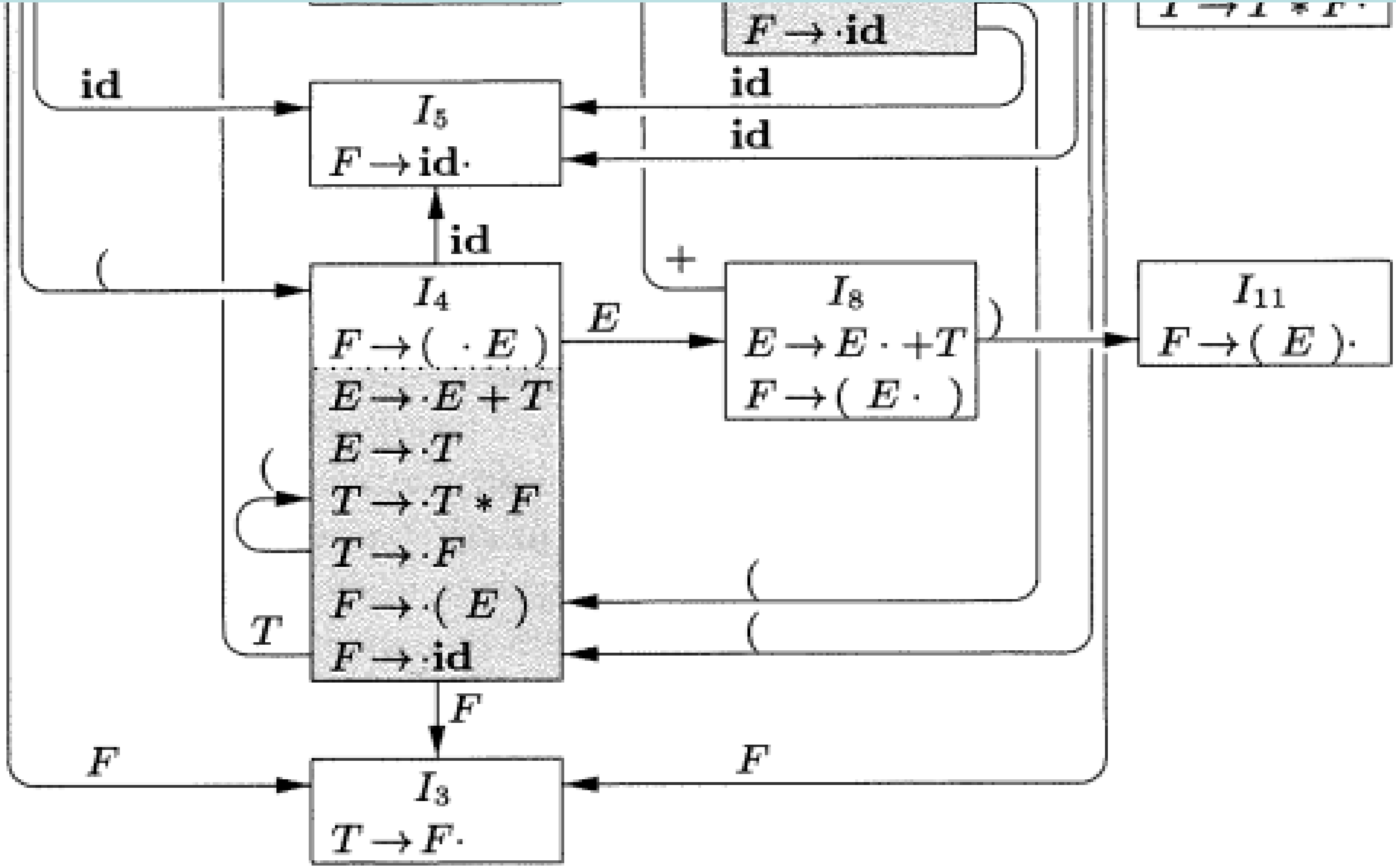
As usual, the ERROR state and transitions to it are not included in the diagram.

From Compilers by Aho, Lam, Sethi, Ullman

(enlarged to improve readability)



(enlarged to improve readability)



How can we avoid shift/reduce conflicts?

Consider I_2

$$\boxed{\begin{array}{l} \quad\quad I_2 \\ E \rightarrow T \bullet \\ T \rightarrow T \bullet *F \end{array}}$$

This inspires one approach called SLR(1) (Simple LR(1)):

1) Shift using if $*$ is the next token.

2) Reduce with $E \rightarrow T$ only if next token is in $\text{FOLLOW}(E) = \{(, +, \$\}$.

Now we can do a DETERMINISTIC SLR(1) parse of *(x+y)*

1) When the stack contains α, the parser is in state $\delta(\mathrm{I}_0, \alpha)$. For example,

$$\delta(\mathrm{I}_0, \mathrm{E} + \mathrm{T}) = \mathrm{I}_9$$

$$\delta(\mathrm{I}_0, (\mathrm{T} *) = \mathrm{I}_7$$

$$\delta(\mathrm{I}_0, \mathrm{E} * \mathrm{T}) = \mathrm{ERROR}$$

2) When the current state is I, the next token is c, and $\mathrm{A} \rightarrow \beta \bullet c\gamma \in \mathrm{I}$, then shift t onto stack

3) When the current state is I, the next token is c, $\mathrm{A} \rightarrow \beta \bullet \in \mathrm{I}$, and $\mathrm{c} \in \mathrm{FOLLOW(A)}$, then reduce with production $\mathrm{A} \rightarrow \beta$

Replay parsing of *(x+y)* using SLR(1) actions (FW(X) abbreviates FOLLOW(X))

stack, input	State action	reason
$\$, \quad (x+y)\$$	I_0 shift	$F \to \bullet(E) \in I_0$
$\$(, \quad x+y)\$$	I_4 shift	$F \to \bullet id \in I_4$
$\$(x, \quad +y)\$$	I_5 reduce $F \to id$	$"+" \in FW(F)$
$\$(F, \quad +y)\$$	I_3 reduce $T \to F$	$"+" \in FW(T)$
$\$(T, \quad +y)\$$	I_2 reduce $E \to T$	$"+" \in FW(E)$
$\$(E, \quad +y)\$$	I_8 shift	$E \to E \bullet +T \in I_8$
$\$(E+, \quad y)\$$	I_6 shift	$F \to \bullet id \in I_6$

stack,	input	State	action	reason
$\$(E+y,$	$)\$$	I_5	reduce $F \rightarrow id$	$")" \in \mathrm{FW(F)}$
$\$(E+F,$	$)\$$	I_3	reduce $T \rightarrow F$	$")" \in \mathrm{FW(T)}$
$\$(E+T,$	$)\$$	I_9	reduce $E \rightarrow E+T$	$")" \in \mathrm{FW(E)}$
$\$(E,$	$)\$$	I_8	shift	$\mathrm{E} \rightarrow (\mathrm{E}\bullet) \in \mathrm{I}_8$
$\$(E),$	$\$$	I_{11}	reduce $F \rightarrow (E)$	$"\$" \in \mathrm{FW(F)}$
$\$F,$	$\$$	I_3	reduce $T \rightarrow F$	$"\$" \in \mathrm{FW(T)}$
$\$T,$	$\$$	I_2	reduce $F \rightarrow E$	$"\$" \in \mathrm{FW(F)}$
$\$E,$	$\$$	I_1	reduce $\mathrm{E'} \rightarrow \mathrm{E}$	$"\$" \in \mathrm{FW(E')}$
$\$E',$	$\$$	accept!		

Better idea: Replace the stack contents with state numbers!

	0
$($	04
$(id$	045
$(F$	043
$(T$	042
$(E$	048
$(E+$	0486

$(E+id$	04865
$(E+F$	04863
$(E+T$	04869
$(E$	048
(E)	$04\ 11$
F	03
T	02
E	01

LR parsing with DFA states on the stack

$a :=$ first symbol of input w$

while(true)

  $s :=$ state at top of stack

  if ACTION$[s, a] =$ shift t

  then push t on stack

    $a :=$ next input token

  else if ACTION$[s, a] =$ reduce $A \rightarrow \beta$

    then pop $|\beta|$ states off the stack

      $t :=$ state at top of stack

      push GOTO$[t, A]$ onto the stack

  else if ACTION$[s, a] =$ accept

    then accept and exit

  else ERROR

ACTION and GOTO for SLR(1)

$$\text{If } [\mathrm{A} \rightarrow \alpha \bullet a\beta] \in \mathrm{I_i} \text{ and } \delta(\mathrm{I_i}, a) = \mathrm{I_j} \text{ then ACTION[i, a]} = \text{shift j}$$

$$\text{If } [\mathrm{A} \rightarrow \alpha \bullet] \in \mathrm{I_i} \text{ and } \mathrm{A} \neq \mathrm{S'}$$

$$\text{then for all } a \in \text{FOLLOW(A)},$$

$$\text{ACTION[i, a]} = \text{reduce } \mathrm{A} \rightarrow \alpha$$

Note: there may still be shift/reduce or reduce/reduce conflicts!

$$\text{If } [\mathrm{S'} \rightarrow \mathrm{S}\bullet] \in \mathrm{I_i} \text{ then ACTION[i,\$]} = \text{accept}$$

$$\text{If } \delta(\mathrm{I_i}, \mathrm{A}) = \mathrm{I_j} \text{ then GOTO[i, A]} = \mathrm{j}$$

(Now do you see why I prefer to use δ rather than GOTO()?)

ACTION and GOTO for SLR(1)

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

From Compilers by Aho, Lam, Sethi, Ullman

Example parse

	STACK	SYMBOLS	INPUT	ACTION
(1)	0		$\mathbf{id} * \mathbf{id} + \mathbf{id}\,\$$	shift
(2)	0 5	$\mathbf{id}$	$* \mathbf{id} + \mathbf{id}\,\$$	reduce by $F \to \mathbf{id}$
(3)	0 3	F	$* \mathbf{id} + \mathbf{id}\,\$$	reduce by $T \to F$
(4)	0 2	T	$* \mathbf{id} + \mathbf{id}\,\$$	shift
(5)	0 2 7	$T *$	$\mathbf{id} + \mathbf{id}\,\$$	shift
(6)	0 2 7 5	$T * \mathbf{id}$	$+ \mathbf{id}\,\$$	reduce by $F \to \mathbf{id}$
(7)	0 2 7 10	$T * F$	$+ \mathbf{id}\,\$$	reduce by $T \to T * F$
(8)	0 2	T	$+ \mathbf{id}\,\$$	reduce by $E \to T$
(9)	0 1	E	$+ \mathbf{id}\,\$$	shift
(10)	0 1 6	$E +$	$\mathbf{id}\,\$$	shift
(11)	0 1 6 5	$E + \mathbf{id}$	$\$$	reduce by $F \to \mathbf{id}$
(12)	0 1 6 3	$E + F$	$\$$	reduce by $T \to F$
(13)	0 1 6 9	$E + T$	$\$$	reduce by $E \to E + T$
(14)	0 1	E	$\$$	accept

From Compilers by Aho, Lam, Sethi, Ullman

Beyond SLR(1)?

$$G_3 = (N_3, T_3, P_3, S')$$

$$N_3 = \{\mathrm{S'}, \mathrm{S}, \mathrm{L}, \mathrm{R}\}$$

$$T_3 = \{*, =, \mathrm{id}\}$$

$$P_3 : \mathrm{S'} \rightarrow \mathrm{S\$}$$

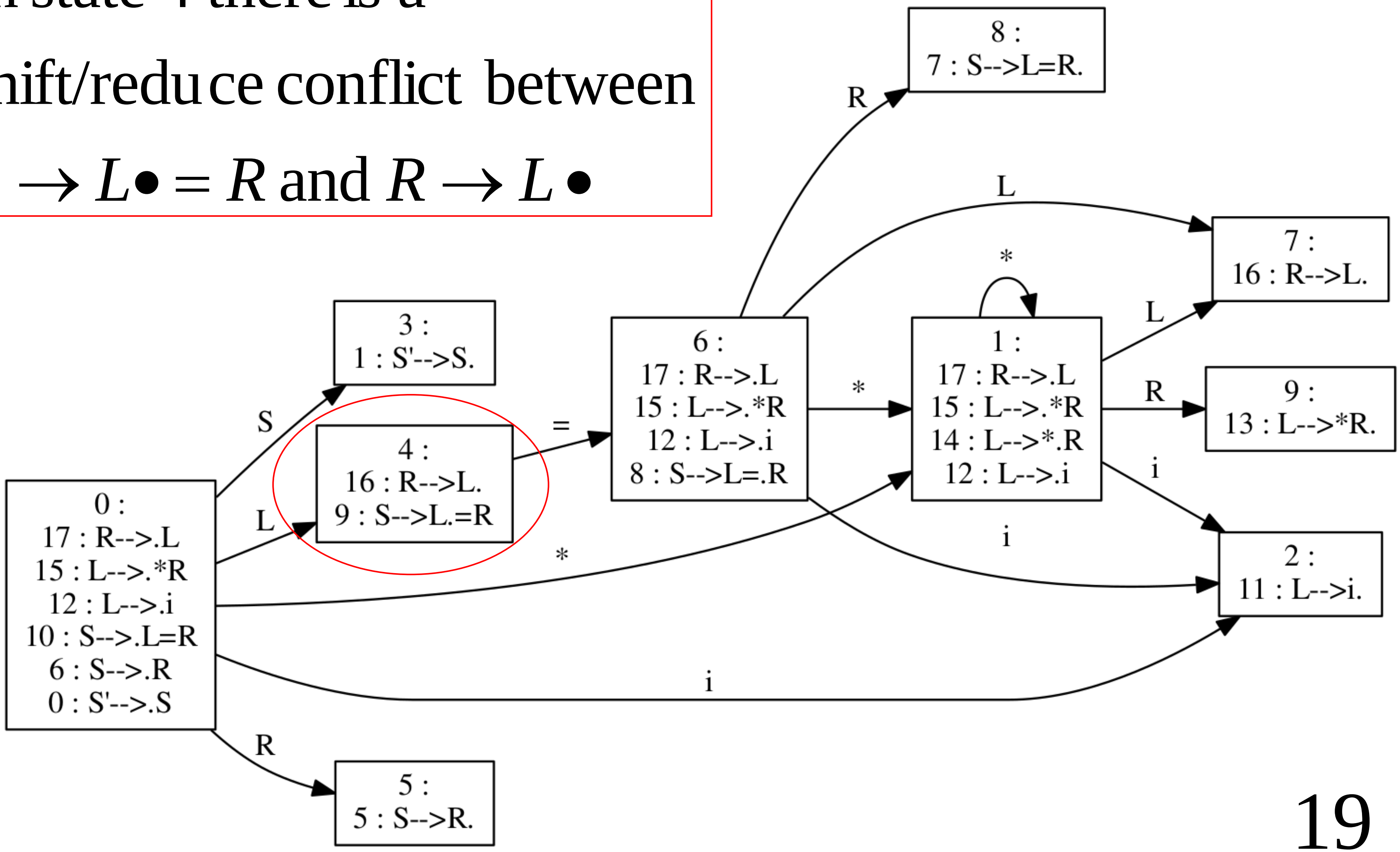
$$\mathrm{S} \rightarrow \mathrm{L} = \mathrm{R} \mid \mathrm{R}$$

$$\mathrm{L} \rightarrow *\mathrm{R} \mid \mathrm{id}$$

$$\mathrm{R} \rightarrow \mathrm{L}$$

LR(0) DFA for grammar G_3

In state 4 there is a shift/reduce conflict between $S \rightarrow L\bullet = R$ and $R \rightarrow L\bullet$



SLR(1) cannot resolve this conflict.

$$[\mathrm{S} \to L\bullet = R] \in \mathrm{I}_4 \text{ so } \delta(\mathrm{I}_4, "=") = \mathrm{I}_6$$

$$\text{and so ACTION}[4, "="] = \text{shift } 6$$

$$\text{However}, [\mathrm{R} \to L\bullet] \in \mathrm{I}_4$$

$$\text{and } "=" \in \mathrm{FOLLOW(R)} = \{"=", \$\},$$

$$\text{so ACTION}[4, "="] = \text{reduce R} \to L$$

Beyond SLR(1)? LR(1)!

Problems: with SLR(1) there may be shift-reduce or reduce-reduce conflicts when ACTION and GOTO are not uniquely defined.

Either fix the grammar or use a more powerful technique.

LR(1) parsing starts with items of the form

$$[A \rightarrow \alpha \bullet \beta, \text{a}]$$

where a is an explicit look-ahead token.

Define an NFA with $LR(1)$ items as states

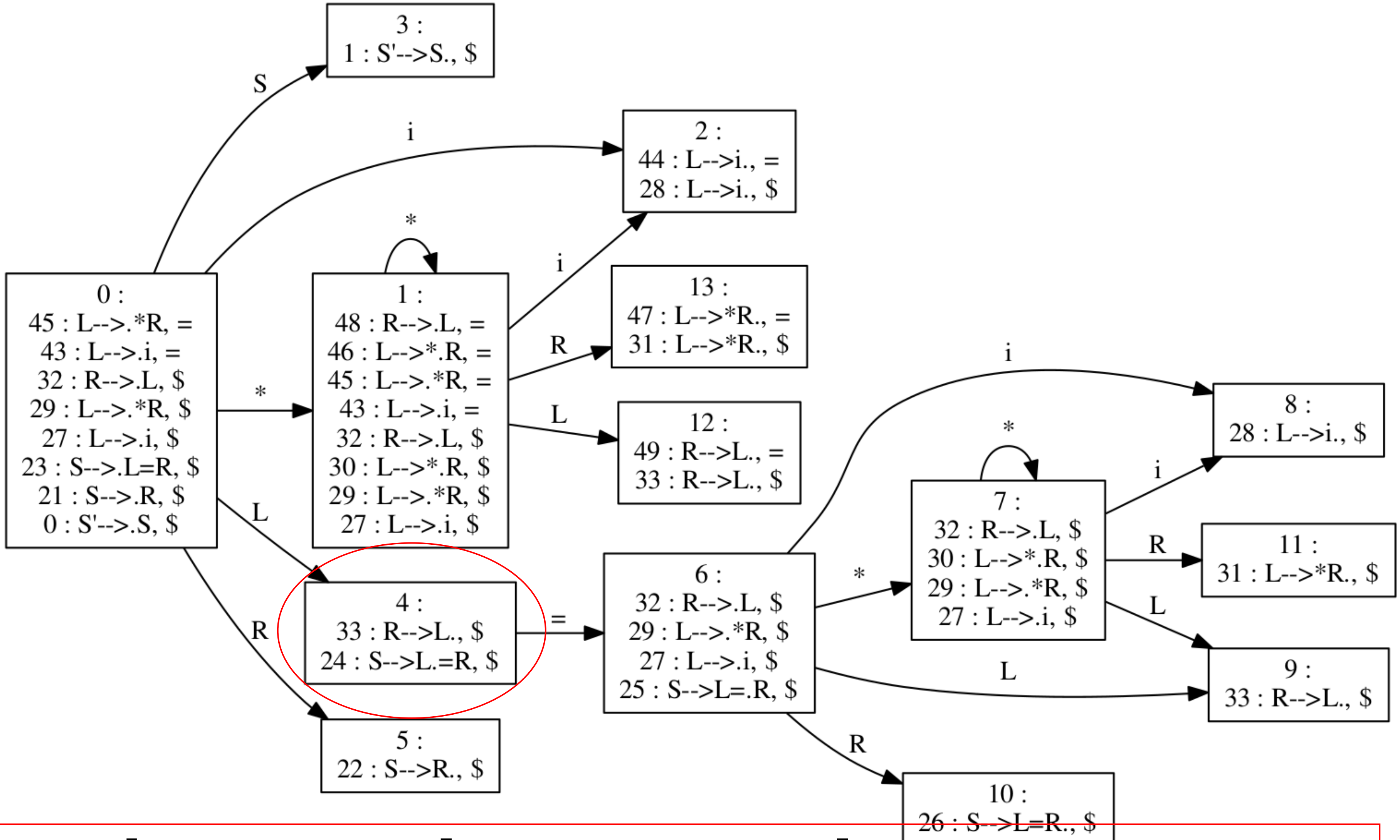
$$\boxed{\mathrm{A} \to \alpha \bullet c\beta,\ a} \xrightarrow{c} \boxed{\mathrm{A} \to \alpha c \bullet \beta,\ a}$$

$$\boxed{\mathrm{A} \to \alpha \bullet B\beta,\ a} \xrightarrow{B} \boxed{\mathrm{A} \to \alpha B \bullet \beta,\ a}$$

For each $b \in \mathrm{FIRST}(\beta a)$:

$$\boxed{\mathrm{A} \to \alpha \bullet B\beta,\ a} \xrightarrow{\varepsilon} \boxed{\mathrm{B} \to \bullet\gamma,\ b}$$

LR(1) DFA for grammar G_3



No ambiguity. Reduce $R \to L$ only if next token is \$. Otherwise shift if next token is $=$.

ACTION and GOTO for LR(1)

If $[\mathrm{A} \rightarrow \alpha \bullet a\beta, a] \in \mathrm{I}_{\mathrm{i}}$ and $\delta(\mathrm{I}_{\mathrm{i}}, a) = \mathrm{I}_{\mathrm{j}}$ then $\mathrm{ACTION}[\mathrm{i}, \mathrm{a}] = \text{shift j}$

If $[\mathrm{A} \rightarrow \alpha \bullet, b] \in \mathrm{I}_{\mathrm{i}}$ and $\mathrm{A} \neq \mathrm{S}'$, then

$\mathrm{ACTION}[\mathrm{i}, \mathrm{b}] = \text{reduce } \mathrm{A} \rightarrow \alpha$

If $[\mathrm{S}' \rightarrow \mathrm{S}\bullet, \$] \in \mathrm{I}_{\mathrm{i}}$ then $\mathrm{ACTION}[\mathrm{i}, \$] = \text{accept}$

If $\delta(\mathrm{I}_{\mathrm{i}}, \mathrm{A}) = \mathrm{I}_{\mathrm{j}}$ then $\mathrm{GOTO}[\mathrm{i}, \mathrm{A}] = \mathrm{j}$

SLR(1) vs LR(1)

$\text{SLR}(1):$

$\text{If } [A \to \alpha\bullet] \in I_i \text{ and } A \neq S'$

$\text{then for all } a \in \text{FOLLOW}(A),$

$\qquad \text{ACTION}[i, a] = \text{reduce } A \to \alpha$

$\text{LR}(1):$

$\text{If } [A \to \alpha\bullet, b] \in I_i \text{ and } A \neq S', \text{ then}$

$\qquad \text{ACTION}[i, b] = \text{reduce } A \to \alpha$

Note that the look-ahead symbol b is used ONLY for reductions, not for shifts.

SLR(1) vs LR(1)

1. **LR(1) is more powerful than SLR(1)**
2. **The DFA associated with a LR(1) parser may have a very large number of states**
3. **This inspired an optimisation (collapsing states) resulting in a the class of LALR papers normally implemented as YACC. These parsers have fewer states but can produce very strange error messages.**
4. **Ocaml’s Menhir is based on LR(1) and claims to overcome many YACC problems.**
5. **We will not cover LALR parsing.**

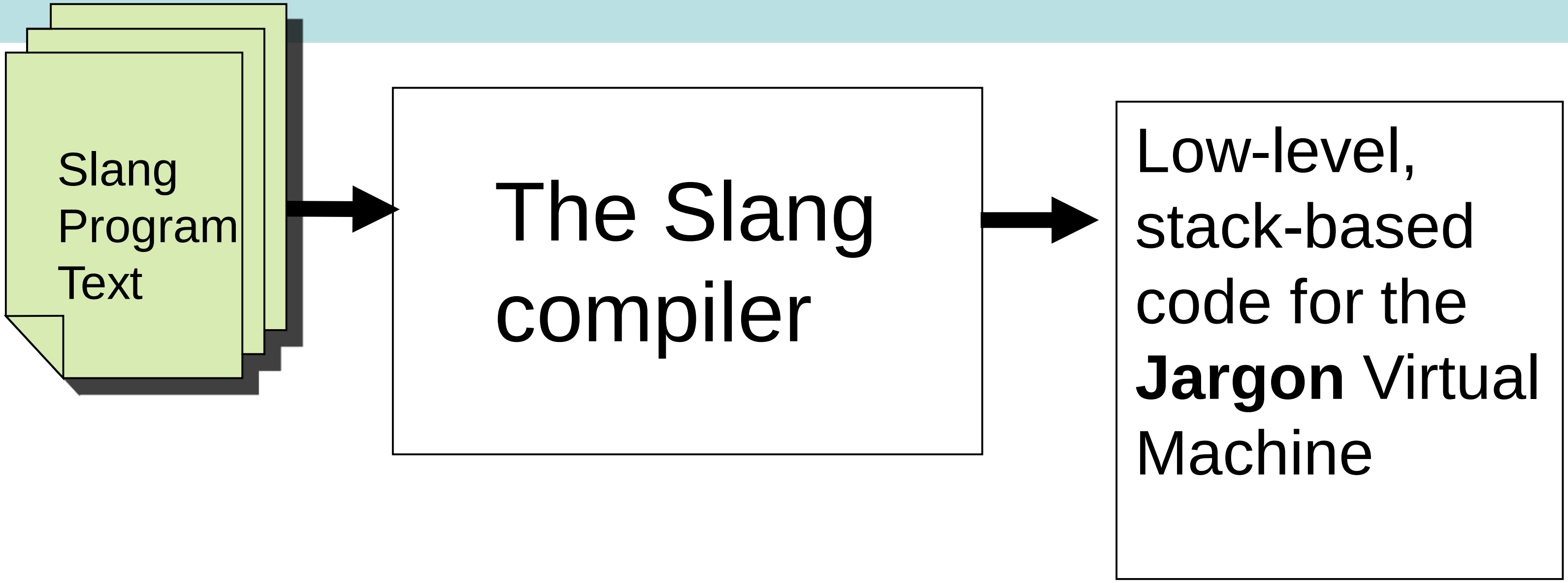
LECTURE 7
Slang front end and interpreter 0

- **Slang (= <u>S</u>imple <u>LANG</u>uage)**
  - **A subset of L3 from Semantics …**
  - **… with <u>very</u> ugly concrete syntax**
  - **You are invited to experiment with improvements to this concrete syntax.**
- **Slang : concrete syntax, types**
- **Abstract Syntax Trees (ASTs)**
- **The Front End**
- **Interpreter 0 : The high-level “definitional” interpreter**
  1. **Slang/L3 values represented directly as OCaml values**
  2. **Recursive interpreter implements a denotational semantics**
  3. **The interpreter implicitly uses OCaml’s runtime stack and heap**

The Slang compiler

- The compiler is available from the course web site.
- It is written in Ocaml
- Slang = **S**imple **Lang**uage. Based on L3 from Semantics of Programming Languages, Part 1B.
- The best way to learn about compilers is to modify one.
- There are several suggested improvements listed on the course web site. I hope that some of you will implement these. If they work, I'll let you commit your changes to the repository. Fame! Fortune!

Bridging the Gap?



Question : How do we leap from the mathematical semantics of L3 to a low-level stack machine?

Answer : We will start with a high-level interpreter based on semantics, and then **derive** the stack machine by a sequence of semantics preserving transformations!

Lectures 7 – 11 : the derivation

Note : this is **not** the traditional way of teaching compilers! Many textbooks will start with a stack machine and bridge the gap informally. We will develop a deeper understanding!

Interpreter 0

Explicit stack via CPS+DFS → Interpreter 1

Split stack into two, refactor → Interpreter 2

Linearise code → Interpreter 3

Low-level addressable stack → Jargon VM

Clunky Slang Syntax (informal)

uop := - | ~

(~ is boolean negation)

bop ::= + | - | * | < | = | && | ||

t ::= bool | int | unit | (t) | t * t | t + t | t -> t | t ref

e ::= () | n | true | false | x | (e) | ? |
e bop e | uop e |
if e then else e end |
e e | fun (x : t) -> e end |
let x : t = e in e end |
let f(x : t) : t = e in e end |
!e | ref e | e := e | while e do e end |
begin e; e; … e end |
(e, e) | snd e | fst e |
inl t e | inr t e |
case e of inl(x : t) -> e **|** inr(x:t) -> e end

(? requests an integer input from terminal)

(notice type annotation on inl and inr constructs)

From slang/examples

```
let fib( m : int) : int =
    if m = 0
    then 1
    else if m = 1
            then 1
             else fib (m - 1) +
                     fib (m -2)
              end
     end
in
    fib(?)
end
```

```
let gcd( p : int * int) : int =
    let m : int = fst p
    in let  n : int = snd p
    in  if m = n
          then m
          else if m < n
                 then gcd(m, n - m)
                 else  gcd(m - n, n)
                 end
           end
         end
     end
in gcd(?, ?) end
```

The ? requests an integer input from the terminal

Slang Front End

Input file foo.slang

Parse (we use Ocaml versions of LEX and YACC, covered in Lectures 3 --- 6)

Parsed AST (Past.expr)

Static analysis : check types, and context-sensitive rules, resolve overloaded operators

Parsed AST (Past.expr)

Remove “syntactic sugar”, file location information, and most type information

Intermediate AST (Ast.expr)

Parsed AST (past.ml)

```
type var = string

type loc = Lexing.position

type type_expr =
   | TEint
   | TEbool
   | TEunit
   | TEref of type_expr
   | TEarrow of type_expr * type_expr
   | TEproduct of type_expr * type_expr
   | TEunion of type_expr * type_expr

type oper = ADD | MUL | SUB | LT |
            AND | OR | EQ | EQB | EQI

type unary_oper = NEG | NOT
```

Locations (loc) are used in generating error messages.

```
type expr =
      | Unit of loc
      | What of loc
      | Var of loc * var
      | Integer of loc * int
      | Boolean of loc * bool
      | UnaryOp of loc * unary_oper * expr
      | Op of loc * expr * oper * expr
      | If of loc * expr * expr * expr
      | Pair of loc * expr * expr
      | Fst of loc * expr
      | Snd of loc * expr
      | Inl of loc * type_expr * expr
      | Inr of loc * type_expr * expr
      | Case of loc * expr * lambda * lambda
      | While of loc * expr * expr
      | Seq of loc * (expr list)
      | Ref of loc * expr
      | Deref of loc * expr
      | Assign of loc * expr * expr
      | Lambda of loc * lambda
      | App of loc * expr * expr
      | Let of loc * var * type_expr * expr * expr
      | LetFun of loc * var * lambda
                        * type_expr * expr
      | LetRecFun of loc * var * lambda
                         * type_expr * expr
```

static.mli, static.ml

```
val infer : (Past.var * Past.type_expr) list
                                -> (Past.expr * Past.type_expr)

val check : Past.expr -> Past.expr   (* infer on empty environment *)
```

- Check type correctness
- Rewrite expressions to resolve EQ to EQI (for integers) or EQB (for bools).
- Only LetFun is returned by parser. Rewrite to LetRecFun when function is actually recursive.

Lesson : while enforcing “context-sensitive rules” we can resolve ambiguities that cannot be specified in context-free grammars.

Internal AST (ast.ml)

```
type var = string

type oper = ADD | MUL | SUB | LT |
            AND | OR | EQB | EQI

type unary_oper = NEG | NOT | READ
```

No locations, types.
No Let, EQ.

Is getting rid of types a bad idea? Perhaps a full answer would be language-dependent…

```
type expr =
      | Unit
      | Var of var
      | Integer of int
      | Boolean of bool
      | UnaryOp of unary_oper * expr
      | Op of expr * oper * expr
      | If of expr * expr * expr
      | Pair of expr * expr
      | Fst of expr
      | Snd of expr
      | Inl of expr
      | Inr of expr
      | Case of expr * lambda * lambda
      | While of expr * expr
      | Seq of (expr list)
      | Ref of expr
      | Deref of expr
      | Assign of expr * expr
      | Lambda of lambda
      | App of expr * expr
      | LetFun of var * lambda * expr
      | LetRecFun of var * lambda * expr

and lambda = var * expr
```

past_to_ast.ml

val translate_expr : Past.expr -> Ast.expr

```
let x : t  = e1 in e2 end
```

```
(fun (x: t) -> e2 end) e1
```

This is done to simplify some of our code.
Is it a good idea?   Perhaps not!
See 2021 paper 4 question 3.

Approaches to Mathematical Semantics

- Axiomatic: Meaning defined through logical specifications of behaviour.
  - Hoare Logic (Part II)
  - Separation Logic
- Operational: Meaning defined in terms of transition relations on states in an abstract machine.
  - Semantics (Part 1B)
- Denotational: Meaning is defined in terms of mathematical objects such as functions.
  - Denotational Semantics (Part II)

A denotational semantics for L3?

$\mathbf{N}$ = set of integers $\mathbf{B}$ = set of booleans $\mathbf{A}$ = set of addresses

$\mathbf{I}$ = set of identifiers Expr = set of L3 expressions

$\mathbf{E}$ = set of environments = $\mathbf{I} \rightarrow \mathbf{V}$ $\mathbf{S}$ = set of stores = $\mathbf{A} \rightarrow \mathbf{V}$

$$
\left.
\begin{aligned}
\mathbf{V} &= \text{set of value} \\
&\approx \mathbf{A} \\
&\quad + \mathbf{N} \\
&\quad + \mathbf{B} \\
&\quad + \{\, () \,\} \\
&\quad + \mathbf{V} \times \mathbf{V} \\
&\quad + (\mathbf{V} + \mathbf{V}) \\
&\quad + (\mathbf{V} \times \mathbf{S}) \rightarrow (\mathbf{V} \times \mathbf{S})
\end{aligned}
\right\}
$$

Set of values $\mathbf{V}$ solves this "domain equation" (here + means disjoint union).

Solving such equations is where some difficult maths is required …

$\mathbf{M}$ = the meaning function

$$\mathbf{M} : (\text{Expr} \times \mathbf{E} \times \mathbf{S}) \rightarrow (\mathbf{V} \times \mathbf{S})$$

Interpreter 0 : An OCaml approximation

$\mathbf{A}$ = set of addresses

$\mathbf{S}$ = set of stores = $\mathbf{A} \rightarrow \mathbf{V}$

$\mathbf{V}$ = set of value

$\approx \mathbf{A}$
$+ \mathbf{N}$
$+ \mathbf{B}$
$+ \{\ ()\ \}$
$+ \mathbf{V} \times \mathbf{V}$
$+ (\mathbf{V} + \mathbf{V})$
$+ (\mathbf{V} \times \mathbf{S}) \rightarrow (\mathbf{V} \times \mathbf{S})$

$\mathbf{E}$ = set of environments = $\mathbf{A} \rightarrow \mathbf{V}$

$\mathbf{M}$ = the meaning function

$\mathbf{M}$: $(\mathrm{Expr} \times \mathbf{E} \times \mathbf{S}) \rightarrow (\mathbf{V} \times \mathbf{S})$

```
type address

type store = address -> value

and value =
     | REF of address
     | INT of int
     | BOOL of bool
     | UNIT
     | PAIR of value * value
     | INL of value
     | INR of value
     | FUN of ((value * store)
         -> (value * store))

type env = Ast.var -> value

val interpret :
       Ast.expr * env * store
         -> (value * store)
```

Most of the code is obvious!

```
let rec interpret (e, env, store) =
    match e with
    | If(e1, e2, e3) ->
       let (v, store') = interpret(e1, env, store) in
            (match v with
            | BOOL true -> interpret(e2, env, store')
            | BOOL false -> interpret(e3, env, store')
            | v -> complain "runtime error.  Expecting a boolean!")
    | Pair(e1, e2)  ->
       let (v1, store1) = interpret(e1, env, store) in
       let (v2, store2) = interpret(e2, env, store1) in (PAIR(v1, v2), store2)
    | Fst e ->
        (match interpret(e, env, store) with
        | (PAIR (v1, _), store') -> (v1, store')
        | (v, _) -> complain "runtime error.  Expecting a pair!")
    | Snd e  ->
       (match interpret(e, env, store) with
        | (PAIR (_, v2), store') -> (v2, store')
        | (v, _) -> complain "runtime error.  Expecting a pair!")
    | Inl e   -> let (v, store') = interpret(e, env, store) in (INL v, store')
    | Inr e  -> let (v, store') = interpret(e, env, store) in (INR v, store')
    :
    .
```

Tricky bits : Slang functions mapped to OCaml functions!

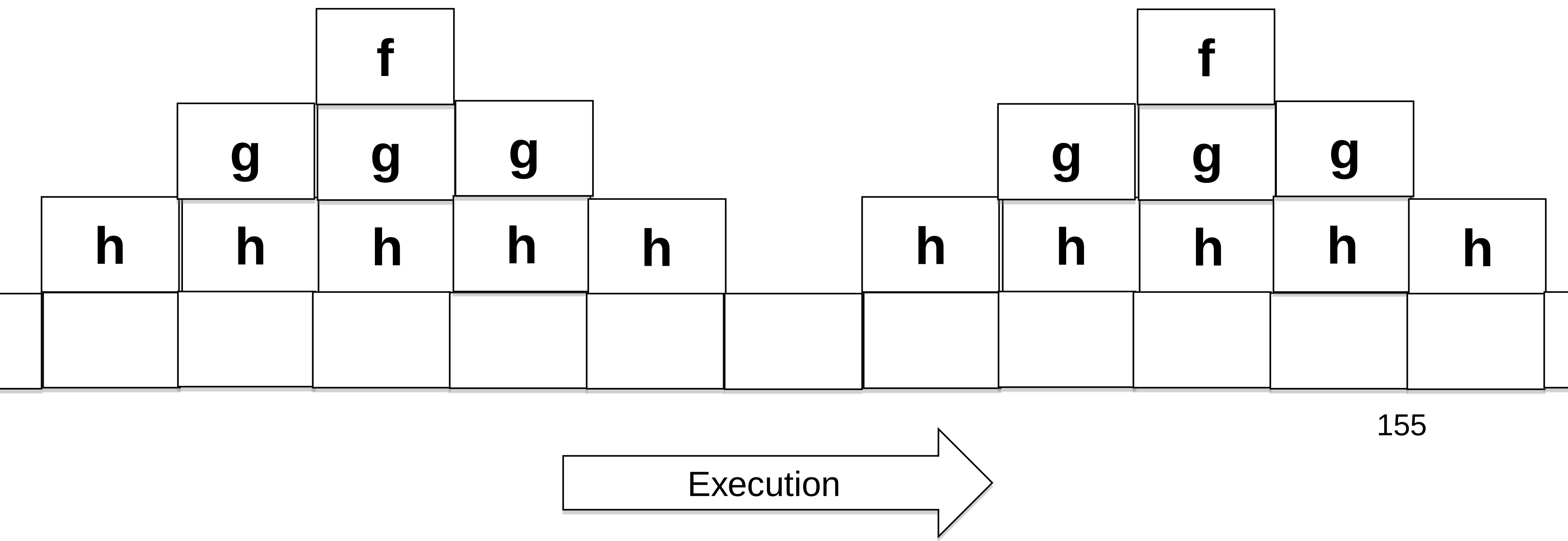
```
let rec interpret (e, env, store) =
    match e with
    :
    :
    | Lambda(x, e)  -> (FUN (fun (v, s) -> interpret(e, update(env, (x, v)), s)), store)
    | App(e1, e2) -> (* I chose to evaluate argument first!  *)
      let (v2, store1) = interpret(e2, env, store) in
      let (v1, store2) =  interpret(e1, env, store1) in
          (match v1 with
          | FUN f -> f (v2, store2)
          | v -> complain "runtime error.  Expecting a function!")
    | LetFun(f, (x, body), e) ->
       let new_env =
           update(env, (f, FUN (fun (v, s) -> interpret(body, update(env, (x, v)), s))))
       in interpret(e, new_env, store)
    | LetRecFun(f, (x, body), e) ->
       let rec new_env g = (* a recursive environment!!! *)
          if g = f then FUN (fun (v, s) -> interpret(body, update(new_env, (x, v)), s))
                   else env g
       in interpret(e, new_env, store)
```

update : env * (var * value) -> env

Interpreter 0 is using OCaml's runtime stack. How can we move toward the Jargon VM?

```
let fun f (x) = x + 1
    fun g(y) = f(y+2)+2
    fun h(w) = g(w+1)+3
in
   h(h(17))
end
```

The run-time data structure is the <u>call stack</u> containing an <u>activation record</u> for each function invocation.



Recall tail recursion : fold_left vs fold_right

From ocaml-4.01.0/stdlib/list.ml :

```
(* fold_left :   ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a

     fold_left f a [b1; ...; bn]]  = f (... (f (f a b1) b2) ...) bn
*)
let rec fold_left f a l =
  match l with
  | []           -> a
  | b :: rest -> fold_left f (f a b) rest

(* fold_right : ('a -> 'b -> 'b) -> 'a list -> 'b -> 'b

    fold_right f [a1; ...; an] b = f a1 (f a2 (... (f an b) ...))
 *)
let rec fold_right f l b =
  match l with
  | []          -> b
  | a::rest -> f a (fold_right f rest b)
```

This is tail recursive

This is NOT tail recursive

Convert tail-recursion to iteration

```
(* gcd : int * int -> int *)
let rec gcd(m, n) =
   if m = n
   then m
   else if m < n
      then gcd(m,     n - m)
      else  gcd(m - n,      n)
```

Here we have illustrated tail-recursion elimination as a source-to-source transformation. However, the OCaml compiler will do something similar to a lower-level intermediate representation. Upshot : we will consider all tail-recursive OCaml functions as representing <u>iterative</u> programs.

```
(* gcd_iter : int * int -> int *)
let gcd_iter (m, n) =
   let rm = ref m
   in let rn = ref n
   in let result = ref 0
   in let not_done = ref true
   in let _ =
       while !not_done
        do
            if !rm = !rn
            then (not_done := false;
                      result := !rm)
            else if !rm < !rn
                 then rn := !rn - !rm
                 else rm := !rm - !rn
         done
   in !result
```

Question: can we transform any recursive function (such as interpreter 0) into a tail recursive function?

The answer is YES!

- **We add an extra argument, called a *continuation*, that represents "the rest of the computation"**
- **This is called the Continuation Passing Style (CPS) transformation.**
- **We will then "defunctionalize" (DFC) these continuations and represent them with a stack.**
- **Finally, we obtain a tail recursive function that carries its own stack as an extra argument!**

We will apply this kind of transformation to the code of interpreter 0 as the first steps towards deriving interpreter 1.

LECTURES 8 & 9
Derivation of Interpreters 1 & 2

- **Continuation Passing Style (CPS) : transform any recursive function to a tail-recursive function**
- **“Defunctionalisation” (DFC) : replace higher-order functions with a data structure**
- **Putting it all together:**
  - **Derive the Fibonacci Machine**
  - **Derive the Expression Machine, and “compiler”!**
- **This provides a roadmap for the interp_0 → interp_1 → interp_2 derivations.**

(CPS) transformation of fib

```
(* fib : int -> int *)
let rec fib m =
    if m = 0
    then 1
    else if m = 1
             then 1
             else fib(m - 1) + fib (m - 2)

(* fib_cps : int * (int -> int)  -> int *)
 let rec fib_cps (m,  cnt) =
    if m = 0
    then cnt 1
    else if m = 1
            then cnt 1
            else fib_cps(m -1,
                          fun a -> fib_cps(m - 2 ,
                                            fun b  -> cnt (a + b)))
```

A closer look

The rest of the computation after computing “fib(m)”. That is, cnt is a function expecting the result of “fib(m)” as its argument.

```
let rec fib_cps (m,  cnt) =
    if m = 0
    then cnt 1
    else if m = 1
          then cnt 1
          else fib_cps(m -1,  fun a -> fib_cps(m - 2 , fun b  -> cnt (a + b)))
```

The computation waiting for the result of “fib(m-1)”

The computation waiting for the result of “fib(m-2)”

This makes explicit the order of evaluation that is implicit in the original “fib(m-1) + fib(m-2)” :
-- first compute fib(m-1)
-- then compute fib(m-2)
-- then add results together
-- then return

Expressed with “let” rather than “fun”

```
(* fib_cps_v2 : (int -> int) * int -> int *)
let rec fib_cps_v2 (m, cnt) =
    if m = 0
    then cnt 1
    else if m = 1
          then cnt 1
          else let cnt2 a b = cnt (a + b)
                in let cnt1 a = fib_cps_v2(m - 2, cnt2 a)
                in fib_cps_v2(m - 1, cnt1)
```

Some prefer writing CPS forms without explicit funs ….

Use the identity continuation …

```
(* fib_cps : int * (int -> int)  -> int *)
 let rec fib_cps (m, cnt) =
    if m = 0
    then cnt 1
    else if m = 1
           then cnt 1
           else fib_cps(m -1,  fun a -> fib_cps(m - 2 , fun b  -> cnt (a + b)))



let id (x : int) = x

let fib_1 x = fib_cps(x, id)
```

```
List.map fib_1 [0; 1; 2; 3; 4; 5; 6; 7; 8; 9; 10];;

   =  [1; 1; 2; 3; 5; 8; 13; 21; 34; 55; 89]
```

Correctness?

For all c : int -> int, for all m, 0 <= m,
we have,   c(fib m) = fib_cps(m, c).

NB: This proof pretends that we can treat OCaml functions as ideal mathematical functions, which of course we cannot. OCaml functions might raise exceptions like "stack overflow" or "you burned my toast", and so on.   But this is a convenient fiction as long as we remember to be careful.

Proof: assume c : int -> int. By Induction on m. Base case : m = 0:

fib_cps(0, c) = c(1) = c(fib(0).

Induction step: Assume for all n < m, c(fib n) = fib_cps(n, c).
(That is, we need course-of-values induction!)

```
fib_cps(m + 1, c)
 = if m + 1 = 1
   then c 1
   else fib_cps((m+1) -1, fun a -> fib_cps((m+1) -2, fun b -> c (a + b)))
 = if m + 1 = 1
   then c 1
   else fib_cps(m, fun a -> fib_cps(m-1, fun b -> c (a + b)))
 = (by induction)
   if m + 1 = 1
   then c 1
   else (fun a -> fib_cps(m -1, fun b -> c (a + b))) (fib m)
```

Correctness?

```
= if m + 1 = 1
  then c 1
  else fib_cps(m-1, fun b -> c ((fib m) + b))
= (by induction)
  if m + 1 = 1
  then c 1
  else (fun b -> c ((fib m) + b)) (fib (m-1))
 = if m + 1 = 1
  then c 1
  else c ((fib m) + (fib (m-1)))
 = c (if m + 1 = 1
     then 1
     else ((fib m) + (fib (m-1))))
 = c(if m +1 = 1
     then 1
     else fib((m + 1) - 1) + fib ((m + 1) - 2))
 = c (fib(m + 1))
```


QED.

Can with express fib_cps without a functional argument ?

```
(* fib_cps_v2 : (int -> int) * int -> int *)
let rec fib_cps_v2 (m, cnt) =
   if m = 0
   then cnt 1
   else if m = 1
         then cnt 1
         else let cnt2 a b = cnt (a + b)
               in let cnt1 a =
                     fib_cps_v2(m - 2, cnt2 a)
               in fib_cps_v2(m - 1, cnt1)
```

Idea of "defunctonalisation" (DFC): replace id, cnt1 and cnt2 with instances of a new data type:

```
type cnt = ID | CNT1 of int * cnt | CNT2 of int * cnt
```

Now we need an "apply" function of type   cnt * int -> int

“Defunctionalised” version of fib_cps

```
(* datatype to represent continuations *)
type cnt = ID | CNT1 of int * cnt | CNT2 of int * cnt

(* apply_cnt : cnt * int -> int *)
let rec apply_cnt = function
  | (ID, a)                   -> a
  | (CNT1 (m, cnt), a) -> fib_cps_dfc(m - 2, CNT2 (a, cnt))
  | (CNT2 (a, cnt), b)  -> apply_cnt (cnt, a + b)

(*  fib_cps_dfc : (cnt * int) -> int *)
and fib_cps_dfc (m, cnt) =
    if m = 0
    then apply_cnt(cnt, 1)
    else if m = 1
           then apply_cnt(cnt, 1)
           else fib_cps_dfc(m -1, CNT1(m, cnt))

(*  fib_2 : int -> int *)
let fib_2 m = fib_cps_dfc(m, ID)
```

Correctness?

Let < c > be of type cnt representing
a continuation c : int -> int constructed by fib_cps.

Then
  apply_cnt(< c >, m) = c(m)
and
  fib_cps(n, c) = fib_cps_dfc(n, < c >).

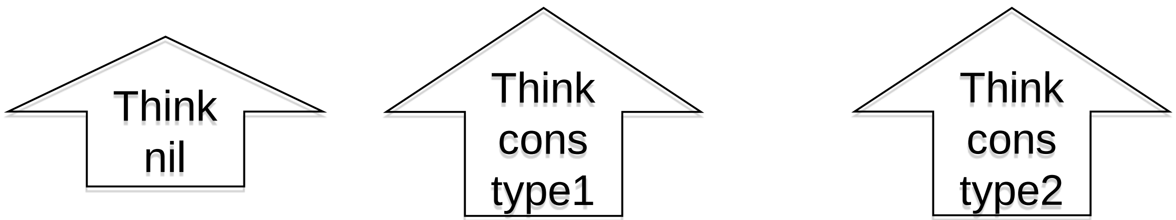
Proof left as an exercise!

Functional continuation c	Representation < c >
fun a -> fib_cps(m - 2 , fun b -> cnt (a + b))	CNT1(m, < cnt >)
fun b -> cnt (a + b)	CNT2(a, < cnt >)
fun x -> x	ID

Eureka! Continuations are just lists (used like a stack)

```
type int_list = NIL | CONS of int * int_list
```

```
type cnt = ID | CNT1 of int * cnt | CNT2 of int * cnt
```



Replace the above continuations with lists! (I’ve selected more suggestive names for the constructors.)

```
type tag = SUB2 of int | PLUS of int

type tag_list_cnt = tag list
```

The continuation lists are used like a stack!

```
type tag = SUB2 of int | PLUS of int
type tag_list_cnt = tag list

(* apply_tag_list_cnt : tag_list_cnt * int -> int *)
let rec apply_tag_list_cnt = function
  | ([], a)                         -> a
  | ((SUB2 m) :: cnt, a) -> fib_cps_dfc_tags(m - 2, (PLUS a):: cnt)
  | ((PLUS a) :: cnt, b)  -> apply_tag_list_cnt (cnt, a + b)

(* fib_cps_dfc_tags : (tag_list_cnt * int) -> int *)
and fib_cps_dfc_tags (m, cnt) =
    if m = 0
    then apply_tag_list_cnt(cnt, 1)
    else if m = 1
          then apply_tag_list_cnt(cnt, 1)
          else fib_cps_dfc_tags(m - 1, (SUB2 m) :: cnt)

(*  fib_3 : int -> int *)
let fib_3 m = fib_cps_dfc_tags(m, [])
```

Combine Mutually tail-recursive functions into a single function

```
type state_type =
  | SUB1 (* for right-hand-sides starting with fib_   *)
  | APPL  (* for right-hand-sides starting with apply_ *)

type state = (state_type * int * tag_list_cnt) -> int

(* eval : state -> int             A two-state transition function*)
let rec eval = function
  | (SUB1, 0,                    cnt) -> eval (APPL, 1,                        cnt)
  | (SUB1, 1,                    cnt) -> eval (APPL, 1,                        cnt)
  | (SUB1, m,                    cnt) -> eval (SUB1, (m-1), (SUB2 m) :: cnt)
  | (APPL, a, (SUB2 m) :: cnt) -> eval (SUB1, (m-2), (PLUS a) :: cnt)
  | (APPL, b,  (PLUS a) :: cnt) -> eval (APPL, (a+b),                    cnt)
  | (APPL, a,                     [])  -> a

(*  fib_4 : int -> int *)
let fib_4 m = eval (SUB1, m, [])
```

Eliminate tail recursion to obtain The Fibonacci Machine!

```
(* step : state -> state *)
let step = function
  | (SUB1, 0,                 cnt) -> (APPL, 1,                    cnt)
  | (SUB1, 1,                 cnt) -> (APPL, 1,                    cnt)
  | (SUB1, m,                 cnt) -> (SUB1, (m-1), (SUB2 m) :: cnt)
  | (APPL, a, (SUB2 m) :: cnt) -> (SUB1, (m-2),  (PLUS a) :: cnt)
  | (APPL, b,  (PLUS a) :: cnt) -> (APPL, (a+b),                  cnt)
  | _ -> failwith "step : runtime error!”

(* clearly TAIL RECURSIVE! *)
let rec driver state = function
    | (APPL, a, []) -> a
    |  state           -> driver (step state)

(*  fib_5 : int -> int *)
let fib_5 m = driver  (SUB1, m, [])
```

In this version we have simply made the tail-recursive structure very explicit.

Here is a trace of fib_5 6.

```
 1 SUB1 || 6 || []
 2 SUB1 || 5 || [SUB2 6]
 3 SUB1 || 4 || [SUB2 6, SUB2 5]
 4 SUB1 || 3 || [SUB2 6, SUB2 5, SUB2 4]
 5 SUB1 || 2 || [SUB2 6, SUB2 5, SUB2 4, SUB2 3]
 6 SUB1 || 1 || [SUB2 6, SUB2 5, SUB2 4, SUB2 3, SUB2 2]
 7 APPL || 1 || [SUB2 6, SUB2 5, SUB2 4, SUB2 3, SUB2 2]
 8 SUB1 || 0 || [SUB2 6, SUB2 5, SUB2 4, SUB2 3, PLUS 1]
 9 APPL || 1 || [SUB2 6, SUB2 5, SUB2 4, SUB2 3, PLUS 1]
10 APPL || 2 || [SUB2 6, SUB2 5, SUB2 4, SUB2 3]
11 SUB1 || 1 || [SUB2 6, SUB2 5, SUB2 4, PLUS 2]
12 APPL || 1 || [SUB2 6, SUB2 5, SUB2 4, PLUS 2]
13 APPL || 3 || [SUB2 6, SUB2 5, SUB2 4]
14 SUB1 || 2 || [SUB2 6, SUB2 5, PLUS 3]
15 SUB1 || 1 || [SUB2 6, SUB2 5, PLUS 3, SUB2 2]
16 APPL || 1 || [SUB2 6, SUB2 5, PLUS 3, SUB2 2]
17 SUB1 || 0 || [SUB2 6, SUB2 5, PLUS 3, PLUS 1]
18 APPL || 1 || [SUB2 6, SUB2 5, PLUS 3, PLUS 1]
19 APPL || 2 || [SUB2 6, SUB2 5, PLUS 3]
20 APPL || 5 || [SUB2 6, SUB2 5]
21 SUB1 || 3 || [SUB2 6, PLUS 5]
22 SUB1 || 2 || [SUB2 6, PLUS 5, SUB2 3]
23 SUB1 || 1 || [SUB2 6, PLUS 5, SUB2 3, SUB2 2]
24 APPL || 1 || [SUB2 6, PLUS 5, SUB2 3, SUB2 2]
25 SUB1 || 0 || [SUB2 6, PLUS 5, SUB2 3, PLUS 1]
26 APPL || 1 || [SUB2 6, PLUS 5, SUB2 3, PLUS 1]
27 APPL || 2 || [SUB2 6, PLUS 5, SUB2 3]
28 SUB1 || 1 || [SUB2 6, PLUS 5, PLUS 2]
29 APPL || 1 || [SUB2 6, PLUS 5, PLUS 2]
30 APPL || 3 || [SUB2 6, PLUS 5]
31 APPL || 8 || [SUB2 6]
32 SUB1 || 4 || [PLUS 8]
33 SUB1 || 3 || [PLUS 8, SUB2 4]
34 SUB1 || 2 || [PLUS 8, SUB2 4, SUB2 3]
35 SUB1 || 1 || [PLUS 8, SUB2 4, SUB2 3, SUB2 2]
36 APPL || 1 || [PLUS 8, SUB2 4, SUB2 3, SUB2 2]
37 SUB1 || 0 || [PLUS 8, SUB2 4, SUB2 3, PLUS 1]
38 APPL || 1 || [PLUS 8, SUB2 4, SUB2 3, PLUS 1]
39 APPL || 2 || [PLUS 8, SUB2 4, SUB2 3]
40 SUB1 || 1 || [PLUS 8, SUB2 4, PLUS 2]
41 APPL || 1 || [PLUS 8, SUB2 4, PLUS 2]
42 APPL || 3 || [PLUS 8, SUB2 4]
43 SUB1 || 2 || [PLUS 8, PLUS 3]
44 SUB1 || 1 || [PLUS 8, PLUS 3, SUB2 2]
45 APPL || 1 || [PLUS 8, PLUS 3, SUB2 2]
46 SUB1 || 0 || [PLUS 8, PLUS 3, PLUS 1]
47 APPL || 1 || [PLUS 8, PLUS 3, PLUS 1]
48 APPL || 2 || [PLUS 8, PLUS 3]
49 APPL || 5 || [PLUS 8]
50 APPL ||13|| []
```

The OCaml file in basic_transformations/fibonacci_machine.ml contains some code for pretty printing such traces….

Pause to reflect

- What have we accomplished?
- We have taken a recursive function and turned it into an iterative function that does not require "stack space" for its evaluation (in OCaml)
- However, this function now carries its own evaluation stack as an extra argument!
- We have derived this iterative function in a step-by-step manner where each tiny step is easily proved correct.
- Wow!

That was fun! Let's do it again!

```
type expr =
  | INT of int
  | PLUS of expr * expr
  | SUBT of expr * expr
  | MULT of expr * expr
```

This time we will derive a stack-machine AND a "compiler" that translates expressions into a list of instructions for the machine.

```
(* eval : expr -> int
   a simple recusive evaluator for expressions *)
let rec eval = function
  | INT a           -> a
  | PLUS(e1, e2)    -> (eval e1) + (eval e2)
  | SUBT(e1, e2)    -> (eval e1) - (eval e2)
  | MULT(e1, e2)    -> (eval e1) * (eval e2)
```

Here we go again : CPS

```
type cnt_2  = int -> int

type state_2 = expr * cnt_2

(* eval_aux_2 : state_2 -> int *)
let rec eval_aux_2 (e, cnt) =
   match e with
   | INT a        -> cnt a
   | PLUS(e1, e2) ->
      eval_aux_2(e1, fun v1 -> eval_aux_2(e2, fun v2 -> cnt(v1 + v2)))
   | SUBT(e1, e2) ->
      eval_aux_2(e1, fun v1 -> eval_aux_2(e2, fun v2 -> cnt(v1 - v2)))
   | MULT(e1, e2) ->
      eval_aux_2(e1, fun v1 -> eval_aux_2(e2, fun v2 -> cnt(v1 * v2)))

 (* id_cnt : cnt_2 *)
let id_cnt (x : int) = x

(*  eval_2 : expr -> int *)
let eval_2 e = eval_aux_2(e, id_cnt)
```

Defunctionalise!

```
type cnt_3 =
  | ID
  | OUTER_PLUS of expr * cnt_3
  | OUTER_SUBT of expr * cnt_3
  | OUTER_MULT of expr * cnt_3
  | INNER_PLUS of int * cnt_3
  | INNER_SUBT of int * cnt_3
  | INNER_MULT of int * cnt_3

type state_3 = expr * cnt_3

(* apply_3 : cnt_3 * int -> int *)
let rec apply_3 = function
   | (ID,                 v)              -> v
   | (OUTER_PLUS(e2, cnt), v1) -> eval_aux_3(e2, INNER_PLUS(v1, cnt))
   | (OUTER_SUBT(e2, cnt), v1) -> eval_aux_3(e2, INNER_SUBT(v1, cnt))
   | (OUTER_MULT(e2, cnt), v1) -> eval_aux_3(e2, INNER_MULT(v1, cnt))
   | (INNER_PLUS(v1, cnt), v2) -> apply_3(cnt, v1 + v2)
   | (INNER_SUBT(v1, cnt), v2) -> apply_3(cnt, v1 - v2)
   | (INNER_MULT(v1, cnt), v2) -> apply_3(cnt, v1 * v2)
```

Defunctionalise!

```
(* eval_aux_2 : state_3 -> int *)
and eval_aux_3 (e, cnt) =
   match e with
   | INT a        -> apply_3(cnt, a)
   | PLUS(e1, e2) -> eval_aux_3(e1, OUTER_PLUS(e2, cnt))
   | SUBT(e1, e2) -> eval_aux_3(e1, OUTER_SUBT(e2, cnt))
   | MULT(e1, e2) -> eval_aux_3(e1, OUTER_MULT(e2, cnt))

(* eval_3 : expr -> int *)
let eval_3 e = eval_aux_3(e, ID)
```

Eureka! Again we have a stack!

```
type tag =
  | O_PLUS of expr
  | I_PLUS of int
  | O_SUBT of expr
  | I_SUBT of int
  | O_MULT of expr
  | I_MULT of int

type cnt_4 = tag list
type state_4 = expr * cnt_4

(* apply_4 : cnt_4 * int -> int *)
let rec apply_4 = function
   | ([],          v)                 -> v
   | ((O_PLUS e2) :: cnt, v1) -> eval_aux_4(e2, (I_PLUS v1) :: cnt)
   | ((O_SUBT e2) :: cnt, v1) -> eval_aux_4(e2, (I_SUBT v1) :: cnt)
   | ((O_MULT e2) :: cnt, v1) -> eval_aux_4(e2, (I_MULT v1) :: cnt)
   | ((I_PLUS v1) :: cnt, v2) -> apply_4(cnt, v1 + v2)
   | ((I_SUBT v1) :: cnt, v2) -> apply_4(cnt, v1 - v2)
   | ((I_MULT v1) :: cnt, v2) -> apply_4(cnt, v1 * v2)
```

Eureka! Again we have a stack!

```
(* eval_aux_4 : state_4 -> int *)
and eval_aux_4 (e, cnt) =
   match e with
   | INT a                -> apply_4(cnt, a)
   | PLUS(e1, e2) -> eval_aux_4(e1, O_PLUS(e2) :: cnt)
   | SUBT(e1, e2) -> eval_aux_4(e1, O_SUBT(e2) :: cnt)
   | MULT(e1, e2) -> eval_aux_4(e1, O_MULT(e2) :: cnt)

(* eval_4 : expr -> int *)
let eval_4 e = eval_aux_4(e, [])
```

Eureka! Can combine apply_4 and eval_aux_4

```
type acc =
  | A_INT of int
  | A_EXP of expr

type cnt_5 = cnt_4

type state_5 = cnt_5 * acc

val : step : state_5 -> state_5

val driver : state_5 -> int

val eval_5 : expr -> int
```

Type of an “accumulator” that contains either an int or an expression.

The driver will be clearly tail-recursive …

Rewrite to use driver, accumulator

```
let step_5 = function
  | (cnt,                    A_EXP (INT a)) -> (cnt, A_INT a)
  | (cnt,    A_EXP (PLUS(e1, e2))) -> (O_PLUS(e2) :: cnt, A_EXP e1)
  | (cnt,    A_EXP (SUBT(e1, e2))) -> (O_SUBT(e2) :: cnt, A_EXP e1)
  | (cnt,    A_EXP (MULT(e1, e2))) -> (O_MULT(e2) :: cnt, A_EXP e1)
  | ((O_PLUS e2) :: cnt,  A_INT v1) -> ((I_PLUS v1) :: cnt, A_EXP e2)
  | ((O_SUBT e2) :: cnt,  A_INT v1) -> ((I_SUBT v1) :: cnt, A_EXP e2)
  | ((O_MULT e2) :: cnt, A_INT v1) -> ((I_MULT v1) :: cnt, A_EXP e2)
  | ((I_PLUS v1) :: cnt,   A_INT v2) -> (cnt, A_INT (v1 + v2))
  | ((I_SUBT v1) :: cnt,   A_INT v2) -> (cnt, A_INT (v1 - v2))
  | ((I_MULT v1) :: cnt,   A_INT v2) -> (cnt, A_INT (v1 * v2))
  | ([],                           A_INT v) -> ([], A_INT v)

let rec driver_5 = function
   | ([], A_INT v) -> v
   | state          -> driver_5 (step_5 state)

let eval_5 e = driver_5([], A_EXP e)
```

Eureka! There are really two independent stacks here --- one for “expressions” and one for values

```
type directive =
  | E of expr
  | DO_PLUS
  | DO_SUBT
  | DO_MULT

type directive_stack = directive list

type value_stack = int list

type state_6 = directive_stack * value_stack

val step_6 : state_6 -> state_6

val driver_6 : state_6 -> int

val exp_6 : expr -> int
```

The state is now two stacks!

Split into two stacks

```
let step_6 = function
| (E(INT v) :: ds,                vs) -> (ds, v :: vs)
| (E(PLUS(e1, e2)) :: ds,    vs) -> ((E e1) :: (E e2) :: DO_PLUS :: ds, vs)
| (E(SUBT(e1, e2)) :: ds,    vs) -> ((E e1) :: (E e2) :: DO_SUBT :: ds, vs)
| (E(MULT(e1, e2)) :: ds,    vs) -> ((E e1) :: (E e2) :: DO_MULT :: ds, vs)

| (DO_PLUS :: ds, v2 :: v1 :: vs) -> (ds, (v1 + v2) :: vs)
| (DO_SUBT :: ds, v2 :: v1 :: vs) -> (ds, (v1 - v2) :: vs)
| (DO_MULT :: ds, v2 :: v1 :: vs) -> (ds, (v1 * v2) :: vs)
| _ -> failwith "eval : runtime error!"

let rec driver_6 = function
   | ([], [v]) -> v
   | state    -> driver_6 (step_6 state)

let eval_6 e = driver_6 ([E e], [])
```

An eval_6 trace

e = PLUS(MULT(INT 89, INT 2), SUBT(INT 10, INT 4))

inspect

state 1 DS = [E(PLUS(MULT(INT(89), INT(2)), SUBT(INT(10), INT(4))))]
VS = []
state 2 DS = [DO_PLUS; E(SUBT(INT(10), INT(4))); E(MULT(INT(89), INT(2)))]
VS = []
state 3 DS = [DO_PLUS; E(SUBT(INT(10), INT(4))); DO_MULT; E(INT(2)); E(INT(89))]
VS = []
state 4 DS = [DO_PLUS; E(SUBT(INT(10), INT(4))); DO_MULT; E(INT(2))]
VS = [89]

compute

state 5 DS = [DO_PLUS; E(SUBT(INT(10), INT(4))); DO_MULT]
VS = [89; 2]
state 6 DS = [DO_PLUS; E(SUBT(INT(10), INT(4)))]
VS = [178]

inspect

state 7 DS = [DO_PLUS; DO_SUBT; E(INT(4)); E(INT(10))]
VS = [178]
state 8 DS = [DO_PLUS; DO_SUBT; E(INT(4))]
VS = [178; 10]

compute

state 9 DS = [DO_PLUS; DO_SUBT]
VS = [178; 10; 4]
state 10DS = [DO_PLUS]
VS = [178; 6]
state 11DS = []
VS = [184]

Top of each stack is on the right

Key insight

This evaluator is interleaving two distinct computations:

(1) decomposition of the input expression into sub-expressions
(2) the computation of +, -, and *.

Idea: why not do the decomposition BEFORE the computation?

Key insight: An interpreter can (usually) be **refactored** into a translation (compilation!) followed by a lower-level interpreter.

Interpret_higher (e) = interpret_lower(compile(e))

Note : this can occur at many levels of abstraction: think of machine code being interpreted in micro-code …

Refactor --- compile!

```
(* low-level instructions *)
type instr =
  | Ipush of int
  | Iplus
  | Isubt
  | Imult

type code = instr list

type state_7 = code * value_stack

(* compile : expr -> code *)
let rec compile = function
   | INT a              -> [Ipush a]
   | PLUS(e1, e2)   -> (compile e1) @ (compile e2) @ [Iplus]
   | SUBT(e1, e2)   -> (compile e1) @ (compile e2) @ [Isubt]
   | MULT(e1, e2)  -> (compile e1) @ (compile e2) @ [Imult]
```

> Never put off till run-time what you can do at compile-time.
> -- David Gries

Evaluate compiled code.

```
(* step_7 : state_7 -> state_7 *)
let step_7 = function
  | (Ipush v :: is,          vs) ->  (is, v :: vs)
  | (Iplus :: is, v2::v1::vs) -> (is, (v1 + v2) :: vs)
  | (Isubt :: is, v2::v1::vs) -> (is, (v1 - v2) :: vs)
  | (Imult :: is, v2::v1::vs) -> (is, (v1 * v2) :: vs)
  | _ -> failwith "eval : runtime error!"

let rec driver_7 = function
   | ([], [v]) -> v
   | _ -> driver_7 (step_7 state)

let eval_7 e = driver_7  (compile e, [])
```

An eval_7 trace

inspect

compile (PLUS(MULT(INT 89, INT 2), SUBT(INT 10, INT 4)))
= [push 89; push 2; mult; push 10; push 4; subt; plus]

compute

state 1 IS = [add; sub; push 4; push 10; mul; push 2; push 89]
VS = []
state 2 IS = [add; sub; push 4; push 10; mul; push 2]
VS = [89]
state 3 IS = [add; sub; push 4; push 10; mul]
VS = [89; 2]
state 4 IS = [add; sub; push 4; push 10]
VS = [178]
state 5 IS = [add; sub; push 4]
VS = [178; 10]
state 6 IS = [add; sub]
VS = [178; 10; 4]
state 7 IS = [add]
VS = [178; 6]
state 8 IS = []
VS = [184]

Top of each stack is on the right

interpret **is implicitly using Ocaml's runtime stack**

```
let rec interpret (e, env, store) =
    match e with
    | Integer n          -> (INT n, store)
    | Op(e1, op, e2)   ->
       let (v1, store1) = interpret(e1, env, store) in
       let (v2, store2) = interpret(e2, env, store1) in
            (do_oper(op, v1, v2), store2)
    :
    :
```

- Every invocation of interpret is building an activation record on Ocaml's runtime stack.
- **We will now define interpreter 2 which makes this stack explicit**

Interp_0.ml → interp_1.ml → interp_2.ml

The derivation from eval to compile+eval_7 can be used as a guide to a derivation from Interpreter 0 to interpreter 2.

1. Apply CPS to the code of Interpreter 0
2. Defunctionalise
3. Arrive at interpreter 1, which has a single continuation stack containing expressions, values and environments (analogous to eval_6)
4. Spit this stack into two stacks : one for instructions and the other for values and environments
5. Refactor into compiler + lower-level interpreter
6. Arrive at interpreter 2. (analogous to eval_7)

Interpreter 0 → Interpreter 2

Interpreter 2: A high-level stack-oriented machine

1. **Makes the Ocaml runtime stack explicit**
2. **Complex values pushed onto stacks**
3. **One stack for values and environments**
4. **One stack for instructions**
5. **Heap used only for references**
6. **Instructions have tree-like structure**

(we will not look at the details of interpreter 1 …)

Inpterp_2 data types

```
type address

type store = address -> value

and value =
    | REF of address
    | INT of int
    | BOOL of bool
    | UNIT
    | PAIR of value * value
    | INL of value
    | INR of value
    | FUN of ((value * store)
                    -> (value * store))

type env = Ast.var -> value
```

Interp_0

```
type address = int

type value =
    | REF of address
    | INT of int
    | BOOL of bool
    | UNIT
    | PAIR of value * value
    | INL of value
    | INR of value
    | CLOSURE of bool *
                       closure

and closure = code * env
```

Interp_2

```
and instruction =
  | PUSH of value
  | LOOKUP of var
  | UNARY of unary_oper
  | OPER of oper
  | ASSIGN
  | SWAP
  | POP
  | BIND of var
  | FST
  | SND
  | DEREF
  | APPLY
  | MK_PAIR
  | MK_INL
  | MK_INR
  | MK_REF
  | MK_CLOSURE of code
  | MK_REC of var * code
  | TEST of code * code
  | CASE of code * code
  | WHILE of code * code
```

Interp_2.ml : The Abstract Machine

```
and code = instruction list

and binding = var * value

and env = binding list

type env_or_value = EV of env | V of value

type env_value_stack = env_or_value list

type state = code * env_value_stack

val step : state -> state

val driver : state -> value

val compile : expr -> code

val interpret : expr -> value
```

The state is actually comprised of a heap --- a global array of values --- a pair of the form

(code, evn_value_stack)

Interpreter 2: The Abstract Machine

```
type state = code * env_value_stack

val step : state -> state
```

The state transition function.

```
let step = function
(* (code stack,          value/env stack) -> (code stack,  value/env stack) *)
 | ((PUSH v) :: ds,                        evs) -> (ds, (V v) :: evs)
 | (POP :: ds,                        s :: evs) -> (ds, evs)
 | (SWAP :: ds,                 s1 :: s2 :: evs) -> (ds, s2 :: s1 :: evs)
 | ((BIND x) :: ds,               (V v) :: evs) -> (ds, EV([(x, v)]) :: evs)
 | ((LOOKUP x) :: ds,                      evs) -> (ds, V(search(evs, x)) :: evs)
 | ((UNARY op) :: ds,             (V v) :: evs) -> (ds, V(do_unary(op, v)) :: evs)
 | ((OPER op) :: ds,    (V v2) :: (V v1) :: evs) -> (ds, V(do_oper(op, v1, v2)) :: evs)
 | (MK_PAIR :: ds,      (V v2) :: (V v1) :: evs) -> (ds, V(PAIR(v1, v2)) :: evs)
 | (FST :: ds,             V(PAIR (v, _)) :: evs) -> (ds, (V v) :: evs)
 | (SND :: ds,             V(PAIR (_, v)) :: evs) -> (ds, (V v) :: evs)
 | (MK_INL :: ds,                 (V v) :: evs) -> (ds, V(INL v) :: evs)
 | (MK_INR :: ds,                 (V v) :: evs) -> (ds, V(INR v) :: evs)
 | (CASE (c1,  _) :: ds,           V(INL v)::evs) -> (c1 @ ds, (V v) :: evs)
 | (CASE ( _, c2) :: ds,           V(INR v)::evs) -> (c2 @ ds, (V v) :: evs)
 | ((TEST(c1, c2)) :: ds,  V(BOOL true) :: evs) -> (c1 @ ds, evs)
 | ((TEST(c1, c2)) :: ds, V(BOOL false) :: evs) -> (c2 @ ds, evs)
 | (ASSIGN :: ds,   (V v) :: (V (REF a)) :: evs) -> (heap.(a) <- v; (ds, V(UNIT) :: evs))
 | (DEREF :: ds,              (V (REF a)) :: evs) -> (ds, V(heap.(a)) :: evs)
 | (MK_REF :: ds,                 (V v) :: evs) -> let a = allocate () in (heap.(a) <- v;
                                                   (ds, V(REF a) :: evs))
 | ((WHILE(c1, c2)) :: ds,V(BOOL false) :: evs) -> (ds, evs)
 | ((WHILE(c1, c2)) :: ds, V(BOOL true) :: evs) -> (c1 @ [WHILE(c1, c2)] @ ds, evs)
 | ((MK_CLOSURE c) :: ds,                  evs) -> (ds,  V(mk_fun(c, evs_to_env evs)) :: evs)
 | (MK_REC(f, c) :: ds,                    evs) -> (ds,  V(mk_rec(f, c, evs_to_env evs)) :: evs)
 | (APPLY :: ds,  V(CLOSURE (_, (c, env))) :: (V v) :: evs)
                                               -> (c @ ds, (V v) :: (EV env) :: evs)
 | state -> complain ("step : bad state = " ^ (string_of_state state) ^ "\n")
```

The driver. Correctness

```
(* val driver : state -> value *)
let rec driver state =
    match state with
    | ([], [V v]) -> v
    | _
       -> driver (step state)
```

val compile : expr -> code

The idea: if e passes the frond-end and
    Interp_0.interpret e = v
then
    driver (compile e, []) = v'
where v' (somehow) represents v.

In other words, evaluating compile e should leave the value of e on top of the stack

Implement inter_0 in interp_2

interp_0.ml

```
let rec interpret (e, env, store) =
   match e with
| Pair(e1, e2)  ->
      let (v1, store1) = interpret(e1, env, store) in
      let (v2, store2) = interpret(e2, env, store1) in (PAIR(v1, v2), store2)
   | Fst e ->
       (match interpret(e, env, store) with
       | (PAIR (v1, _), store') -> (v1, store')
       | (v, _) -> complain "runtime error.  Expecting a pair!")
   :
```

```
let step = function
 | (MK_PAIR :: ds,  (V v2) :: (V v1) :: evs)  ->  (ds,   V(PAIR(v1, v2)) :: evs)
 | (FST :: ds,           V(PAIR (v, _)) :: evs)  ->  (ds,   (V v) :: evs)
 :

let rec compile = function
 | Pair(e1, e2)   -> (compile e1) @ (compile e2) @ [MK_PAIR]
 | Fst e           -> (compile e) @ [FST]
 :
```

interp_2.ml

Implement inter_0 in interp_2

interp_0.ml

```
let rec interpret (e, env, store) =
    match e with
    | If(e1, e2, e3) ->
       let (v, store') = interpret(e1, env, store) in
            (match v with
            | BOOL true -> interpret(e2, env, store')
            | BOOL false -> interpret(e3, env, store')
            | v -> complain "runtime error.  Expecting a boolean!”)
     :
```

```
let step = function
 | ((TEST(c1, c2)) :: ds,  V(BOOL true) :: evs) -> (c1 @ ds, evs)
 | ((TEST(c1, c2)) :: ds, V(BOOL false) :: evs) -> (c2 @ ds, evs)
 :

let rec compile = function
 | If(e1, e2, e3) -> (compile e1) @ [TEST(compile e2, compile e3)]
 :
```

interp_2.ml

Tricky bits again!

interp_0.ml

```
let rec interpret (e, env, store) =
   match e with
   | Lambda(x, e)  -> (FUN (fun (v, s) -> interpret(e, update(env, (x, v)), s)), store)
   | App(e1, e2) -> (* I chose to evaluate argument first!  *)
     let (v2, store1) = interpret(e2, env, store) in
     let (v1, store2) =  interpret(e1, env, store1) in
         (match v1 with
         | FUN f -> f (v2, store2)
         | v -> complain "runtime error.  Expecting a function!")
   :
```

interp_2.ml

```
let step = function
 | (POP :: ds,                          s :: evs) -> (ds,  evs)
 | (SWAP :: ds,                  s1 :: s2 :: evs) -> (ds,  s2 :: s1 :: evs)
 | ((BIND x) :: ds,                 (V v) :: evs) -> (ds,  EV([(x, v)]) :: evs)
 | ((MK_CLOSURE c) :: ds,                    evs) -> (ds,   V(mk_fun(c, evs_to_env evs)) :: evs)
 | (APPLY :: ds,  V(CLOSURE (_, (c, env))) :: (V v) :: evs)
                                                  -> (c @ ds,  (V v) :: (EV env) :: evs)
let rec compile = function
 | Lambda(x, e)   -> [MK_CLOSURE((BIND x) :: (compile e) @ [SWAP; POP])]
 | App(e1, e2)      -> (compile e2) @ (compile e1) @ [APPLY; SWAP; POP]
 :
```

Example : Compiled code for rev_pair.slang

```
let rev_pair (p : int * int) : int * int  = (snd p, fst p)
in
    rev_pair (21, 17)
end
```

```
MK_CLOSURE([BIND p; LOOKUP p; SND; LOOKUP p; FST; MK_PAIR; SWAP; POP]);
 BIND rev_pair;
 PUSH 21;
 PUSH 17;
 MK_PAIR;
 LOOKUP rev_pair;
 APPLY;
 SWAP;
 POP;
 SWAP;
 POP
```

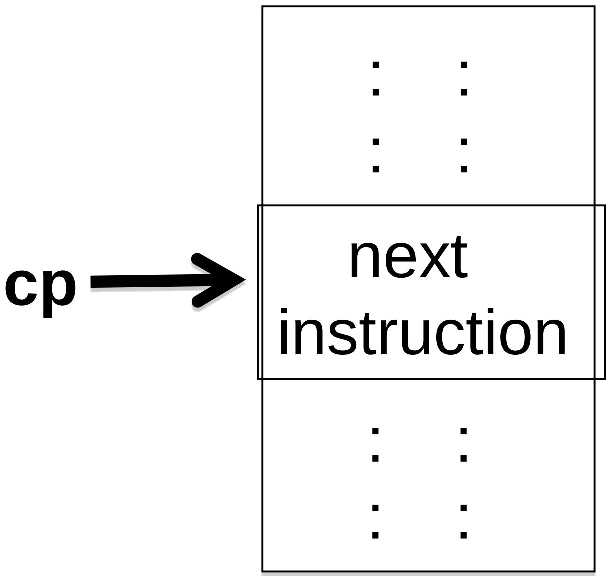
DEMO TIME!!!

LECTURE 10
Derive Interpreter 3

1. “Flatten” code into linear array
2. Add “code pointer” (cp) to machine state
3. New instructions : LABEL, GOTO, RETURN
4. “Compile away” conditionals and while loops

Linearise code

Interpreter 2 copies code on the code stack.
We want to introduce one global array of instructions indexed by a code pointer (**cp**).
At runtime the **cp** points at the next instruction to be executed.



This will require two new instructions:

LABEL L : Associate label L with this location in the code array

GOTO L : Set the **cp** to the code address associated with L

Compile conditionals, loops

If(e1, e2, e3)

code for e1
TEST k
code for e2
GOTO m
k: code for e3
m:

While(e1, e2)

m: code for e1
TEST k
code for e2
GOTO m
k:

If ? = 0 Then 17 else 21 end

interp_2	interp_3	interp_3 (loaded)
PUSH UNIT;	PUSH UNIT;	0: PUSH UNIT;
UNARY READ;	UNARY READ;	1: UNARY READ;
PUSH 0;	PUSH 0;	2: PUSH 0;
OPER EQI;	OPER EQI;	3: OPER EQI;
TEST(	TEST L0;	4: TEST L0 = 7;
[PUSH 17],	PUSH 17;	5: PUSH 17;
[PUSH 21]	GOTO L1;	6: GOTO L1 = 9;
)	LABEL L0;	7: LABEL L0;
	PUSH 21;	8: PUSH 21;
	LABEL L1;	9: LABEL L1;
	HALT	10: HALT
	Symbolic code locations	Numeric code locations

Implement inter_2 in interp_3

```
let step = function
 | ((TEST(c1, c2)) :: ds,  V(BOOL true) :: evs) -> (c1 @ ds, evs)
 | ((TEST(c1, c2)) :: ds, V(BOOL false) :: evs) -> (c2 @ ds, evs)
 :
```

interp_2.ml

```
let step (cp, evs) =
 match (get_instruction cp, evs) with
 | (TEST (_, Some _),  V(BOOL true) :: evs)  ->  (cp + 1, evs)
 | (TEST (_, Some i),  V(BOOL false) :: evs)  ->  (i,         evs)
 | (LABEL l,                                  evs)  ->  (cp + 1, evs)
 | (GOTO (_, Some i),                         evs)  -> (i,          evs)
 :
```

Interp_3.ml

Code locations are represented as

("L", None)   : not yet loaded (assigned numeric address)

("L", Some i) : label "L" has been assigned numeric address i

Tricky bits again!

interp_2.ml

```
let step = function
 | (POP :: ds,                        s :: evs) -> (ds,  evs)
 | (SWAP :: ds,                s1 :: s2 :: evs) -> (ds,  s2 :: s1 :: evs)
 | ((BIND x) :: ds,               (V v) :: evs) -> (ds,  EV([(x, v)]) :: evs)
 | ((MK_CLOSURE c) :: ds,                   evs) -> (ds,   V(mk_fun(c, evs_to_env evs)) :: evs)
 | (APPLY :: ds,  V(CLOSURE (_, (c, env))) :: (V v) :: evs)
                                               -> (c @ ds, (V v) :: (EV env) :: evs)
```

interp_3.ml

```
let step (cp, evs) =
 match (get_instruction cp, evs) with
 | (POP,                               s :: evs) -> (cp + 1, evs)
 | (SWAP,                       s1 :: s2 :: evs) -> (cp + 1, s2 :: s1 :: evs)
 | (BIND x,                      (V v) :: evs) -> (cp + 1, EV([(x, v)]) :: evs)
 | (MK_CLOSURE loc,                          evs)        -> (cp + 1,
                                V(CLOSURE(loc, evs_to_env evs)) :: evs)
 | (RETURN,    (V v) :: _ :: (RA i) :: evs)  -> (i, (V v) :: evs)
 | (APPLY,  V(CLOSURE ((_, Some i), env)) :: (V v) :: evs)
                                        ->
                                   (i, (V v) :: (EV env) :: (RA (cp + 1)) :: evs)
```

Note that in interp_2 the body of a closure is consumed from the code stack. But in interp_3 we need to save the return address on the stack (here i is the location of the closure's code).

Tricky bits again!

interp_2.ml

```
let rec compile = function
 | Lambda(x, e)   -> [MK_CLOSURE((BIND x) :: (compile e) @ [SWAP; POP])]
 | App(e1, e2)     -> (compile e2) @ (compile e1) @ [APPLY; SWAP; POP]
 :
```

Interp_3.ml

```
let rec comp = function
 | App(e1, e2)    ->
   let (defs1, c1) = comp e1 in
   let (defs2, c2) = comp e2 in
       (defs1 @ defs2, c2 @ c1 @ [APPLY])
 | Lambda(x, e)    ->
   let (defs, c) = comp e in
   let f = new_label () in
   let def = [LABEL f ; BIND x] @ c @ [SWAP; POP; RETURN] in
      (def @ defs, [MK_CLOSURE((f, None))])
```

Interp_3.ml

```
let compile e =
   let (defs, c) = comp e in
     c                   (* body of program *)
    @ [HALT]      (* stop the interpreter *)
    @ defs          (*  function definitions *)
```

Interpreter 3
(very similar to interpreter 2)

```
let step (cp, evs) =
 match (get_instruction cp, evs) with
 | (PUSH v,                                  evs) -> (cp + 1, (V v) :: evs)
 | (POP,                                s :: evs) -> (cp + 1, evs)
 | (SWAP,                        s1 :: s2 :: evs) -> (cp + 1, s2 :: s1 :: evs)
 | (BIND x,                         (V v) :: evs) -> (cp + 1, EV([(x, v)]) :: evs)
 | (LOOKUP x,                                evs) -> (cp + 1, V(search(evs, x)) :: evs)
 | (UNARY op,                       (V v) :: evs) -> (cp + 1, V(do_unary(op, v)) :: evs)
 | (OPER op,            (V v2) :: (V v1) :: evs) -> (cp + 1, V(do_oper(op, v1, v2)) :: evs)
 | (MK_PAIR,            (V v2) :: (V v1) :: evs) -> (cp + 1, V(PAIR(v1, v2)) :: evs)
 | (FST,                   V(PAIR (v, _)) :: evs) -> (cp + 1, (V v) :: evs)
 | (SND,                   V(PAIR (_, v)) :: evs) -> (cp + 1, (V v) :: evs)
 | (MK_INL,                         (V v) :: evs) -> (cp + 1, V(INL v) :: evs)
 | (MK_INR,                         (V v) :: evs) -> (cp + 1, V(INR v) :: evs)
 | (CASE (_, Some _),             V(INL v)::evs) -> (cp + 1, (V v) :: evs)
 | (CASE (_, Some i),             V(INR v)::evs) -> (i,      (V v) :: evs)
 | (TEST (_, Some _),       V(BOOL true) :: evs) -> (cp + 1, evs)
 | (TEST (_, Some i),      V(BOOL false) :: evs) -> (i,      evs)
 | (ASSIGN,        (V v) :: (V (REF a)) :: evs) -> (heap.(a) <- v; (cp + 1, V(UNIT) :: evs))
 | (DEREF,                  (V (REF a)) :: evs) -> (cp + 1, V(heap.(a)) :: evs)
 | (MK_REF,                         (V v) :: evs) -> let a = new_address () in (heap.(a) <- v;
                                                      (cp + 1, V(REF a) :: evs))
 | (MK_CLOSURE loc,                          evs) -> (cp + 1, V(CLOSURE(loc, evs_to_env evs)) :: evs)
 | (APPLY,  V(CLOSURE ((_, Some i), env)) :: (V v) :: evs)
                                                 -> (i, (V v) :: (EV env) :: (RA (cp + 1)) :: evs)
(* new intructions *)
 | (RETURN,        (V v) :: _ :: (RA i) :: evs) -> (i, (V v) :: evs)
 | (LABEL l,                                 evs) -> (cp + 1, evs)
 | (HALT,                                    evs) -> (cp, evs)
 | (GOTO (_, Some i),                        evs) -> (i, evs)
 | _ -> complain ("step : bad state = " ^ (string_of_state (cp, evs)) ^ "\n")
```

Some observations

- A very clean machine!
- But it still has a **very** inefficient treatment of environments.
- Also, pushing complex values on the stack is not what most virtual machines do. In fact, we are still using OCaml's runtime memory management to manipulate complex values.

Example : Compiled code for rev_pair.slang

```
let rev_pair (p : int * int) : int * int  = (snd p, fst p)
in
    rev_pair (21, 17)
end
```

```
MK_CLOSURE(
   [BIND p; LOOKUP p; SND;
    LOOKUP p; FST; MK_PAIR;
    SWAP; POP]);
 BIND rev_pair;
 PUSH 21;
 PUSH 17;
 MK_PAIR;
 LOOKUP rev_pair;
 APPLY;
 SWAP;
 POP;
 SWAP;
 POP
```

Interp_2

```
MK_CLOSURE(rev_pair)
 BIND rev_pair
 PUSH 21
 PUSH 17
 MK_PAIR
 LOOKUP rev_pair
 APPLY
 SWAP
 POP
 HALT
```

```
LABEL rev_pair
 BIND p
 LOOKUP p
 SND
 LOOKUP p
 FST
 MK_PAIR
 SWAP
 POP
 RETURN
```

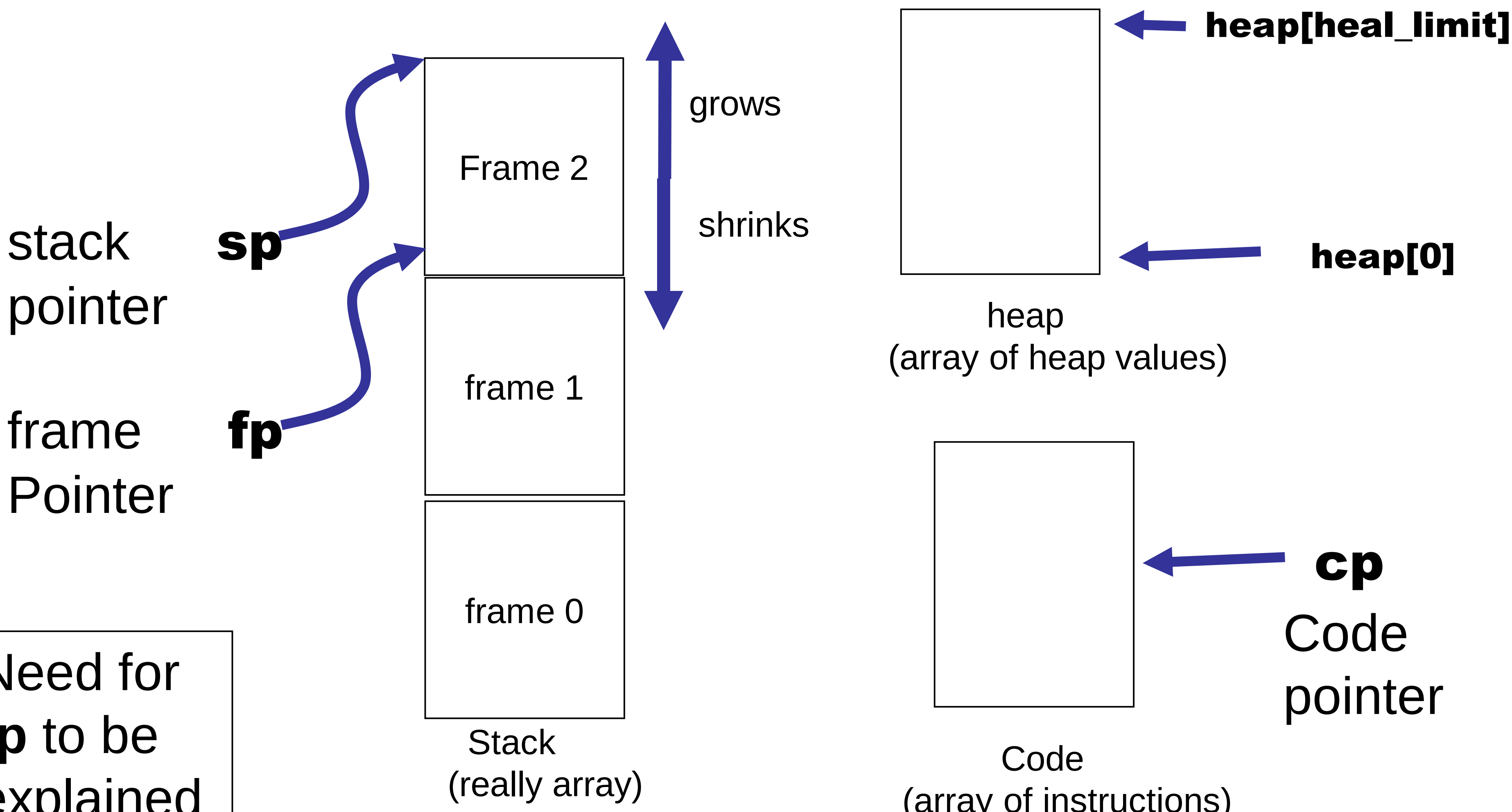
Interp_3

DEMO TIME!!!

LECTURES 11
Deriving The Jargon VM
(interpreter 4)

1. **First change**: Introduce an **addressable stack.**
2. Replace variable lookup by a (relative) location on the stack or heap determined at **compile time**.
3. Relative to what? A **frame pointer** (**fp**) pointing into the stack is needed to keep track of the current **activation record.**
4. **Second change**: Optimise the representation of closures so that they contain **only** the values associated with the **free variables** of the closure and a pointer to code.
5. **Third change**: Restrict values on stack to be simple (ints, bools, heap addresses, etc). Complex data is moved to the heap, leaving pointers into the heap on the stack.
6. How might things look different in a language without first-class functions? In a language with multiple arguments to function calls?

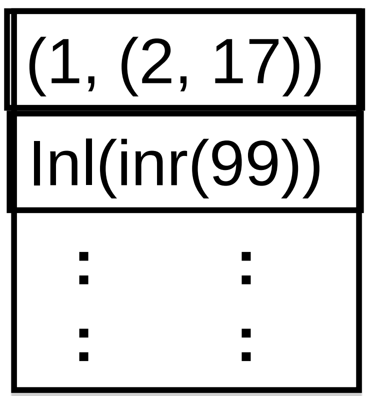
Jargon Virtual Machine



Need for **fp** to be explained soon …

The stack in interpreter 3

A stack
in interpreter 3



"All problems in computer science can be solved by another level of indirection, except of course for the problem of too many indirections."
--- David Wheeler

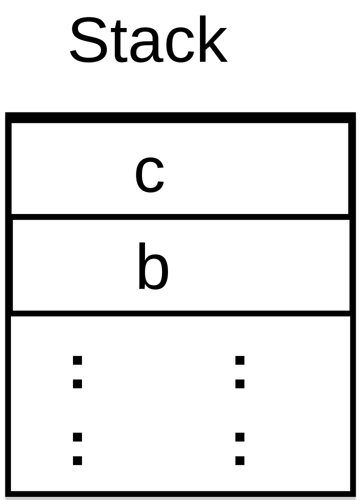
Stack elements in interpreter 3 are not of fixed size.

Virtual machines (JVM, etc) typically restrict stack elements to be of a fixed size

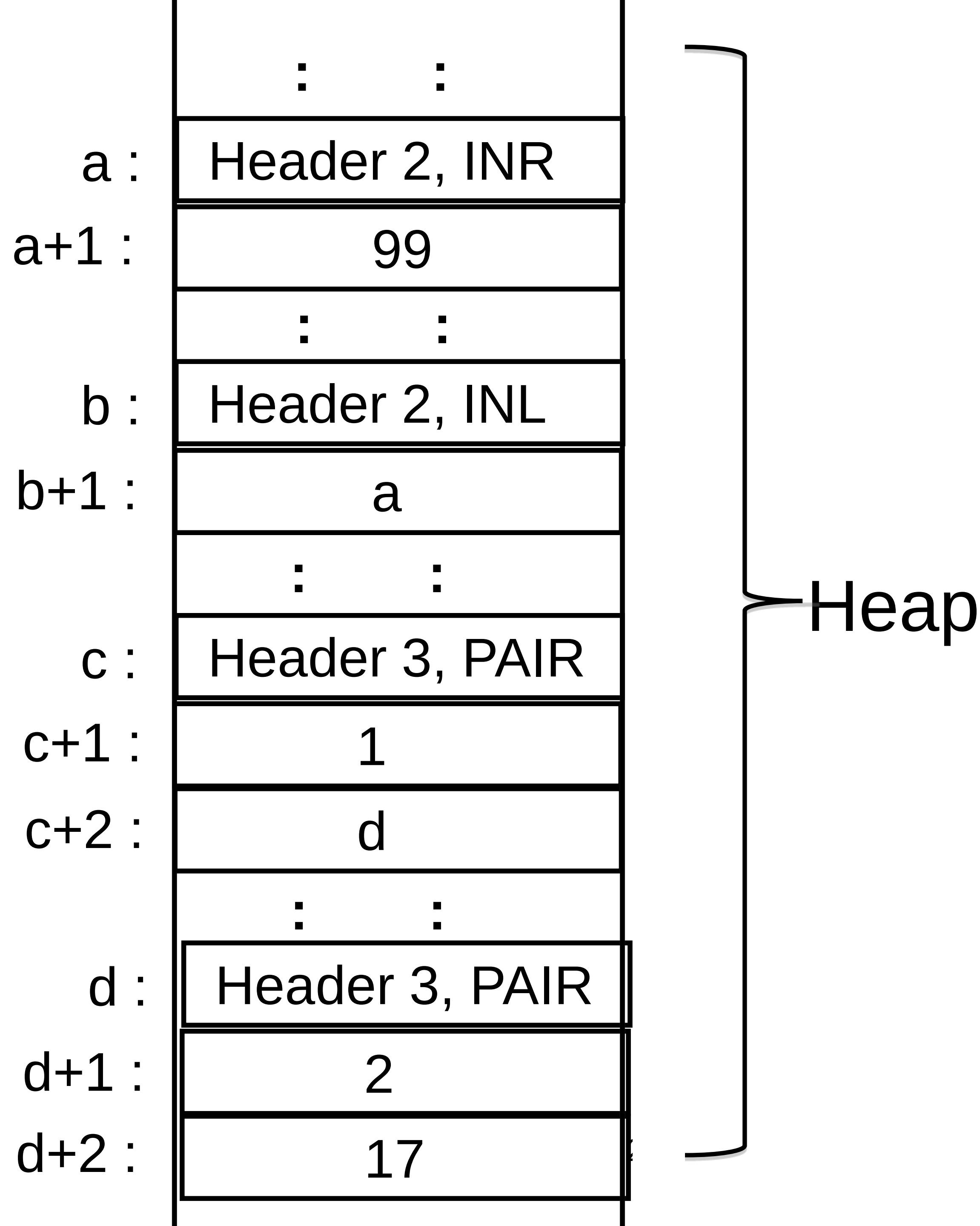
We need to shift data from the high-level stack of interpreter 3 to a lower-level stack with fixed size elements.

Solution : put the data in the heap. Place pointers to the heap on the stack.

The Jargon VM stack



Some stack elements represent pointers into the heap



Small change to instructions

interp_3.mli

```
type instruction =
  | PUSH of value
  | LOOKUP of Ast.var
  | UNARY of Ast.unary_oper
  | OPER of Ast.oper
  | ASSIGN
  | SWAP
  | POP
  | BIND of Ast.var
  | FST
  | SND
  | DEREF
  | APPLY
  | RETURN
  | MK_PAIR
  | MK_INL
  | MK_INR
  | MK_REF
  | MK_CLOSURE of location
  | TEST of location
  | CASE of location
  | GOTO of location
  | LABEL of label
  | HALT
```

jargon.mli

```
type instruction =
  | PUSH of stack_item           (* modified *)
  | LOOKUP of value_path         (* modified *)
  | UNARY of Ast.unary_oper
  | OPER of Ast.oper
  | ASSIGN
  | SWAP
  | POP
(*  | BIND of var           not needed *)
  | FST
  | SND
  | DEREF
  | APPLY
  | RETURN
  | MK_PAIR
  | MK_INL
  | MK_INR
  | MK_REF
  | MK_CLOSURE of location * int   (* modified *)
  | TEST of location
  | CASE of location
  | GOTO of location
  | LABEL of label
  | HALT
```

A word about implementation

Interpreter 3

```
type value = | REF of address | INT of int | BOOL of bool | UNIT
  | PAIR of value * value | INL of value | INR of value | CLOSURE of location * env
type env_or_value = | EV of env | V of value | RA of address
type env_value_stack = env_or_value list
```

Jargon VM

```
type stack_item =
  | STACK_INT of int
  | STACK_BOOL of bool
  | STACK_UNIT
  | STACK_HI of heap_index    (* Heap Inde           *)
  | STACK_RA of code_index    (* Return Address      *)
  | STACK_FP of stack_index   (* (saved) Frame Pointer *)
```

```
type heap_type =
  | HT_PAIR
  | HT_INL
  | HT_INR
  | HT_CLOSURE
```

The headers will be essential for garbage collection!

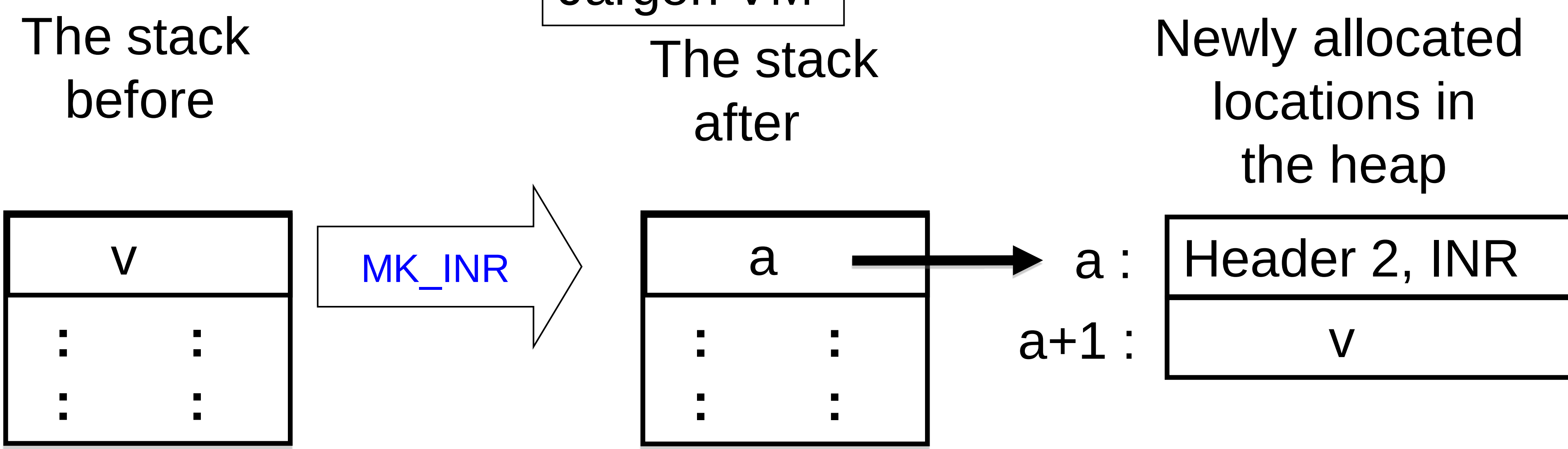
```
type heap_item =
  | HEAP_INT of int
  | HEAP_BOOL of bool
  | HEAP_UNIT
  | HEAP_HI of heap_index             (* Heap  Index                      *)
  | HEAP_CI of code_index             (* Code pointer for closures        *)
  | HEAP_HEADER of int * heap_type    (* int is number items in heap block *)
```

MK_INR (MK_INL is similar)

In interpreter 3

(MK_INR, (V v) :: evs)
-> (cp + 1, V(INR(v)) :: evs)

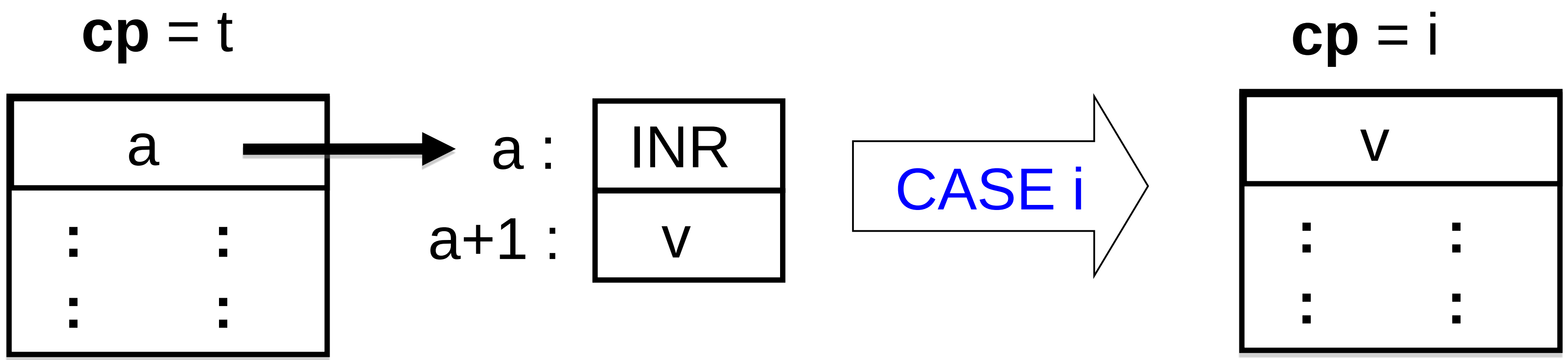
Jargon VM

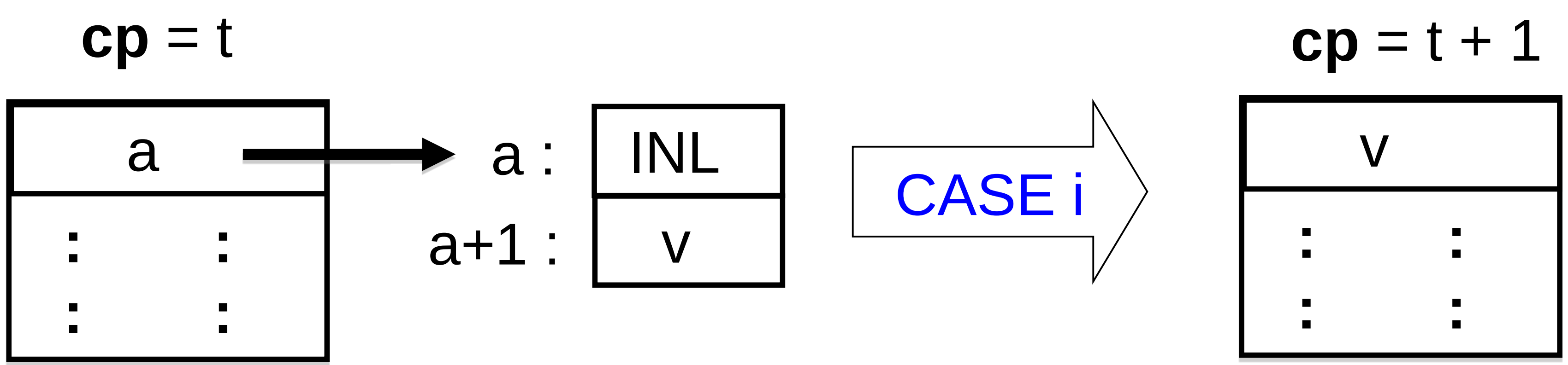


Note: The header types are not really required. We could instead add an extra field here (for example, 0 or 1). However, header types aid in understanding the code and traces of runtime execution.

CASE (TEST is similar)

```
(CASE (_, Some _),  V(INL v)::evs) -> (cp + 1, (V v) :: evs)
(CASE (_, Some i),  V(INR v)::evs) -> (i,      (V v) :: evs)
```



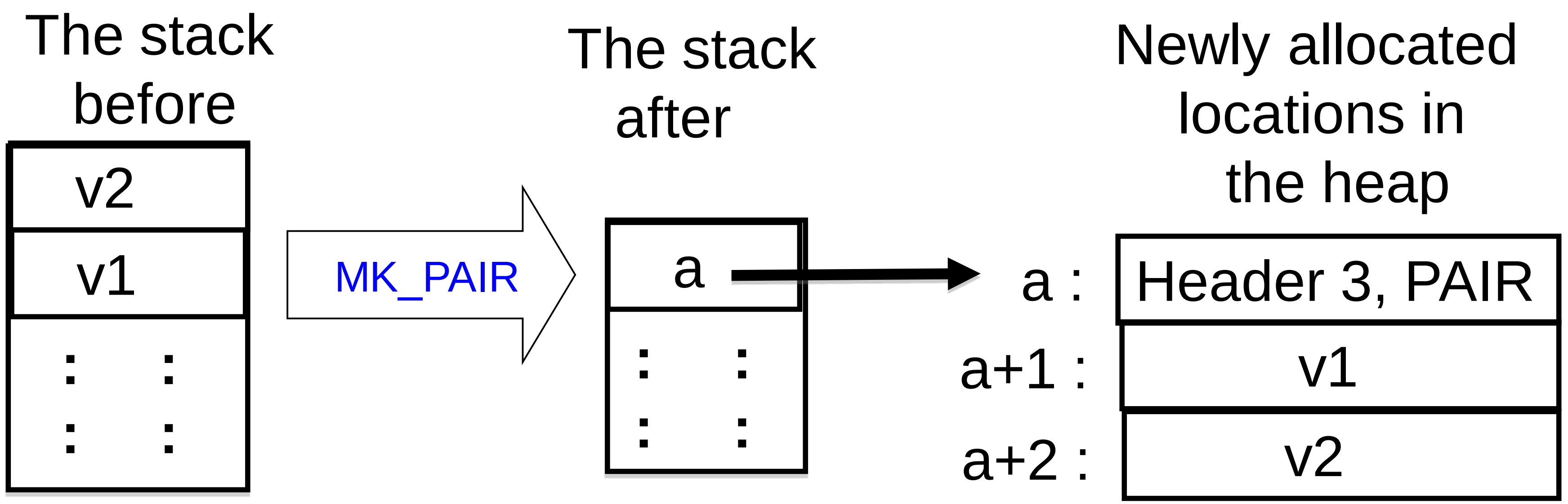


MK_PAIR

In interpreter 3:

(MK_PAIR, (V v2) :: (V v1) :: evs) -> (cp + 1, V(PAIR(v1, v2)) :: evs)

In Jargon VM:

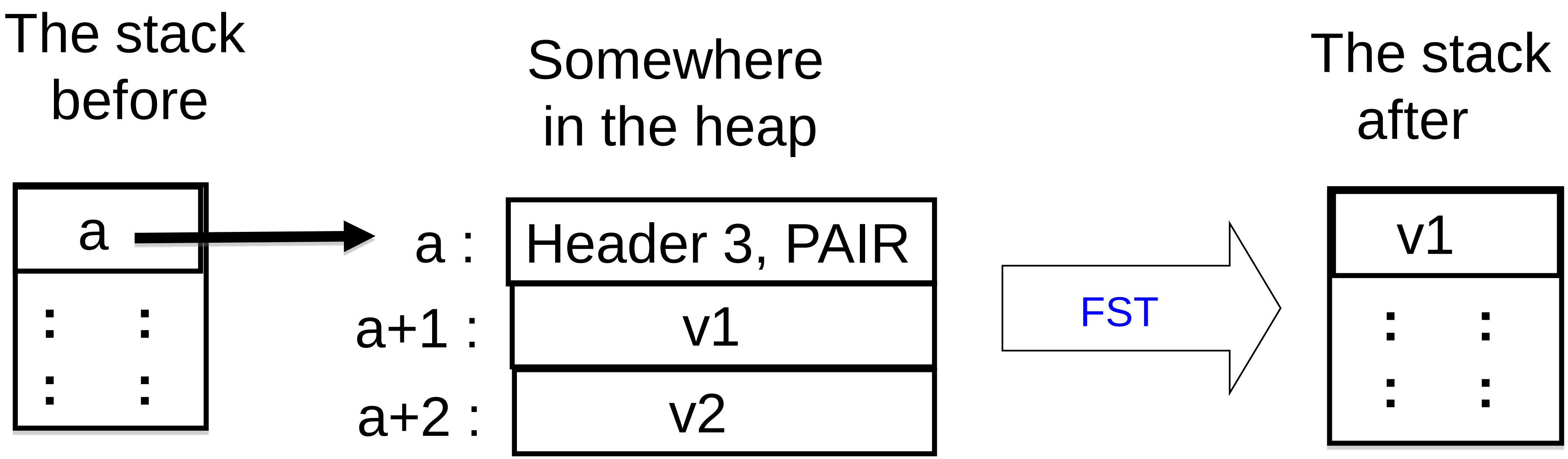


FST (similar for SND)

In interpreter 3:

(FST, V (PAIR(v1, v2)) :: evs)
-> (cp + 1, v1 :: evs)

In Jargon VM:



Note that v1 could be a simple value (int or bool), or aother heap address.

These require more care …

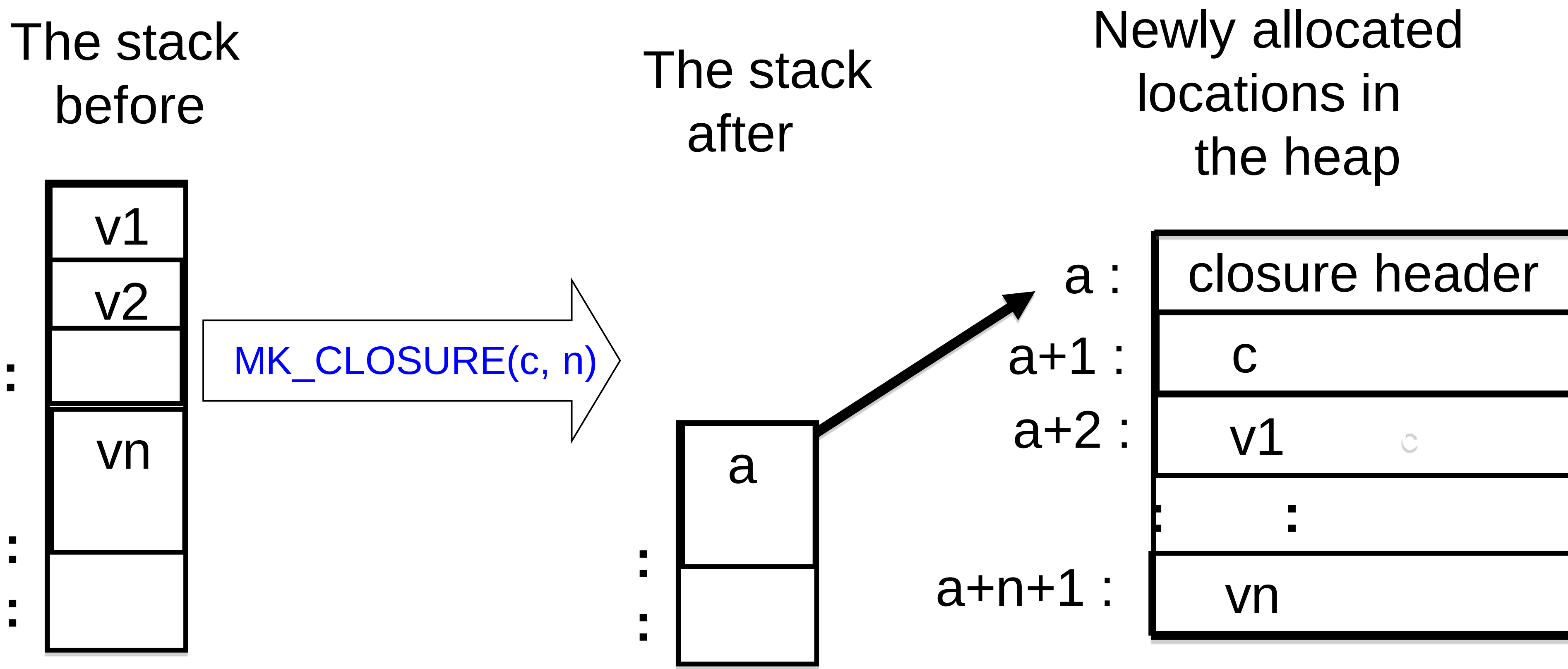
In interpreter 3:

```
let step (cp, evs) =
 match (get_instruction cp, evs) with
| (MK_CLOSURE loc,   evs)
    -> (cp + 1, V(CLOSURE(loc, evs_to_env evs)) :: evs)

| (APPLY,    V(CLOSURE ((_, Some i), env)) :: (V v) :: evs)
    -> (i,  (V v) :: (EV env) :: (RA (cp + 1)) :: evs)

| (RETURN,    (V v) :: _ :: (RA i) :: evs)
    -> (i,  (V v) :: evs)
```

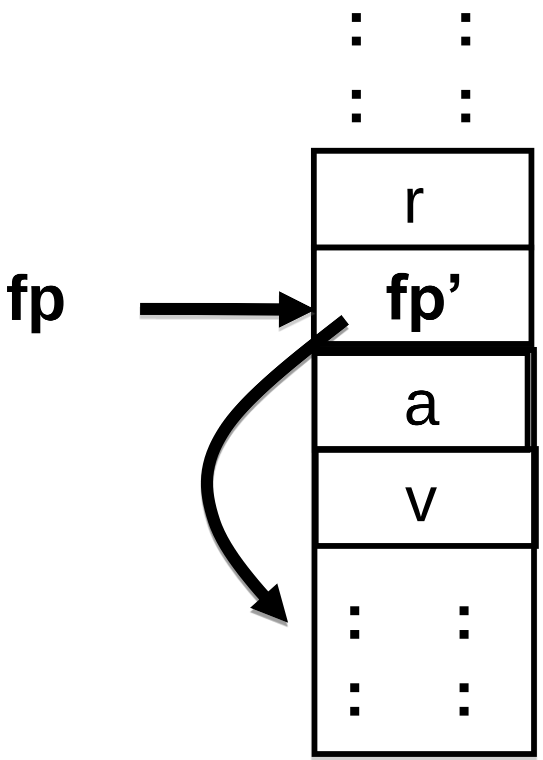
MK_CLOSURE(c, n)

c = code location of start of instructions for closure,
n = number of free variables in the body of closure.

Put values associated with **free variables** on stack,
then construct the closure on the heap



A stack frame



Return address

Saved frame pointer

Pointer to closure

Argument value

Stack frame. (Boundary May vary in the literature.)

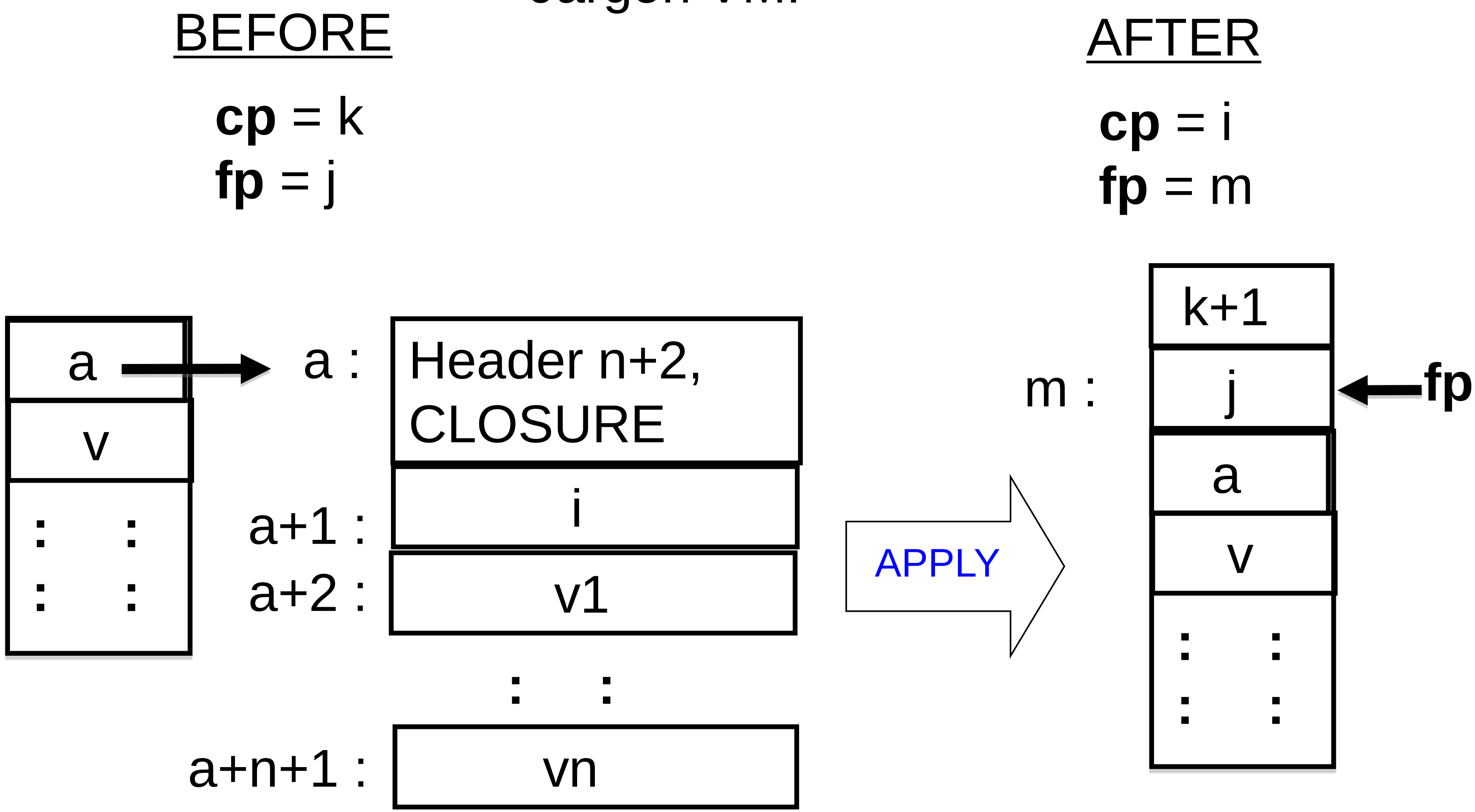
Currently executing code for the closure at heap address “a” after it was applied to argument v.

APPLY

Interpreter 3:

(APPLY, V(CLOSURE ((_, Some i), env)) :: (V v) :: evs)
-> (i, (V v) :: (EV env) :: (RA (cp + 1)) :: evs)

Jargon VM:

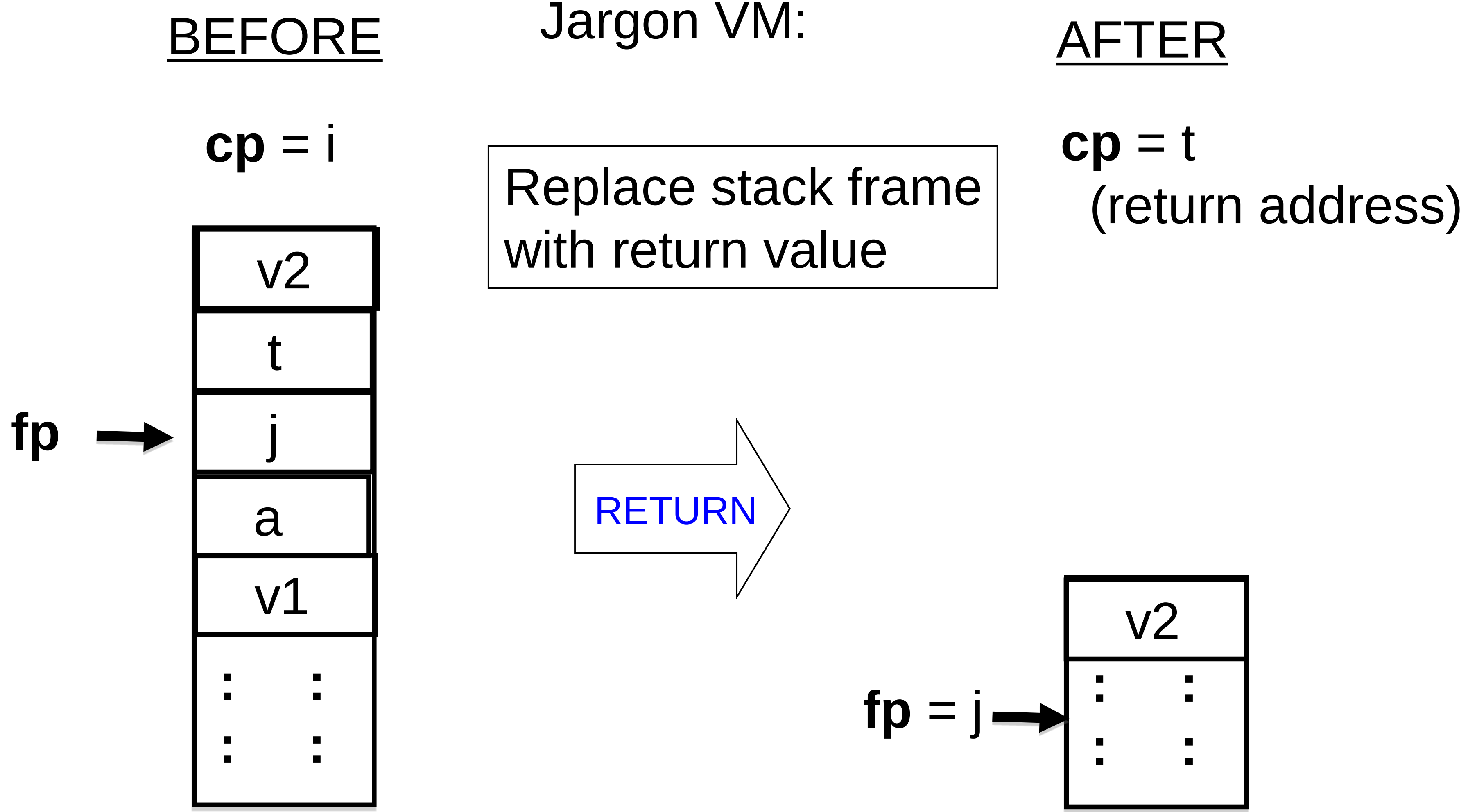


RETURN

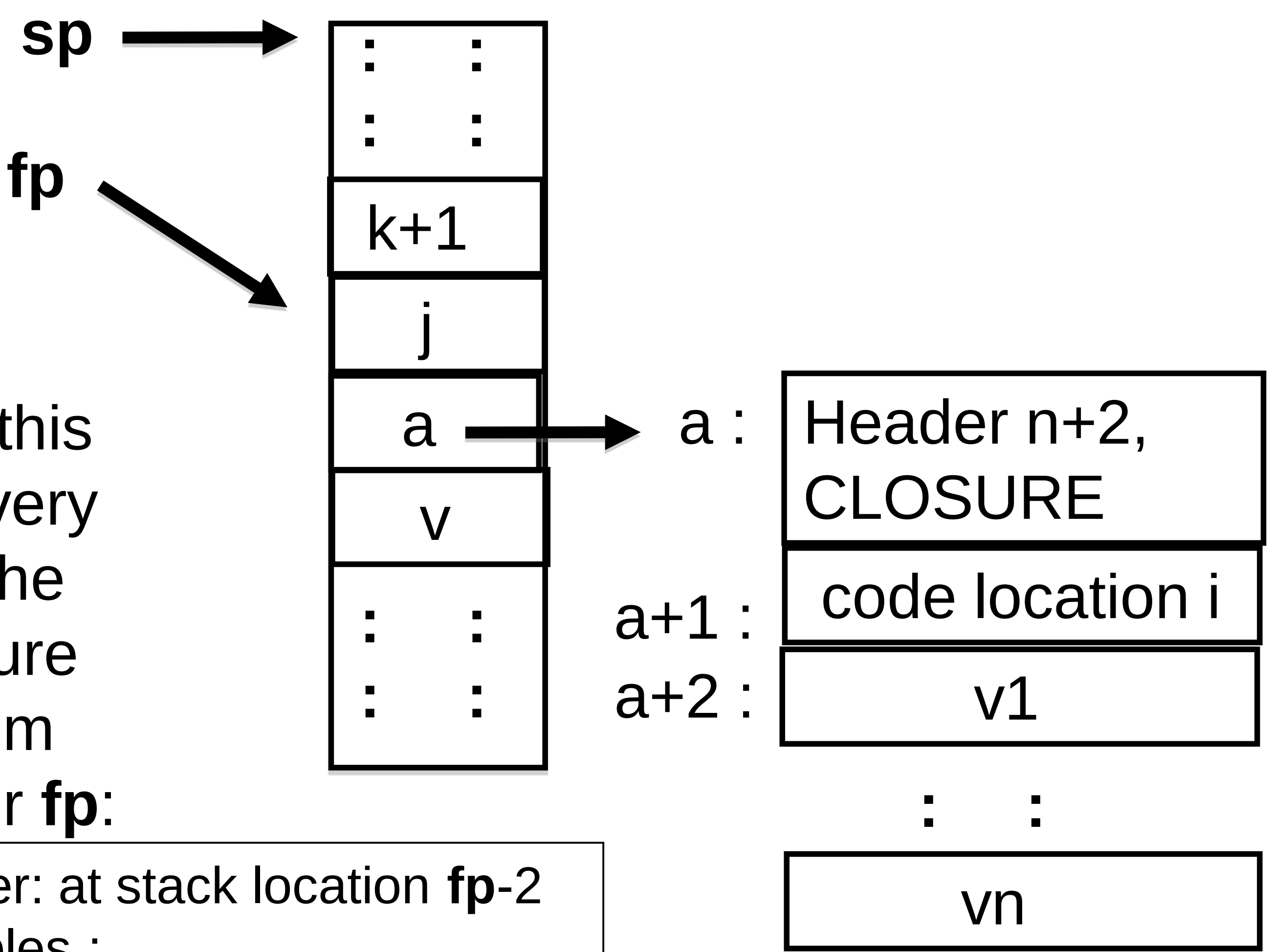
Interpreter 3:

(RETURN, (V v) :: _ :: (RA i) :: evs) -> (i, (V v) :: evs)

Jargon VM:



Finding a variable's value at runtime



Suppose we are executing code associated with this closure. Then every free variable in the body of the closure can be found from the frame pointer **fp**:

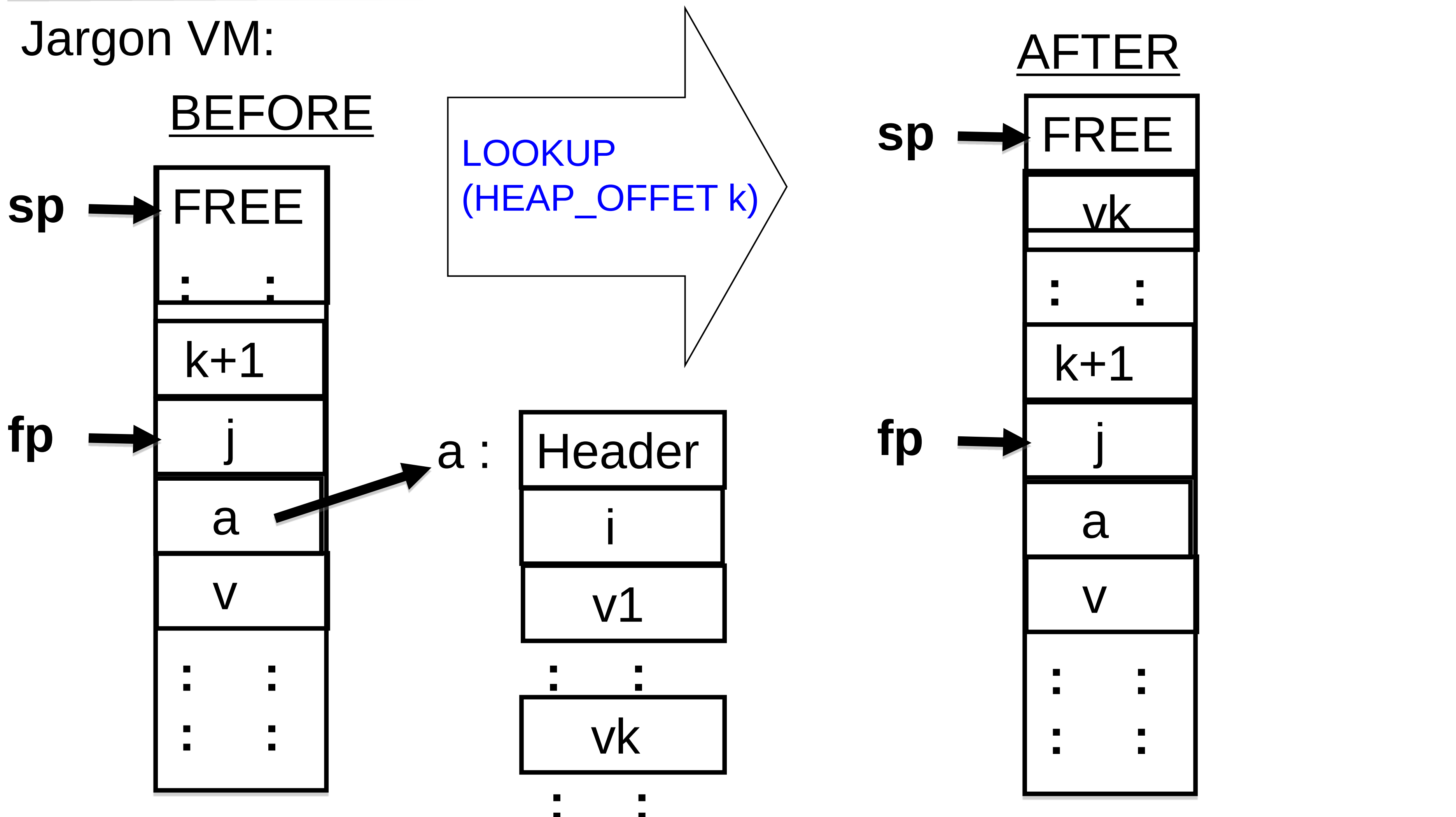
- Formal parameter: at stack location **fp**-2
- Other free variables :
  - Follow heap pointer found at **fp** -1
  - Each free variable can be associated with a fixed offset from this heap address

LOOKUP (HEAP_OFFSET k)

Interpreter 3:

(LOOKUP x, evs) -> (cp + 1, V(search(evs, x)) :: evs)

Jargon VM:



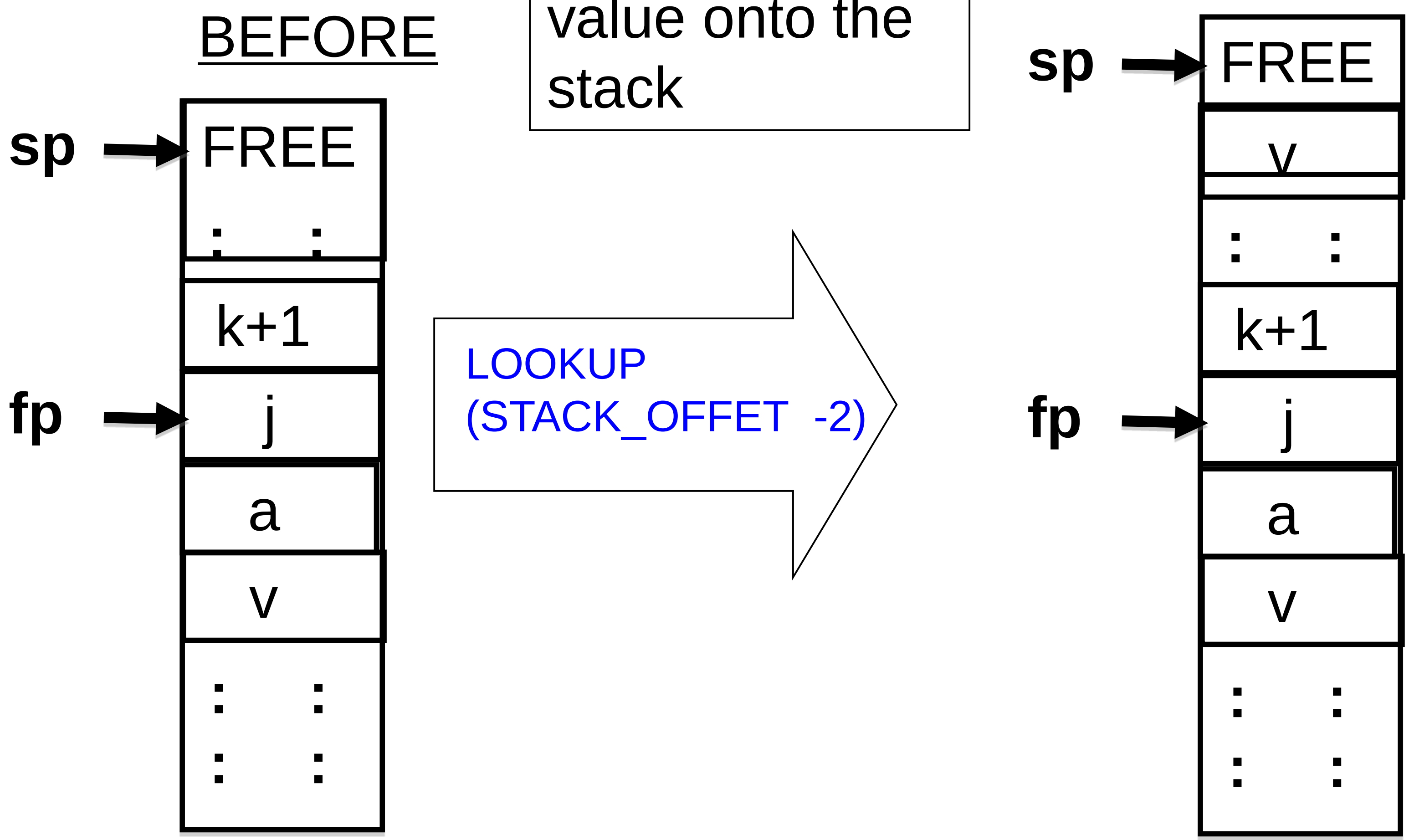
LOOKUP (STACK_OFFSET -2)

Interpreter 3:

(LOOKUP x, evs) -> (cp + 1, V(search(evs, x)) :: evs)

Jargon VM:

push argument value onto the stack



Oh, one problem

interpreter 3

```
let rec comp = function
 :
 | LetFun(f, (x, e1), e2) ->
          let (defs1, c1) = comp e1 in
          let (defs2, c2) = comp e2 in
          let def = [LABEL f; BIND x] @ c1 @ [SWAP; POP; RETURN] in
            (def @ defs1 @ defs2,
             [MK_CLOSURE((f, None)); BIND f] @ c2 @ [SWAP; POP])
 :
```

Problem: Code c2 can be anything --- how are we going to find the closure for f when we need it? It has to be a fixed offset from a frame pointer --- we no longer scan the stack for bindings!

Solution in Jargon VM

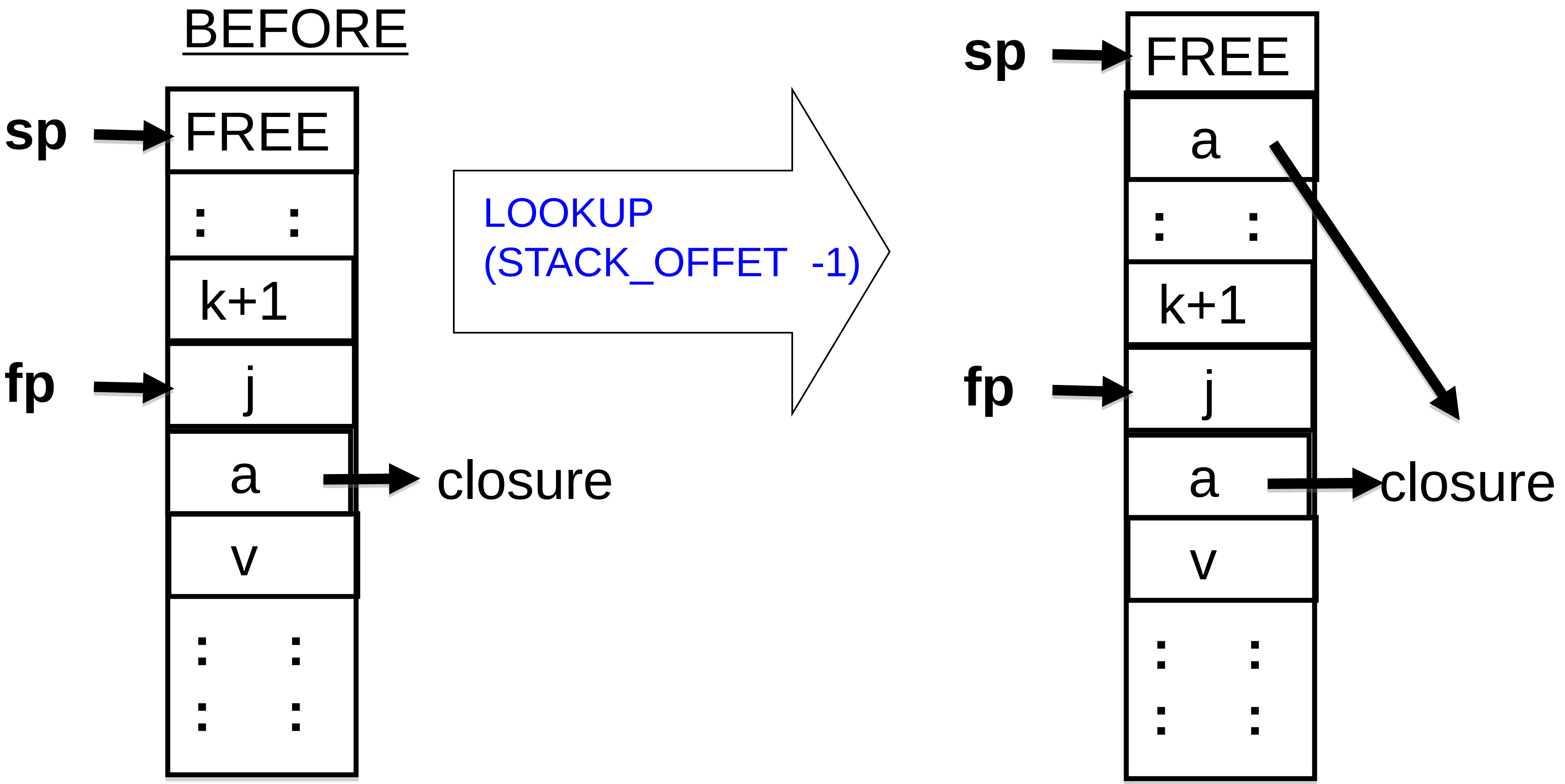
```
let rec comp vmap = function
 :
| LetFun(f, (x, e1), e2) -> comp vmap (App(Lambda(f, e2), Lambda(x, e1)))
:
```

Similar trick for LetRecFun

LOOKUP (STACK_OFFSET -1)

For recursive function calls,
push current closure on to the stack.

Jargon VM:



Example : Compiled code for rev_pair.slang

```
let rev_pair (p : int * int) : int * int  = (snd p, fst p)
in
    rev_pair (21, 17)
end
```

After the front-end, compile treats this as follows.

```
App(
   Lambda(
      ”rev_pair”,
       App(Var ”rev_pair”,  Pair (Integer 21, Integer 17))),
   Lambda(”p”, Pair(Snd (Var ”p”), Fst (Var ”p”))))
```

Example : Compiled code for rev_pair.slang

```
App(
   Lambda(”rev_pair”,
              App(Var ”rev_pair”,  Pair (Integer 21, Integer 17))),
   Lambda(”p”, Pair(Snd (Var ”p”), Fst (Var ”p”))))
```

“first lambda”

“second lambda”

MK_CLOSURE(L1, 0)	-- Make closure for second lambda
MK_CLOSURE(L0, 0)	-- Make closure for first lambda
APPLY	-- do application
HALT	-- the end!
L0 : PUSH STACK_INT 21	-- code for first lambda, push 21
PUSH STACK_INT 17	-- push 17
MK_PAIR	-- make the pair on the heap
LOOKUP STACK_LOCATION -2	-- push closure for second lambda on stack
APPLY	-- apply first lambda
RETURN	-- return from first lambda
L1 : LOOKUP STACK_LOCATION -2	-- code for second lambda, push arg on stack
SND	-- extract second part of pair
LOOKUP STACK_LOCATION -2	-- push arg on stack again
FST	-- extract first part of pair
MK_PAIR	-- construct a new pair
RETURN	-- return from second lambda

Example : trace of rev_pair.slang execution

```
Installed Code =
0: MK_CLOSURE(L1 = 11, 0)
1: MK_CLOSURE(L0 = 4, 0)
2: APPLY
3: HALT
4: LABEL L0
5: PUSH STACK_INT 21
6: PUSH STACK_INT 17
7: MK_PAIR
8: LOOKUP STACK_LOCATION-2
9: APPLY
10: RETURN
11: LABEL L1
12: LOOKUP STACK_LOCATION-2
13: SND
14: LOOKUP STACK_LOCATION-2
15: FST
16: MK_PAIR
17: RETURN
```

```
========== state 1 ==========
cp = 0 -> MK_CLOSURE(L1 = 11, 0)
fp = 0
Stack =
1: STACK_RA 0
0: STACK_FP 0

========== state 2 ==========
cp = 1 -> MK_CLOSURE(L0 = 4, 0)
fp = 0
Stack =
2: STACK_HI 0
1: STACK_RA 0
0: STACK_FP 0

Heap =
0 -> HEAP_HEADER(2, HT_CLOSURE)
1 -> HEAP_CI 11

……
```

Example : trace of rev_pair.slang execution

========== state 15 ==========
cp = 16 -> MK_PAIR
fp = 8
Stack =
11: STACK_INT 21
10: STACK_INT 17
9: STACK_RA 10
8: STACK_FP 4
7: STACK_HI 0
6: STACK_HI 4
5: STACK_RA 3
4: STACK_FP 0
3: STACK_HI 2
2: STACK_HI 0
1: STACK_RA 0
0: STACK_FP 0

Heap =
0 -> HEAP_HEADER(2, HT_CLOSURE)
1 -> HEAP_CI 11
2 -> HEAP_HEADER(2, HT_CLOSURE)
3 -> HEAP_CI 4
4 -> HEAP_HEADER(3, HT_PAIR)
5 -> HEAP_INT 21
6 -> HEAP_INT 17

========== state 19 ==========
cp = 3 -> HALT
fp = 0
Stack =
2: STACK_HI 7
1: STACK_RA 0
0: STACK_FP 0

Heap =
0 -> HEAP_HEADER(2, HT_CLOSURE)
1 -> HEAP_CI 11
2 -> HEAP_HEADER(2, HT_CLOSURE)
3 -> HEAP_CI 4
4 -> HEAP_HEADER(3, HT_PAIR)
5 -> HEAP_INT 21
6 -> HEAP_INT 17
7 -> HEAP_HEADER(3, HT_PAIR)
8 -> HEAP_INT 17
9 -> HEAP_INT 21

Jargon VM :
output> (17, 21)

Example : closure_add.slang

```
let f(y : int) : int -> int = let g(x :int) : int = y + x  in g end
in let add21 : int -> int  = f(21)
    in let add17 : int -> int  = f(17)
        in add17(3) + add21(10)
        end
   end
end
```

Note : we really do need closures on the heap here — the values 21 and 17 do not exist on the stack at this point in the execution.

After the front-end, this becomes represented as follows.

```
App(Lambda(f, App(Lambda(add21,
                 App(Lambda(add17,
                     Op(App(Var(add17), Integer(3)),
                      ADD,
                        App(Var(add21), Integer(10)))),
                       App(Var(f), Integer(17))),
                  App(Var(f), Integer(21)))),
     Lambda(y, App(Lambda(g, Var(g)),
Lambda(x, Op(Var(y), ADD, Var(x))))))
```

Can we make sense of this?

```
MK_CLOSURE(L3, 0)
MK_CLOSURE(L0, 0)
APPLY
HALT
L0 :        PUSH STACK_INT 21
LOOKUP STACK_LOCATION -2
APPLY
LOOKUP STACK_LOCATION -2
MK_CLOSURE(L1, 1)
APPLY
RETURN
L1 :        PUSH STACK_INT 17
LOOKUP HEAP_LOCATION 1
APPLY
LOOKUP STACK_LOCATION -2
MK_CLOSURE(L2, 1)
APPLY
RETURN
L2 :        PUSH STACK_INT 3
LOOKUP STACK_LOCATION -2
APPLY
PUSH STACK_INT 10
LOOKUP HEAP_LOCATION 1
APPLY
OPER ADD
RETURN
L3 :        LOOKUP STACK_LOCATION -2
MK_CLOSURE(L5, 1)
MK_CLOSURE(L4, 0)
APPLY
RETURN
L4 :        LOOKUP STACK_LOCATION -2
RETURN
L5 :        LOOKUP HEAP_LOCATION 1
LOOKUP STACK_LOCATION -2
OPER ADD
RETURN
```

The Gap, illustrated

fib.slang

```
let fib (m :int) : int  =
    if m = 0
    then 1
    else if m = 1
            then 1
            else fib(m - 1) + fib (m - 2)
            end
     end
in fib (?) end
```

slang.byte –c –i4 fib.slang

```
MK_CLOSURE(fib, 0)
MK_CLOSURE(L0, 0)
APPLY
HALT
L0 :        PUSH STACK_UNIT
UNARY READ
LOOKUP STACK_LOCATION -2
APPLY
RETURN
fib :       LOOKUP STACK_LOCATION -2
PUSH STACK_INT 0
OPER EQI
TEST L1
PUSH STACK_INT 1
GOTO L2
L1 :        LOOKUP STACK_LOCATION -2
PUSH STACK_INT 1
OPER EQI
TEST L3
PUSH STACK_INT 1
GOTO L4
L3 :        LOOKUP STACK_LOCATION -2
PUSH STACK_INT 1
OPER SUB
LOOKUP STACK_LOCATION -1
APPLY
LOOKUP STACK_LOCATION -2
PUSH STACK_INT 2
OPER SUB
LOOKUP STACK_LOCATION -1
APPLY
OPER ADD
L4 :
L2 :        RETURN
```

Jargon VM code

Taking stock

Starting from a direct implementation of Slang/L3 semantics, we have **DERIVED** a Virtual Machine in a step-by-step manner. The correctness of aach step is (more or less) easy to check.

Interpreter 0

Explicit stack via CPS+DFS → Interpreter 1

Split stack into two, refactor → Interpreter 2

Linearise code → Interpreter 3

Low-level addressable stack → Jargon VM

Remarks

1. The semantic GAP between a Slang/L3 program and a low-level translation (say x86/Unix) has been significantly reduced.
2. Implementing the Jargon VM at a lower-level of abstraction (in C?, JVM bytecodes? X86/Unix? …) looks like a <u>relatively</u> easy programming problem.
3. However, using a lower-level implementation (say x86, exploiting fast registers) to generate very efficient code is not so easy. See Part II Optimising Compilers.

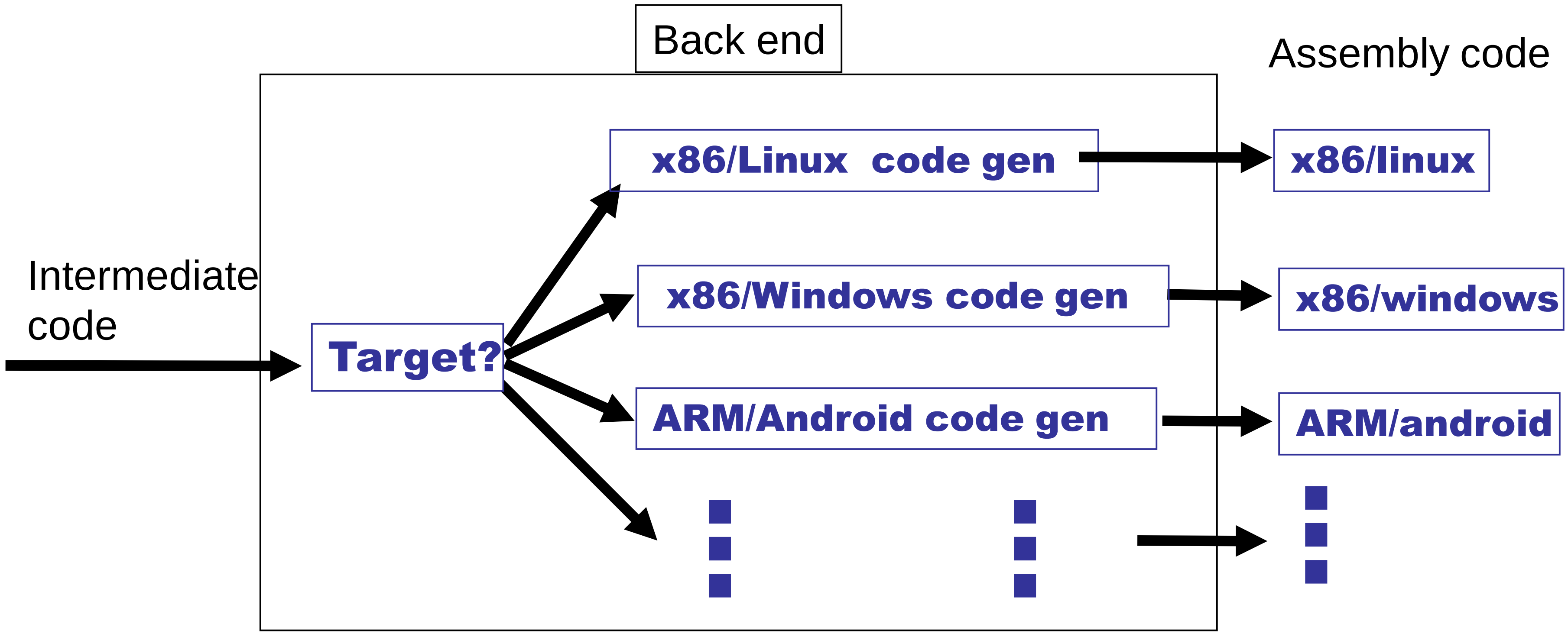
Verification of compilers is an active area of research. See CompCert, CakeML, and DeepSpec.

We could implement a Jargon byte code interpreter …

```
...
...
void vsm_execute_instruction(vsm_state *state, bytecode instruction)
{
  opcode code   = instruction.code;
  argument arg1 = instruction.arg1;
  switch (code) {
       case PUSH: { state->stack[state->sp++] = arg1; state->pc++; break; }
       case POP : { state->sp--; state->pc++; break; }
       case GOTO: { state->pc = arg1; break; }
       case STACK_LOOKUP: {
  state->stack[state->sp++] =
             state->stack[state->fp + arg1];
  state->pc++;  break; }

      ...
      ...
   }
}
...
...
```

- Generate compact byte code for each Jargon instruction.
- Compiler writes byte codes to a file.
- Implement an interpreter in C or C++ for these byte codes.
- Execution is much faster than our jargon.ml implementation.
- **Or, we could generate assembly code from Jargon instructions ….**

Backend could target multiple platforms

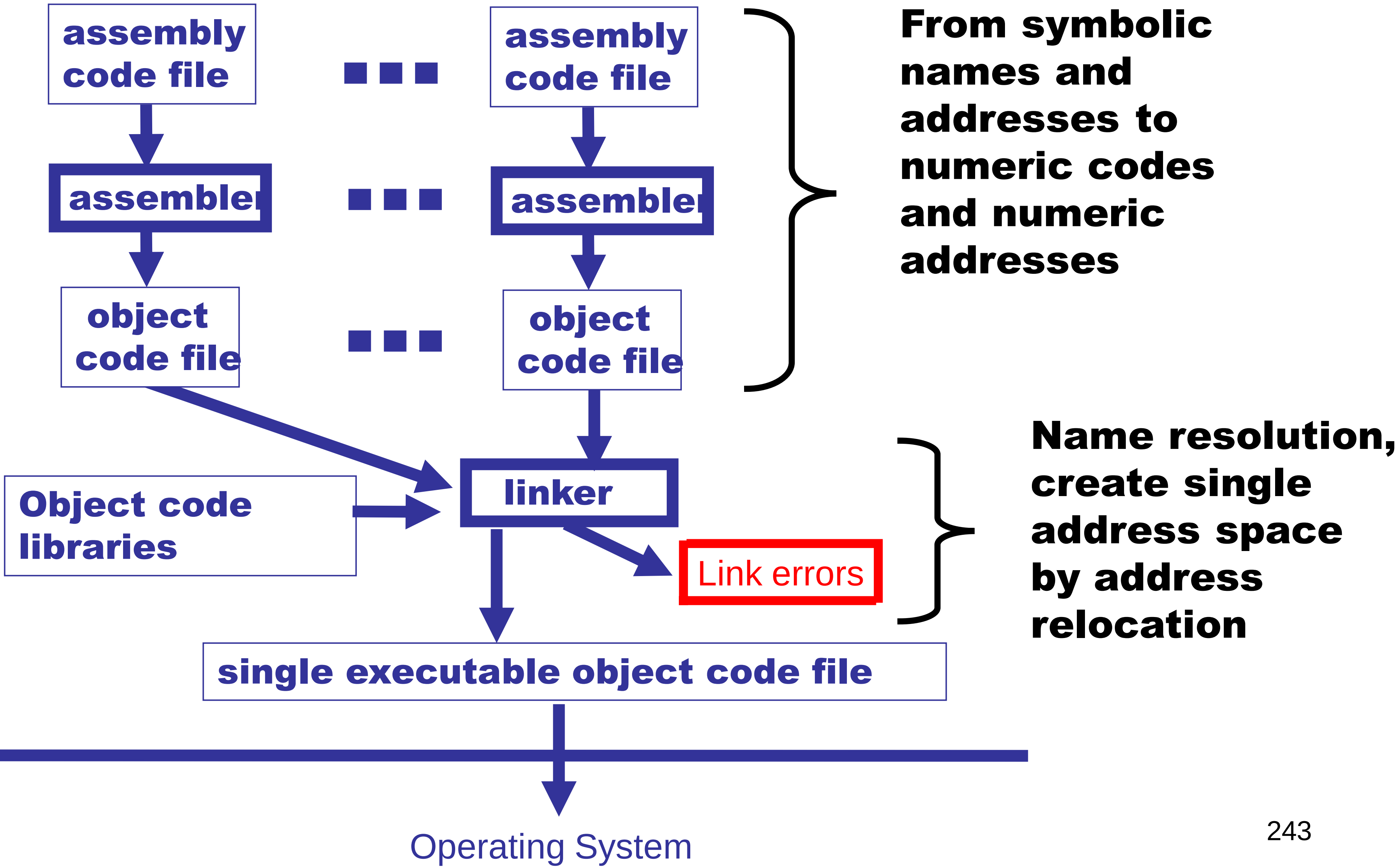


One of the great benefits of Virtual Machines is their portability. However, for more efficient code we may want to compile to assembler. Lost portability can be regained through the extra effort of implementing code generation for every desired target platform.

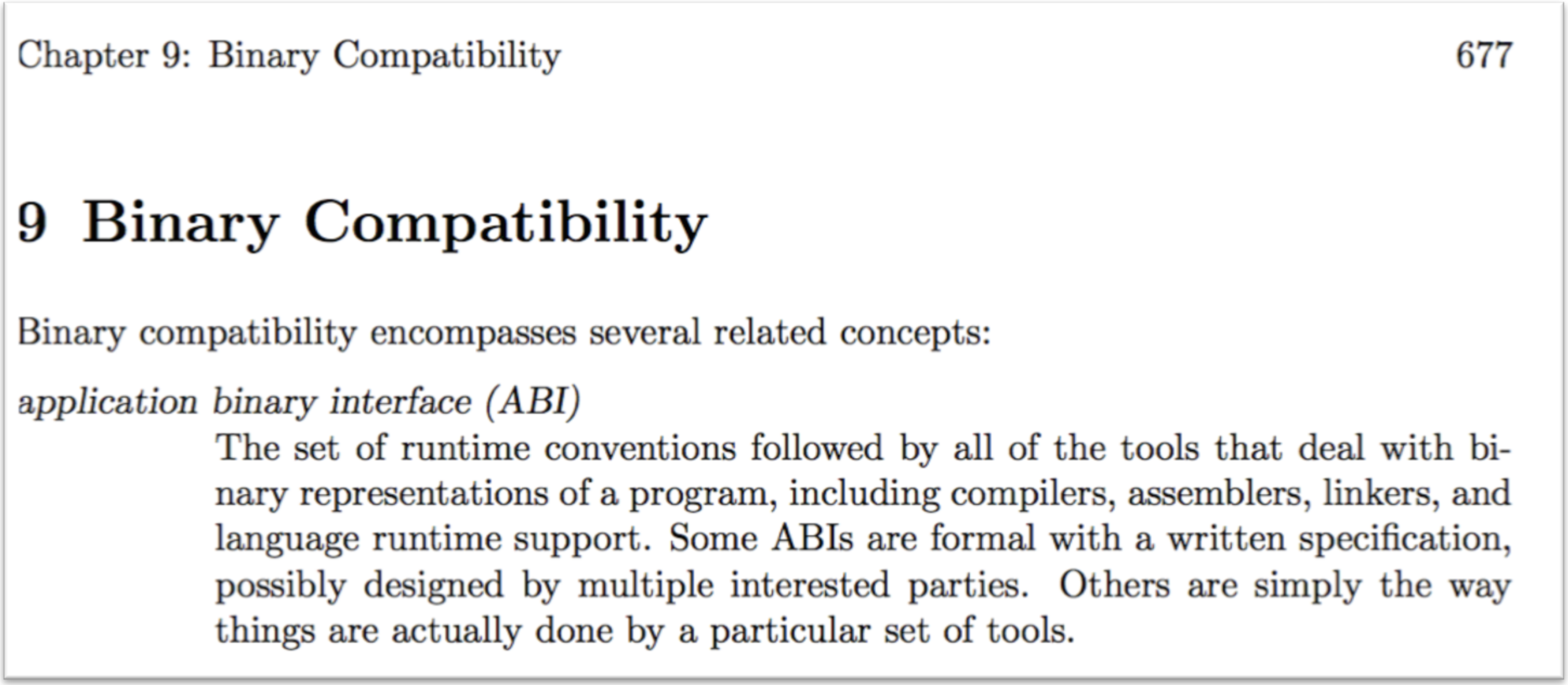
Lectures 12 --- 16
Assorted Topics

1. Separate compilation, linking
2. Interface with OS
3. Stacks vs registers
4. Calling conventions
5. Generating assembler code
6. Simple optimisations
7. The runtime system (automatic memory management, …)
8. Static links (for languages without nested functions/procedures)
9. Implementing OOP with inheritance
10. Implementing exceptions
11. Compiling a compiler, “boot strapping”

Assembly and Linking



The gcc manual (810 pages)
https://gcc.gnu.org/onlinedocs/gcc-5.3.0/gcc.pdf



9 Binary Compatibility

Binary compatibility encompasses several related concepts:

application binary interface (ABI)
: The set of runtime conventions followed by all of the tools that deal with binary representations of a program, including compilers, assemblers, linkers, and language runtime support. Some ABIs are formal with a written specification, possibly designed by multiple interested parties. Others are simply the way things are actually done by a particular set of tools.

Applications Binary Interface (ABI)

We will use x86/Unix as our running example.
Specifies many things, including the following.

- C calling conventions used for systems calls or calls to compiled C code.
  - Register usage and stack frame layout
  - How parameters are passed, results returned
  - Caller/callee responsibilities for placement and cleanup
- Byte-level layout and semantics of object files.
  - Executable and Linkable Format (ELF). Formerly known as Extensible Linking Format.
- Linking, loading, and name mangling

Note: the conventions are required for portable interaction with compiled C. Your compiled language does not have to follow the same conventions!

Object files

Must contain at least

- Program instructions
- Symbols being exported
- Symbols being imported
- Constants used in the program (such as strings)

Executable and Linkable Format (ELF) is a common format for both linker input and output.

ELF details (1)

Header information; positions and sizes of sections
`.text` segment (code segment): binary data
`.data` segment: binary data
`.rela.text` code segment relocation table: list of (offset,symbol) pairs giving: (*i*) offset within `.text` to be relocated; and (*ii*) by which symbol
`.rela.data` data segment relocation table: list of (offset,symbol) pairs giving: (*i*) offset within `.data` to be relocated; and (*ii*) by which symbol
...

ELF details (2)

. . .
`.symtab` symbol table: List of external symbols (as triples) used by the module. Each is (attribute, offset, symname) with attribute: 1. undef: externally defined, offset is ignored; 2. defined in code segment (with offset of definition); 3. defined in data segment (with offset of definition). Symbol names are given as offsets within `.strtab` to keep table entries of the same size.
`.strtab` string table: the string form of all external names used in the module

The (Static) Linker

What does a linker do?

- takes some object files as input, notes all undefined symbols.
- recursively searches libraries adding ELF files which define such symbols until all names defined (“library search”).
- whinges if any symbol is undefined or multiply defined.

Then what?

- concatenates all code segments (forming the output code segment).
- concatenates all data segments.
- performs relocations (updates code/data segments at specified offsets.

Dynamic vs. Static linking

Static linking (compile time)

Problem: a simple “hello world” program may give a 10MB executable if it refers to a big graphics or other library.

Dynamic linking (run time)

For shared libraries, the object files contain stubs, not code, and the operating system loads and links the code on demand.

Pros and Cons of dynamic linking:

(+) Executables are smaller
(+) Bug fixes to libraries don’t require re-linking.
(-) Non-compatible changes to a library can wreck previously working programs (“dependency hell”).

A “runtime system”

A library implementing functionality needed to run compiled code on a given operating system. Normally tailored to the language being compiled.

- Implements interface between OS and language.
- May implement memory management.
- May implement “foreign function” interface (say we want to call compiled C code from Slang code, or vice versa).
- May include efficient implementations of primitive operations defined in the compiled language.
- For some languages, the runtime system may perform runtime type checking, method lookup, security checks, and so on.
- …

Runtime system

Targeting a VM

Generated code

Virtual Machine

Implementation
Includes runtime system

Targeting a platform

Generated code

Run-time system

Linker

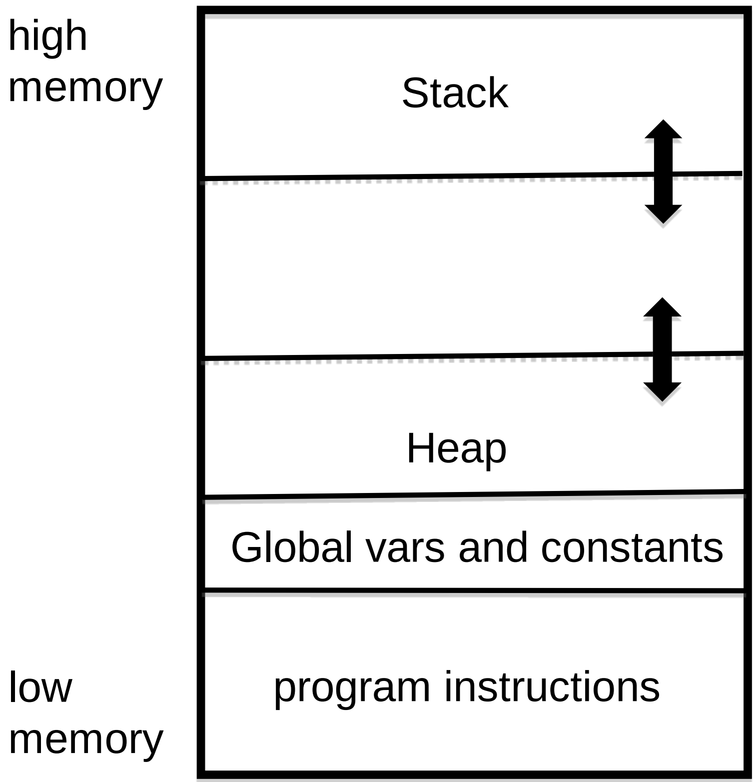
Executable

In either case, implementers of the compiler and the runtime system must agree on many low-level details of memory layout and data representation.

Typical (Low-Level) Memory Layout (UNIX)

Rough schematic of traditional layout in (virtual) memory.

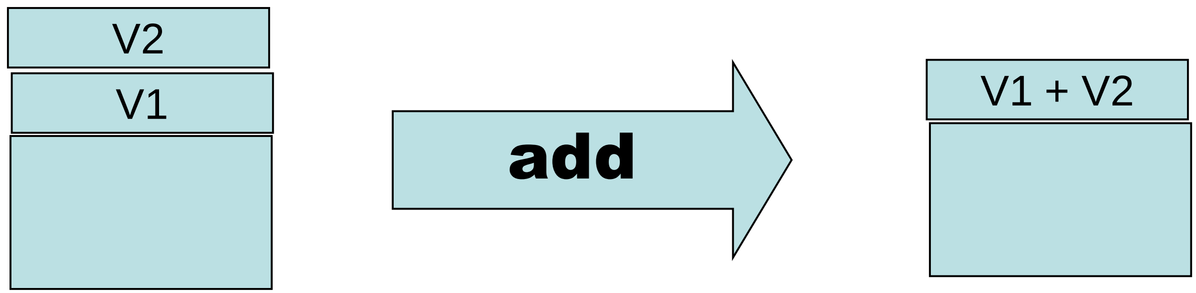
Dealing with Virtual Machines allows us to ignore some of the low-level details….

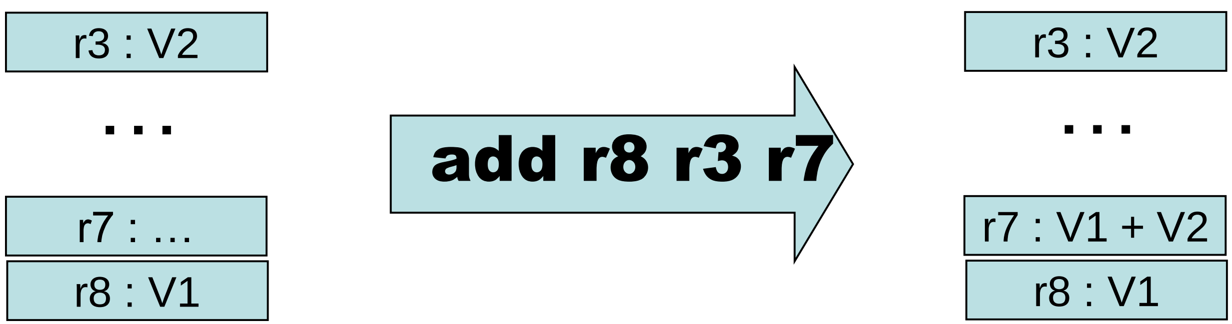


The heap is used for dynamically allocating memory. Typically either for very large objects or for those objects that are returned by functions/procedures and must outlive the associated activation record.

In languages like Java and ML, the heap is managed automatically ("garbage collection")

Stack vs regsisters





Stack-oriented:
(+) argument locations is implicit, so instructions are smaller.
(---) Execution is slower

Register-oriented:
(+++) Execution MUCH faster
(-) argument location is explicit, so instructions are larger

Main dilemma : registers are fast, but are fixed in number. And that number is rather small.

- Manipulating the stack involves RAM access, which can be orders of magnitude slower than register access (the "von Neumann Bottleneck")
- Fast registers are (today) a scarce resource, shared by many code fragments
- How can registers be used most effectively?
  - Requires a careful examination of a program's structure
  - Analysis phase: building data structures (typically directed graphs) that capture definition/use relationships
  - Transformation phase : using this information to rewrite code, attempting to most efficiently utilise registers
  - Problem is NP-complete
  - One of the central topics of Part II Optimising Compilers.
- Here we focus <u>only</u> on general issues : <u>calling conventions</u> and <u>register spilling</u>

Caller/callee conventions

- Caller and callee code may use overlapping sets of registers
- An agreement is needed concerning use of registers
  - Are some arguments passed in specific registers?
  - Is the result returned in a specific register?
  - If the caller and callee are both using a set of registers for "scratch space" then caller or callee must save and restore these registers so that the caller's registers are not obliterated by the callee.
- Standard calling conventions identify specific subsets of registers as "caller saved" or "callee saved"
  - **Caller saved**: if caller cares about the value in a register, then must save it before making any call
  - **Callee saved**: The caller can be assured that the callee will leave the register intact (perhaps by saving and restoring it)

Another C example.
X86, 64 bit, with gcc

```
int
callee(int, int,int,
        int,int,int,int);

int caller(void)
{
  int ret;
  ret =
    callee(1,2,3,4,5,6,7);
  ret += 5;
  return ret;
}
```

```
        _caller:
pushq       %rbp   # save frame pointer
movq %rsp, %rbp  # set new frame pointer
subq $16, %rsp   # make room on stack
movl $7, (%rsp)  # put 7th arg on stack
movl $1, %edi    # put 1st arg on in edi
movl $2, %esi    # put 2nd arg on in esi
movl $3, %edx    # put 3rd arg on in edx
movl $4, %ecx    # put 4th arg on in ecx
movl $5, %r8d    # put 5th arg on in r8d
movl $6, %r9d    # put 6th arg on in r9d
callq _callee    #will put resut in eax
addl $5, %eax    # add 5
addq $16, %rsp   # adjust stack
popq %rbp        # restore  frame pointer
ret              # pop return address, go there
```

Regsiter spilling

- What happens when all registers are in use?
- Could use the stack for scratch space …
- … or (1) move some register values to the stack, (2) use the registers for computation, (3) restore the registers to their original value
- This is called <u>register spilling</u>

A Crash Course in x86 assembler

- A CISC architecture
- There are 16, 32 and 64 bit versions
- 32 bit version :
  - General purpose registers : EAX EBX ECX EDX
  - Special purpose registers : ESI EDI EBP EIP ESP
    - EBP : normally used as the frame pointer
    - ESP : normally used as the stack pointer
    - EDI : often used to pass (first) argument
    - EIP : the code pointer
  - Segment and flag registers that we will ignore …
- 64 bit version:
  - Rename 32-bit registers with “R” (RAX, RBX, RCX, …)
  - More general registers: R8 R9 R10 R11 R12 R13 R14 R15

Register names can indicate “width” of a value.

rax : 64 bit version
eax : 32 bit version (or lower 32 bits of **rax**)
ax : 16 bit version (or lower 16 bits of **eax**)
al : lower 8 bits of ax
ah : upper 8 bits of ax

See https://en.wikibooks.org/wiki/X86_Assembly

The syntax of x86 assembler comes in several flavours.
Here are two examples of "put integer 4 into register eax":

```
movl $4, %eax          // GAS (aka AT&T) notation
mov  eax, 4             // Intel notation
```

I will (mostly) use the GAS syntax, where a suffix is used to indicate width of arguments:

- b (byte) = 8 bits
- w (word) = 16 bits
- l (long) = 32 bits
- q (quad) = 64 bits

For example, we have movb, movw movl, and movq.

Examples (in GAS notation)

```
movl $4, %eax     # put 32 bit integer 4 in register eax
movw $4, %eax    # put 16 bit integer 4 in lower 16 bits of eax
movb $4, %eax    # put 8 bit integer 4 in lowest 8 bits of eax
movl      %esp, %ebp    # put the contents of esp into ebp
movl      (%esp), %ebp  # interpret contents of esp as a memory
              # address. Copy the value at that address
              # into register ebp
movl      %esp, (%ebp)  # interpret contents of ebp as a memory
              # address. Copy the value in esp to
              # that address.
movl      %esp, 4(%ebp)# interpret contents of ebp as a memory
             # address. Add 4 to that address. Copy
             # the value in esp to this new address.
```

A few more examples

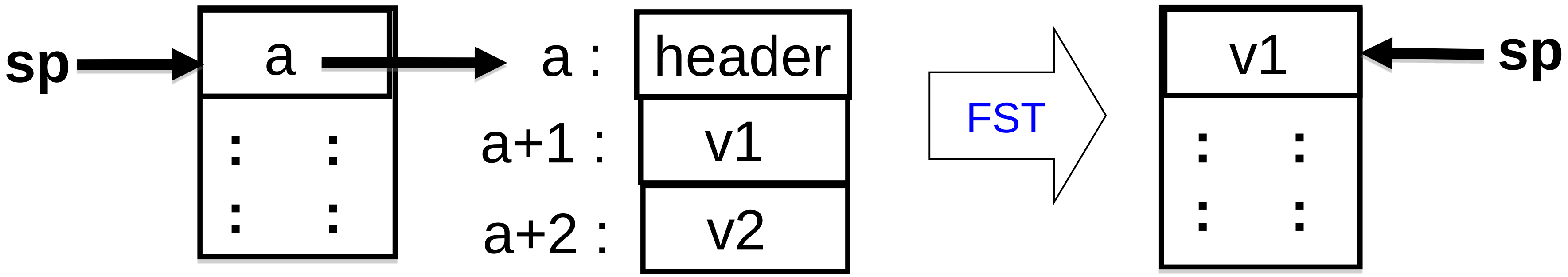
```
call label  # push return address on stack and jump to label
ret         # pop return address off stack and jump there
            # NOTE: managing other bits of the stack frame
            # such as stack and frame pointer must be done
            # explicitly
subl $4, %esp   # subtract 4 from esp. That is, adjust the
                # stack pointer to make room for one 32-bit
                # (4 byte) value. (stack grows downward!)
```

Assume that we have implemented a procedure in C called allocate that will manage heap memory. We will compile and link this in with code generated by the slang compiler. At the x86 level, allocate will expect a header in **edi** and return a heap pointer in **eax**.

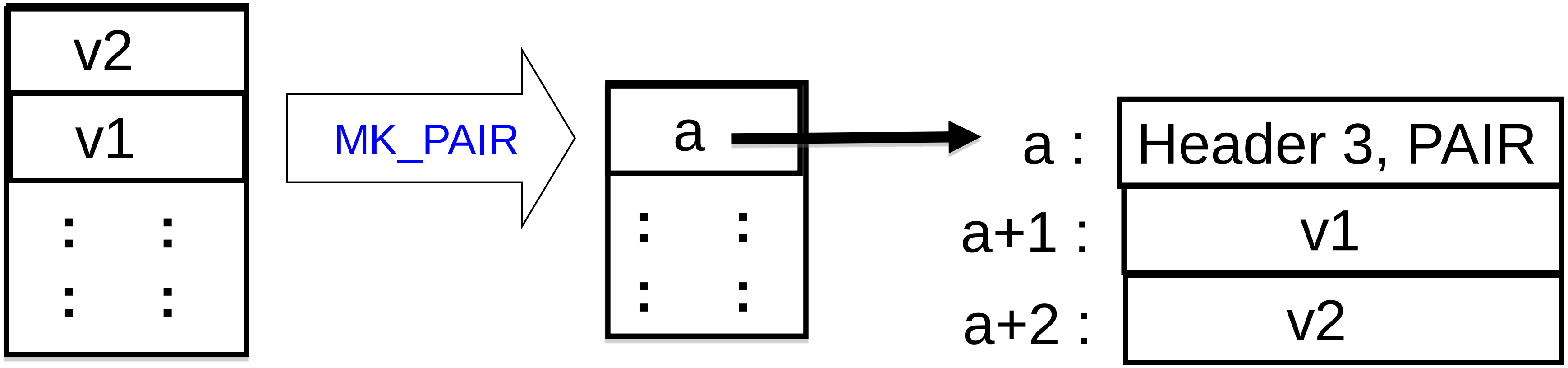
Some Jargon VM instructions are "easy" to translate

Remember: X86 is CISC, so RISC architectures may require more instructions …

GOTO loc	jmp loc
POP	addl $4, %esp // move stack pointer 1 word = 4 bytes
PUSH v	subl $4, %esp // make room on top of stack movl $i, (%esp) // where i is an integer representing v
FST	movl (%esp), %edx //store "a" into edx movl 4(%edx), %edx // load v1, 4 bytes, 1 word, after header movl %edx, (%esp) // replace "a" with "v1" at top of stack
SND	movl (%esp), %edx //store "a" into edx movl 8(%edx), %edx // vload v2, 8 bytes, 2 words, after header movl %edx, (%esp) // replace "a" with "v2" at top of stack



… while others require more work



One possible x86 (32 bit) implementation of MK_PAIR:

```
movl $3, %edi              // construct header in edi
shr $16, %edi,             // … put size in upper 16 bits (shift right)
movw $PAIR, %di            // … put type in lower 16 bits of edi
call allocate              // input: header in ebi, output: “a” in eax
movl (%esp), %edx          // move “v2” to the heap,
movl %edx, 8(%eax)         //  …  using temporary register edx
addl $4, %esp              // adjust stack pointer (pop “v2”)
movl (%esp), %edx          // move “v1” to the heap
movl %edx, 4(%eax)         //  …  using temporary register edx
movl %eax, (%esp)          // copy value “a” to top of stack
```

call function computed at runtime?

For things you don't understand, just experiment!
OK, you need to pull an address out of a closure and call it. Hmm, how does something similar get compiled from C?

```
int func ( int (*f)(int) ) { return (*f)(17); } /* pass a function pointer and apply it /*
```

```
 _func:
pushq   %rbp               # save frame pointer
movq    %rsp, %rbp         # set frame pointer to stack pointer
subq    $16, %rsp          # make some room on stack
movl    $17, %eax          # put 17 in argument register eax
movq    %rdi, -8(%rbp)     # rdi contains the argument f
movl    %eax, %edi         # put 17 in register edi, so f will get it
callq   *-8(%rbp)          # WOW, a computed address for call!
addq    $16, %rsp          # restore stack pointer
popq    %rbp               # restore old frame pointer
ret                        # restore stack
```

X86,
64 bit

without
–O2

What about arithmetic?

Houston, we have a problem….

- It may not be obvious now, but if we want to have automated memory management we need to be able to distinguish between values (say integers) and pointers at runtime.
- Have you ever noticed that integers in SML or Ocaml are either 31 (or 63) bits rather than the native 32 (or 64) bits?
  - That is because these compilers use a the least significant bit to distinguish integers (bit = 1) from pointers (bit = 0).
  - OK, this works. But it may complicate every arithmetic operation!
  - This is another exercise left for you to ponder …

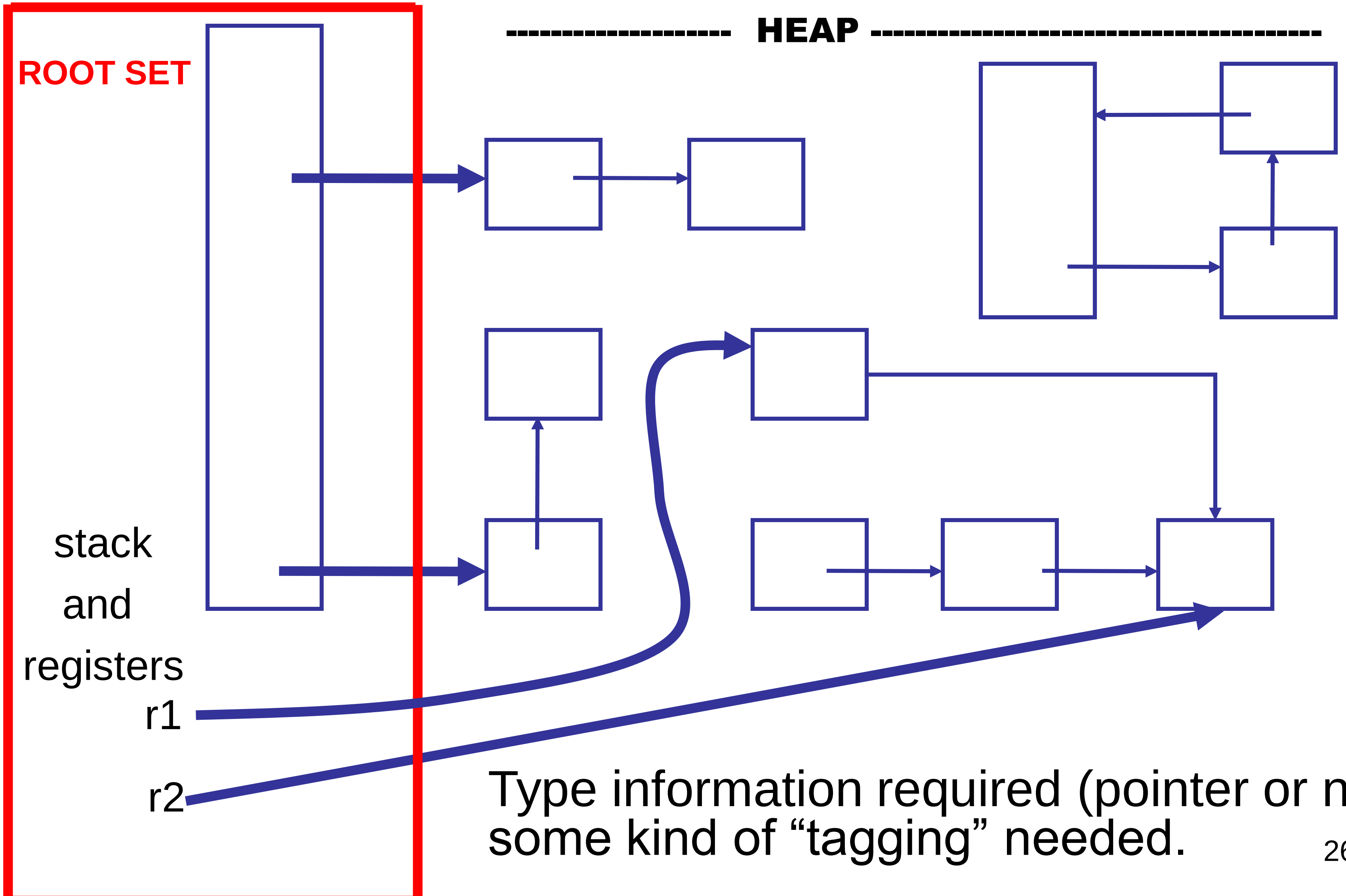
New topic: Memory Management

- Many programming languages allow programmers to (implicitly) allocate new storage dynamically, with no need to worry about reclaiming space no longer used.
  - New records, arrays, tuples, objects, closures, etc.
  - Java, SML, OCaml, Python, JavaScript, Python, Ruby, Go, Swift, SmallTalk, …
- Memory could easily be exhausted without some method of reclaiming and recycling the storage that will no longer be used.
  - Often called “garbage collection”
  - Is really “automated memory management” since it deals with allocation, de-allocation, compaction, and memory-related interactions with the OS.

Explicit (manual) memory management

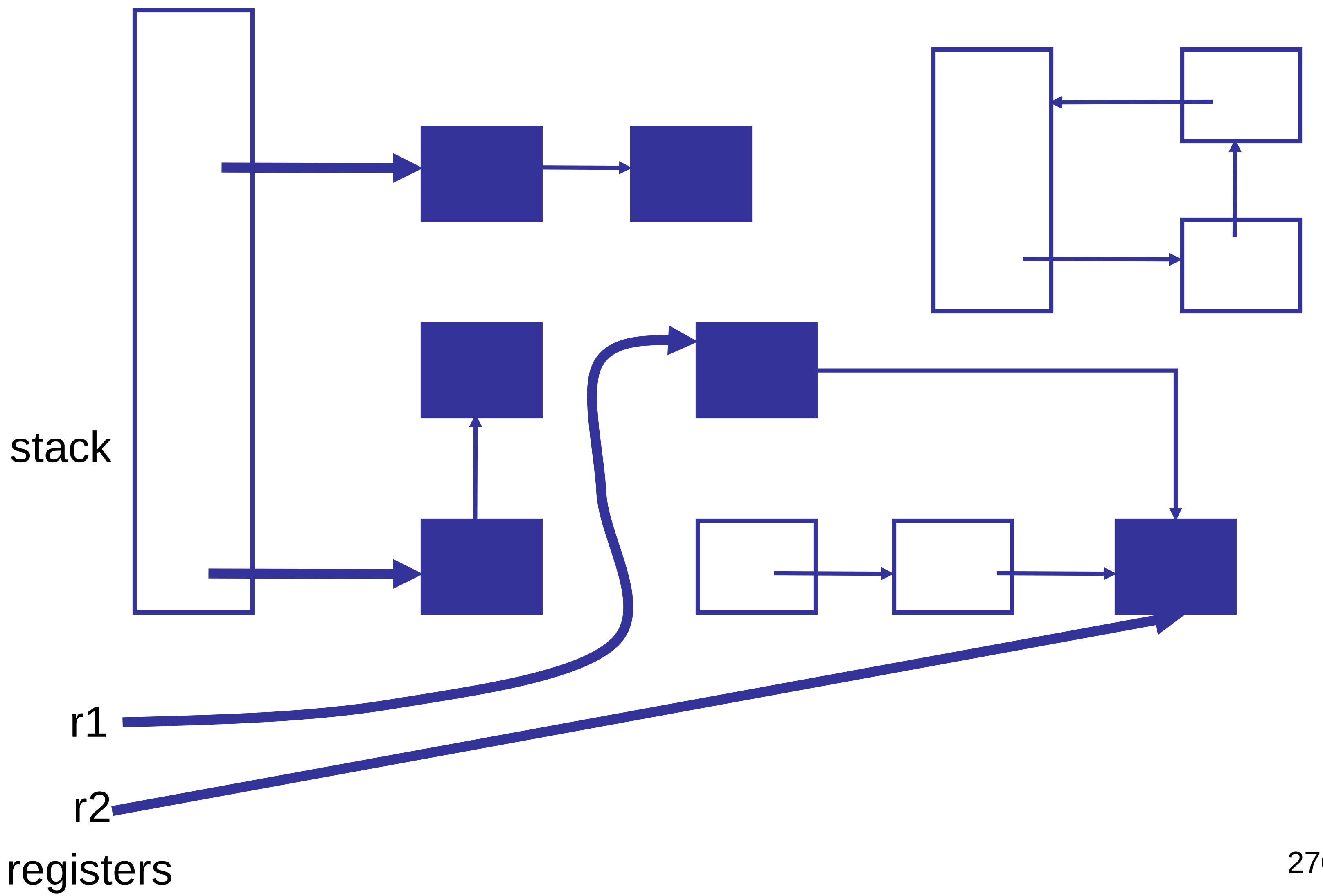
- User library manages memory; programmer decides when and where to allocate and de-allocate
  - void* malloc(long n)
  - void free(void *addr)
  - Library calls OS for more pages when necessary
  - Advantage: Gives programmer a lot of control.
  - Disadvantage: people too clever and make mistakes. Getting it right can be costly. And don't we want to automate-away tedium?
  - Advantage: With these procedures we can implement memory management for "higher level" languages ;-)

Automation is based on an approximation : if data can be reached from a root set, then it is not “garbage”

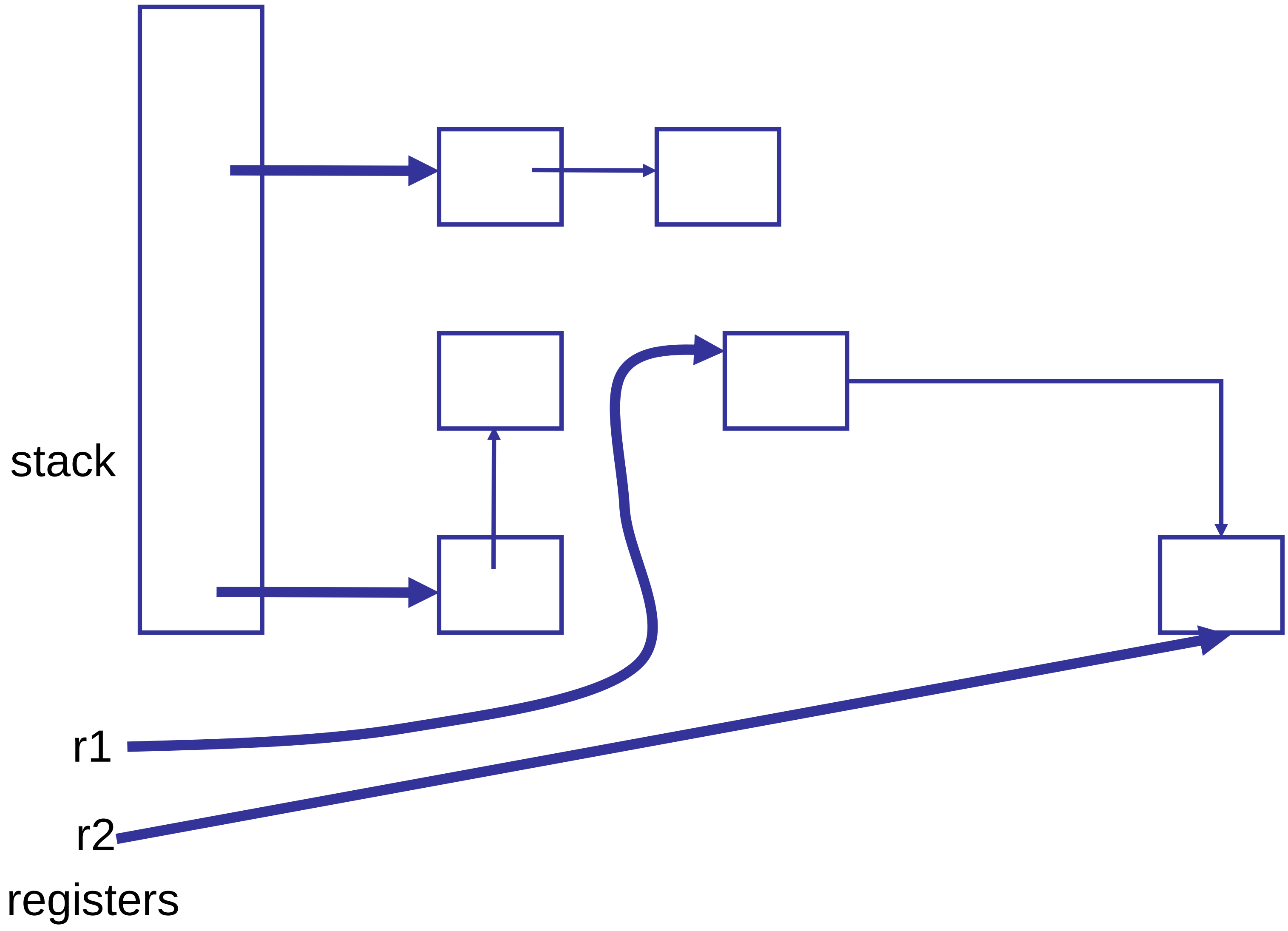


Type information required (pointer or not), some kind of “tagging” needed.

… Identify Cells Reachable From Root Set…



... reclaim unreachable cells



But How? Two basic techniques, and many variations

- **Reference counting** : Keep a reference count with each object that represents the number of pointers to it. Is garbage when count is 0.
- **Tracing** : find all objects reachable from root set. Basically transitive close of pointer graph.

For a very interesting (non-examinable) treatment of this subject see

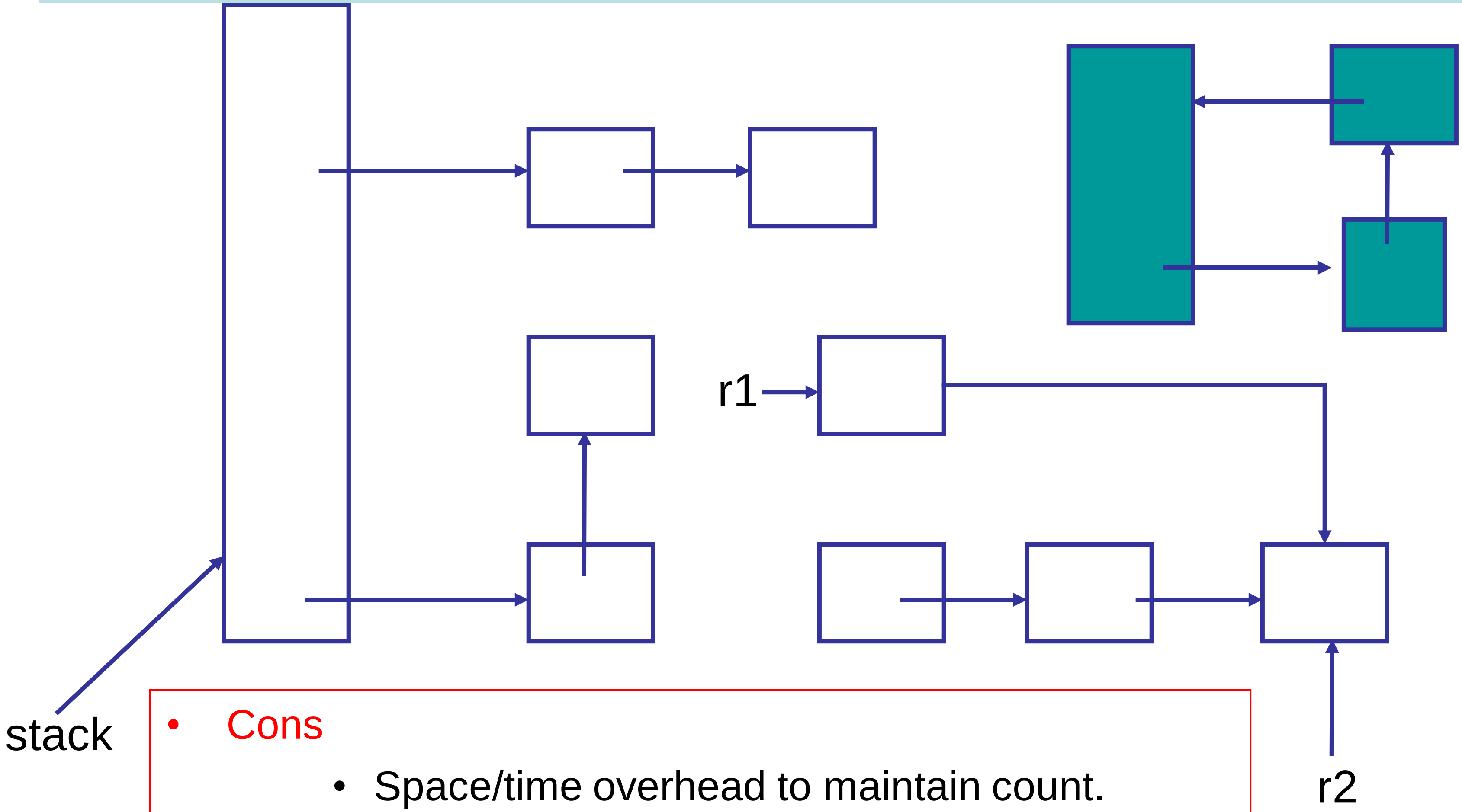
> **A Unified Theory of Garbage Collection**.
> David F. Bacon, Perry Cheng, V.T. Rajan.
> OOPSLA 2004.

In that paper reference counting and tracing are presented as “dual” approaches, and other techniques are hybrids of the two.

Reference Counting, basic idea:

- Keep track of the number of pointers to each object (the reference count).
- When Object is created, set count to 1.
- Every time a new pointer to the object is created, increment the count.
- Every time an existing pointer to an object is destroyed, decrement the count
- When the reference count goes to 0, the object is unreachable garbage

Reference counting can't detect cycles!



- Cons
  - Space/time overhead to maintain count.
  - Memory leakage when have cycles in data.
- Pros
  - <u>Incremental</u> (no long pauses to collect…)

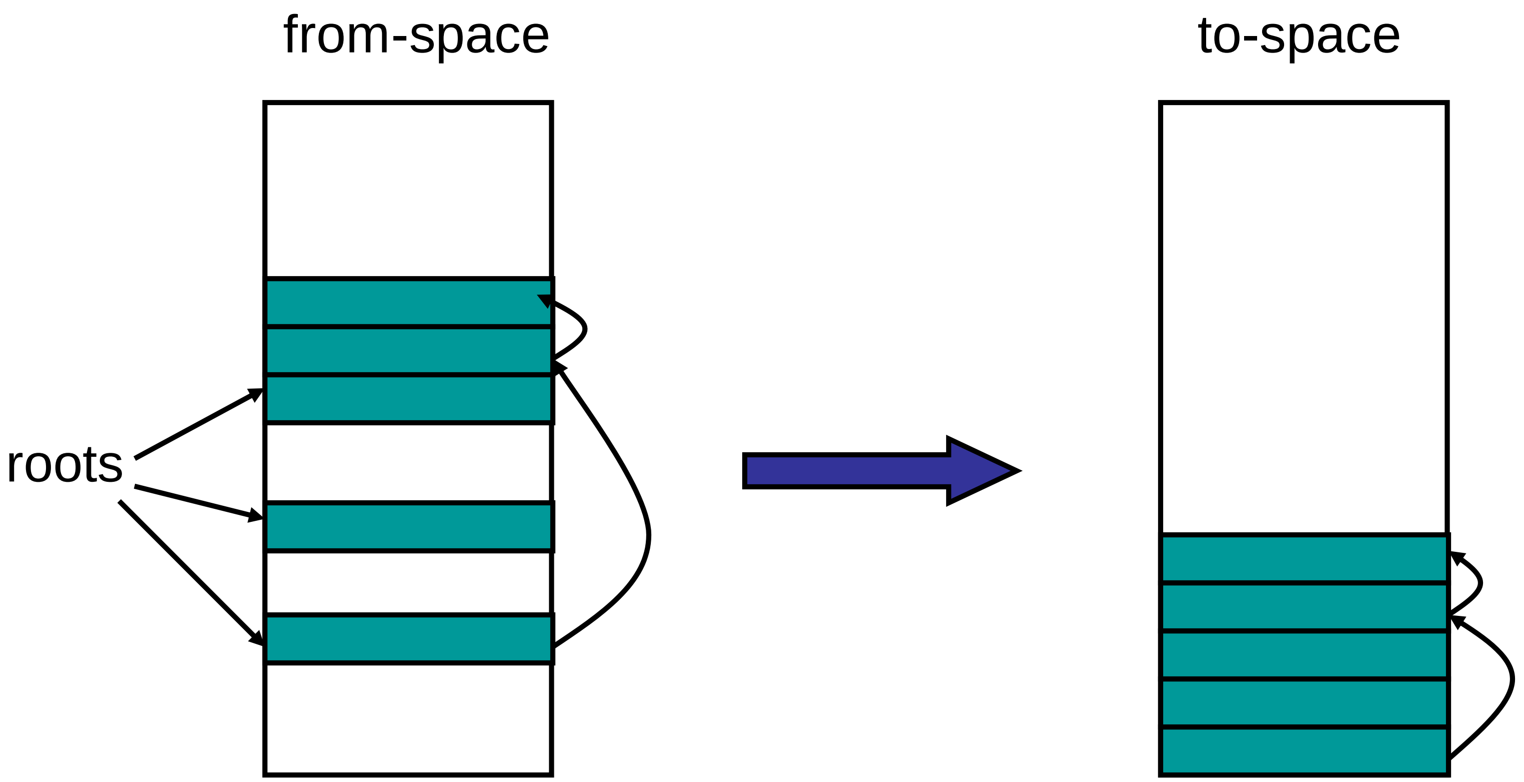
Mark and Sweep

- A two-phase algorithm
  - Mark phase: Depth first traversal of object graph from the roots to mark live data
  - Sweep phase: iterate over entire heap, adding the unmarked data back onto the free list

Copying Collection

- Basic idea: use 2 heaps
  - One used by program
  - The other unused until GC time
- GC:
  - Start at the roots & traverse the reachable data
  - Copy reachable data from the active heap (from-space) to the other heap (to-space)
  - Dead objects are left behind in from space
  - Heaps switch roles

Copying Collection



Copying GC

- Pros
  - Simple & collects cycles
  - Run-time proportional to # live objects
  - Automatic compaction eliminates fragmentation
- Cons
  - Twice as much memory used as program requires
    - Usually, we anticipate live data will only be a small fragment of store
    - Allocate until 70% full
    - From-space = 70% heap; to-space = 30%
  - Long GC pauses = bad for interactive, real-time apps

OBSERVATION: for a copying garbage collector

- 80% to 98% new objects die very quickly.
- An object that has survived several collections has a bigger chance to become a long-lived one.
- It's a inefficient that long-lived objects be copied over and over.

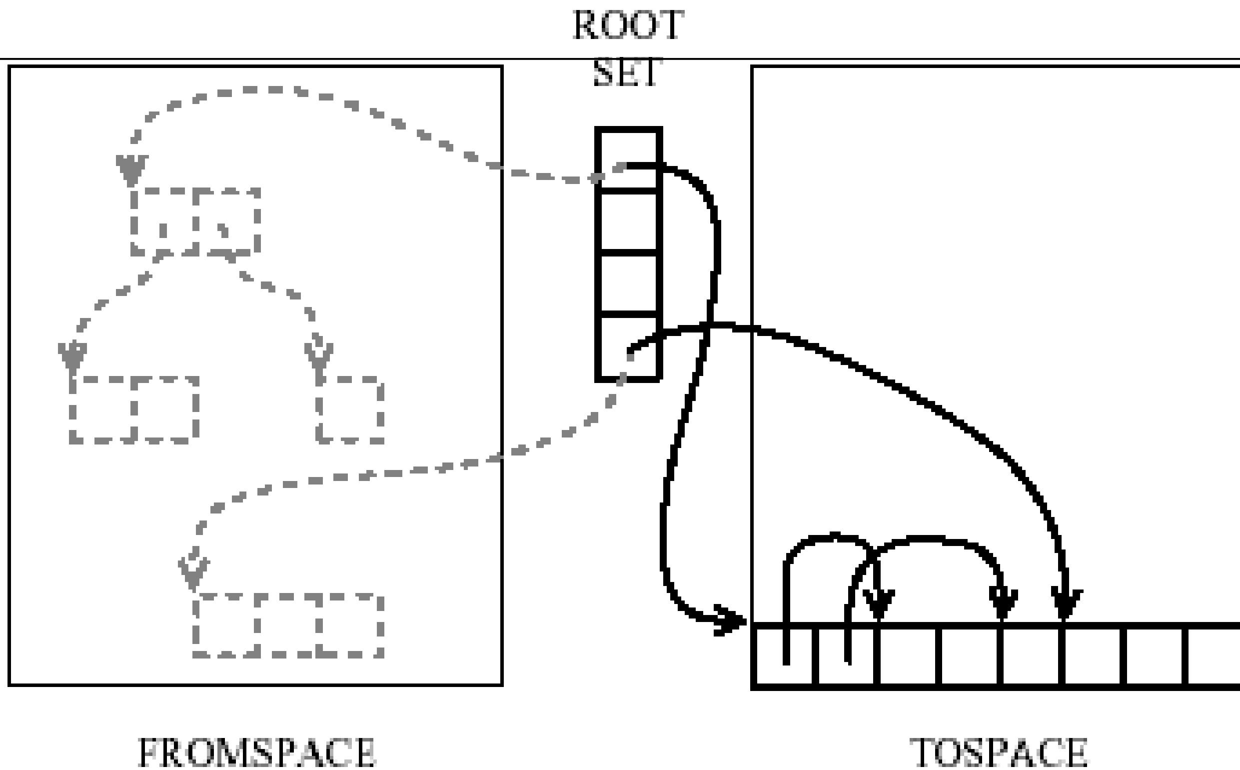


Diagram from Andrew Appel's **Modern Compiler Implementation**

IDEA: Generational garbage collection

Segregate objects into multiple areas by age, and collect areas containing older objects less often than the younger ones.

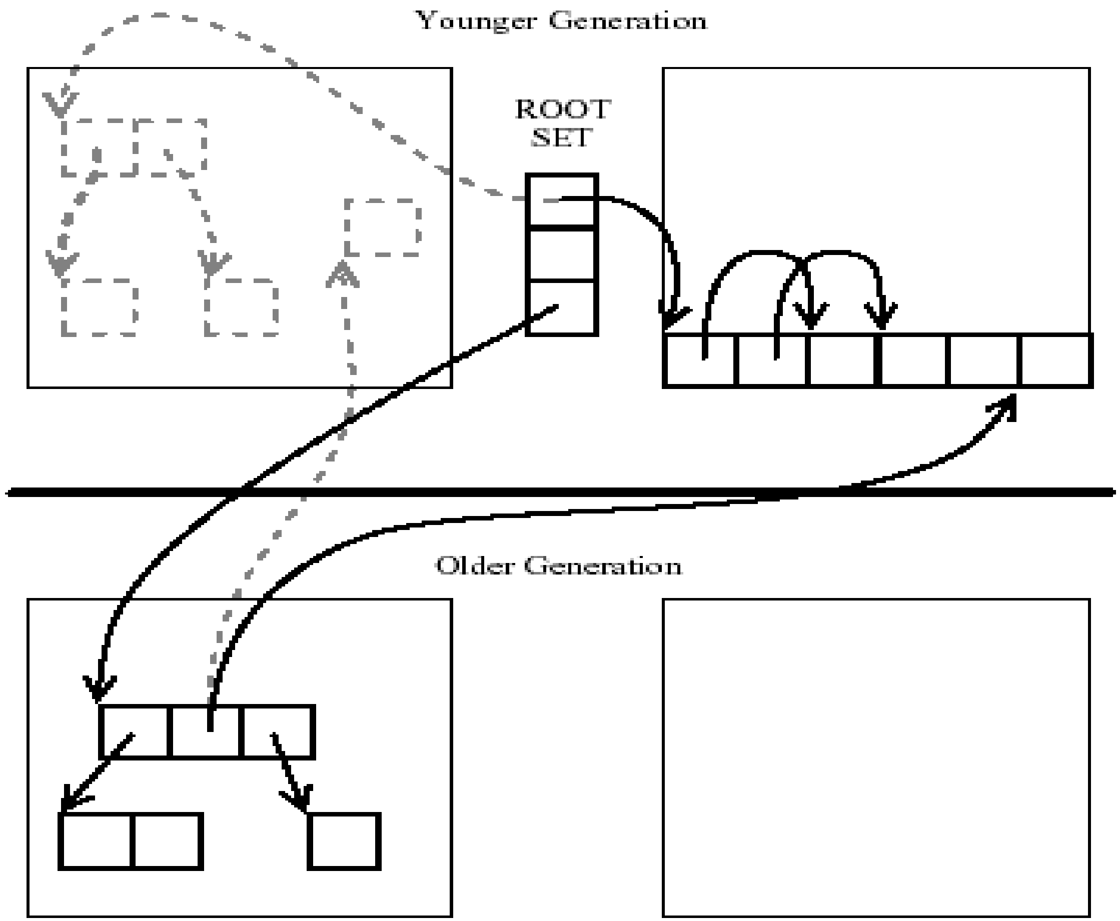


Diagram from Andrew Appel's **Modern Compiler Implementation**

Other issues…

- When do we promote objects from young generation to old generation
  - Usually after an object survives a collection, it will be promoted
- Need to keep track of older objects pointing to newer ones!
- How big should the generations be?
  - When do we collect the old generation?
  - After several minor collections, we do a major collection
- Sometimes different GC algorithms are used for the new and older generations.
  - Why? Because the have different characteristics
  - Copying collection for the new
    - Less than 10% of the new data is usually live
    - Copying collection cost is proportional to the live data
  - Mark-sweep for the old

New topic : Simple optimisations. Inline expansion

```
fun f(x) = x + 1
fun g(x) = x - 1
…
…
fun h(x) = f(x) + g(x)
```

inline f and g

```
fun f(x) = x + 1
fun g(x) = x - 1
…
…
fun h(x) = (x+1) + (x-1)
```

(+) Avoid building activation records at runtime

(+) May allow further optimisations

(-) May lead to "code bloat" (apply only to functions with "small" bodies?)

Question: if we inline all occurrences of a function, can we delete its definition from the code?
What if it is needed at link time?

Be careful with variable scope

Inline g in h

```
let val x = 1
    fun g(y) = x + y
    fun h(x) = g(x) + 1
in
   h(17)
end
```

NO

```
let val x = 1
    fun g(y) = x + y
    fun h(x) = x + y + 1
in
   h(17)
end
```

YES

```
let val x = 1
    fun g(y) = x + y
    fun h(z) = x + z + 1
in
   h(17)
end
```

What kind of care might be needed will depend on the representation level of the Intermediate code involved.

(b) Constant propagation, constant folding

```
let x = 2
let y = x - 1
let z = y * 17
```

```
let x = 2
let y = 2 - 1
let z = y * 17
```

```
let x = 2
let y = 1
let z = y * 17
```

```
let x = 2
let y = 1
let z = 1 * 17
```

```
let x = 2
let y = 1
let z = 17
```

Propagate constants and evaluate simple expressions at compile-time

Note : opportunities are often exposed by inline expansion!

David Gries :
“Never put off till run-time what you can do at compile-time.”

But be careful

How about this?

Replace

x * 0

with

0

OOPS, not if x has type float!

NAN*0 = NAN,

(c) peephole optimisation

Peephole Optimization

W. M. McKeeman
Stanford University, Stanford, California

Communications of the ACM, July 1965

Example 1. Source code:

$$X := Y;$$

$$Z := X + Z$$

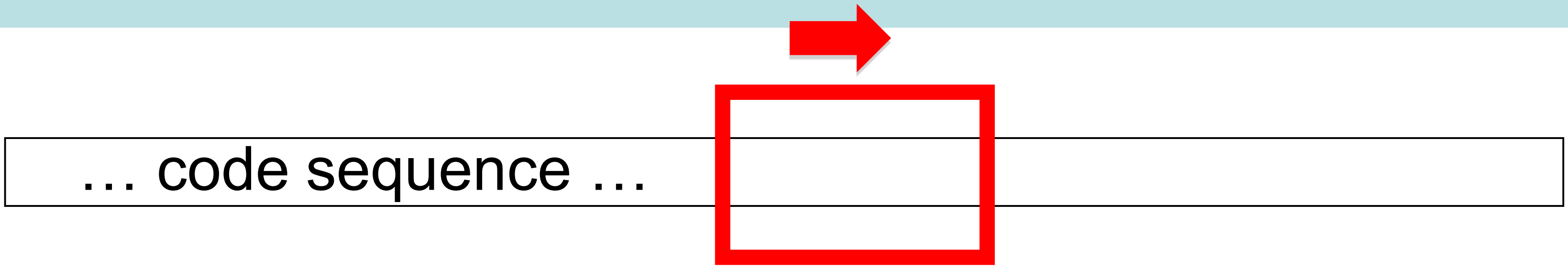
Compiled code:

LDA	Y	load the accumulator from Y
STA	X	store the accumulator in X
LDA	X	load the accumulator from X
ADD	Z	add the contents of Z
STA	Z	store the accumulator in Z

Eliminate!

Results for syntax-directed code generation.

peephole optimisation



Sweep a window over the code sequence looking for instances of simple code patterns that can be rewritten to better code … (might be combined with constant folding, etc, and employ multiple passes)

Examples
-- eliminate useless combinations (push 0; pop)
-- introduce machine-specific instructions
-- improve control flow. For example: rewrite
“GOTO L1 … L1: GOTO L2”
to
“GOTO L2 … L1 : GOTO L2”)

gcc example.
-O<m> turns on optimisation to level m

g.c

```
int h(int n)  { return (0 < n) ? n : 101 ; }

int g(int n)  { return 12 * h(n + 17); }
```

g.s (fragment)

gcc –O2 –S –c g.c

```
_g:
.cfi_startproc
pushq   %rbp
movq    %rsp, %rbp
addl    $17, %edi
imull   $12, %edi, %ecx
testl   %edi, %edi
movl    $1212, %eax
cmovgl  %ecx, %eax
popq    %rbp
ret
.cfi_endproc
```

Wait. What happened to the call to h???

GNU AS (GAS) Syntax
x86, 64 bit

gcc example (-O<m> turns on optimisation)

g.c

```
int h(int n)  { return (0 < n) ? n : 101 ; }

int g(int n)  { return 12 * h(n + 17); }
```

The compiler must have done something similar to this:

```
int g(int n)  { return 12 * h(n + 17); }
➔
 int g(int n)  { int t := n+ 17; return 12 * h(t); }
➔
int g(int n)  { int t := n+ 17; return 12 *((0 < t) ? t : 101 ); }
➔
int g(int n)  { int t := n+ 17; return (0 < t) ? 12 * t : 1212 ; }
➔ …
```

New topic : static links on the call stack.

- Many textbooks on compilers treat only languages with first-order functions --- that is, functions cannot be passes as an argument or returned as a result. In this case, we can avoid allocating environments on the heap since all values associated with free variables will be somewhere on the stack!
- But how do we find these values? We optimise stack search by following a chain of **static links**. Static links are added to every stack frame and points to the stack frame of the last invocation of the defining function.
- One other thing: most languages take multiple arguments for a function/procedure call.

Terminology: Caller and Callee

```
fun f (x, y) = e1

…

fun g(w, v) =
    w + f(v, v)
```

For this invocation of the function f, we say that g is the <u>caller</u> while f is the callee

Recursive functions can play both roles at the same time …

Nesting depth

Pseudo-code

```
fun b(z) = e

 fun g(x1) =
    fun h(x2) =
       fun f(x3) = e3(x1, x2, x3, b, g h, f)
       in
           e2(x1, x2, b, g, h, f)
        end
    in
        e1(x1, b, g, h)
   end
…
b(g(17))
…
```

Nesting depth

code in big box is at nesting depth k

```
fun b(z) = e
```

nesting depth k + 1

```
fun g(x1) =
   fun h(x2) =
      fun f(x3) = e3(x1, x2, x3, b, g h, f)
      in
         e2(x1, x2, b, g, h, f)
       end
   in
      e1(x1, b, g, h)
   end
...
b(g(17))
...
```

nesting depth k + 3

nesting depth k + 2

nesting depth k + 1

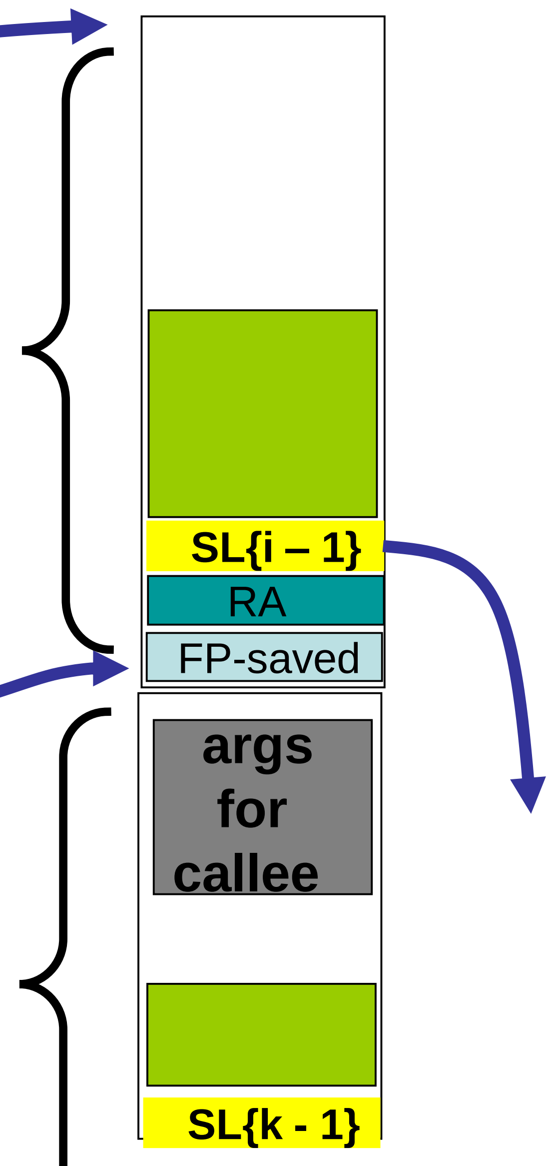
Function g is the **definer** of h. Functions g and b must share a definer defined at depth k-1

Stack with static links and variable number of arguments

sp

stack frame for **callee** defined at nesting depth **i <= k + 1**

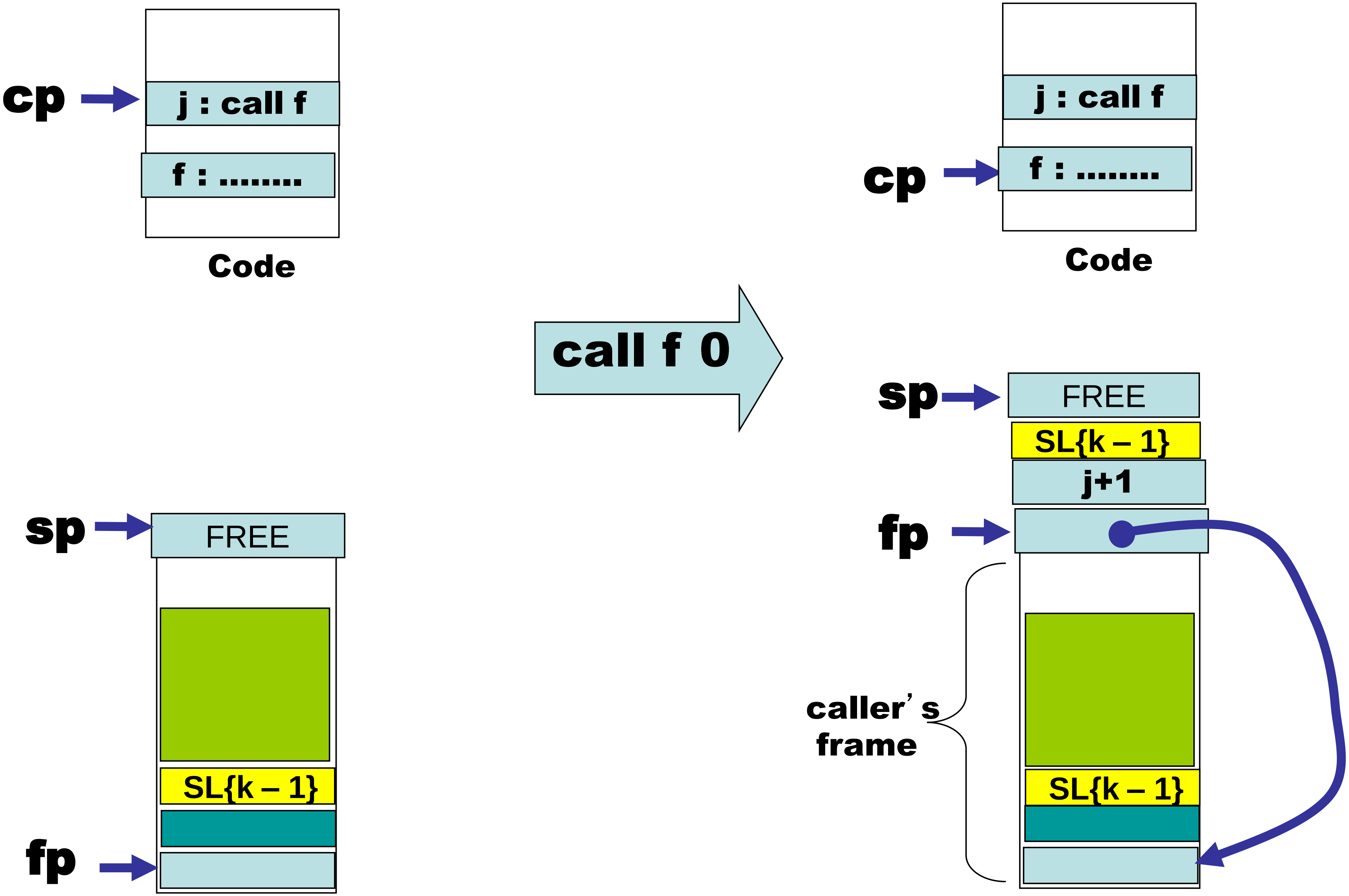
fp



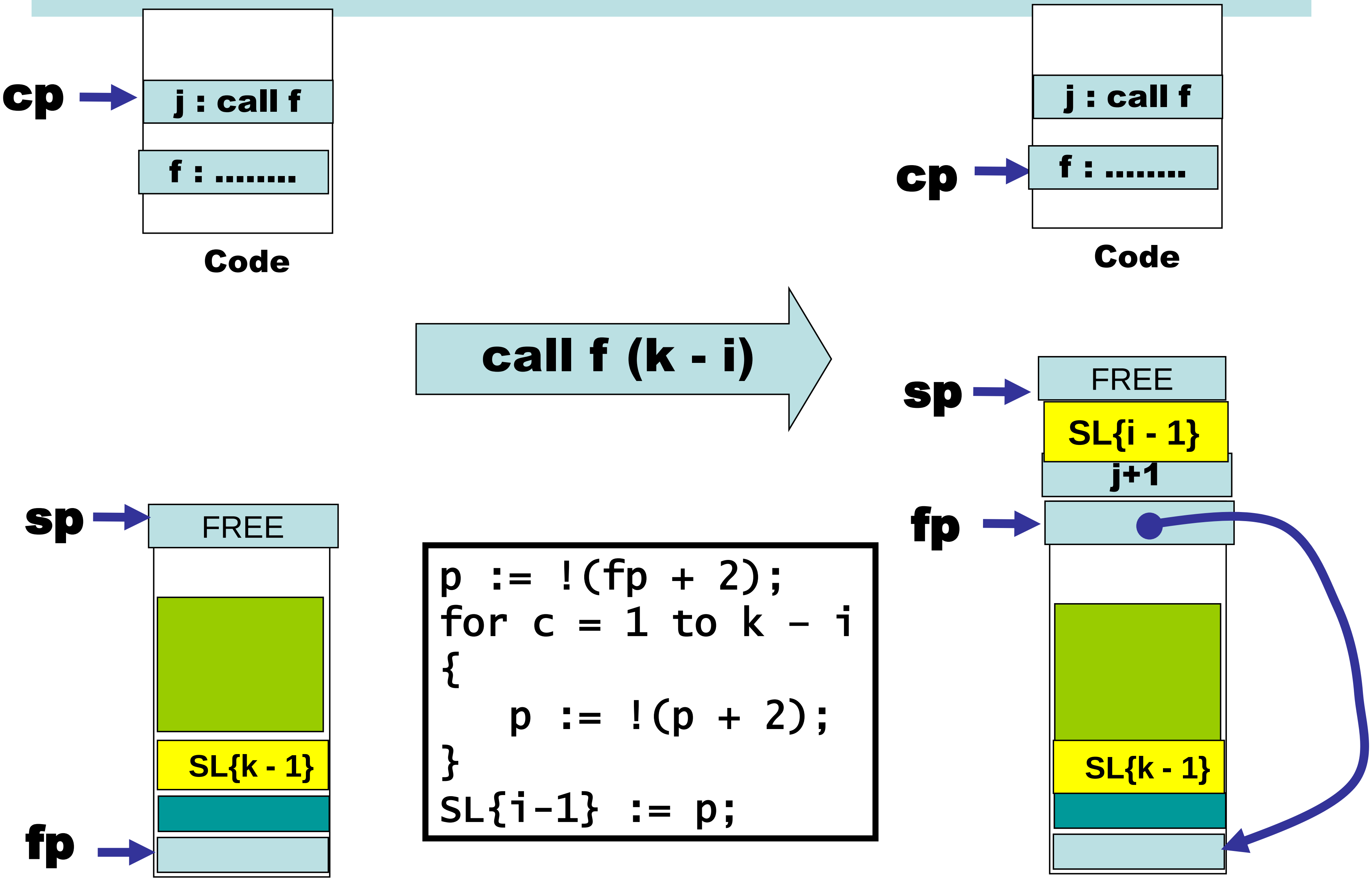
The static link points down to the closest frame **of definer** at nesting depth **i - 1**

stack frame for **caller** defined at nesting **depth k** used to evaluate code at **depth k + 1**.

caller and callee at same nesting depth k

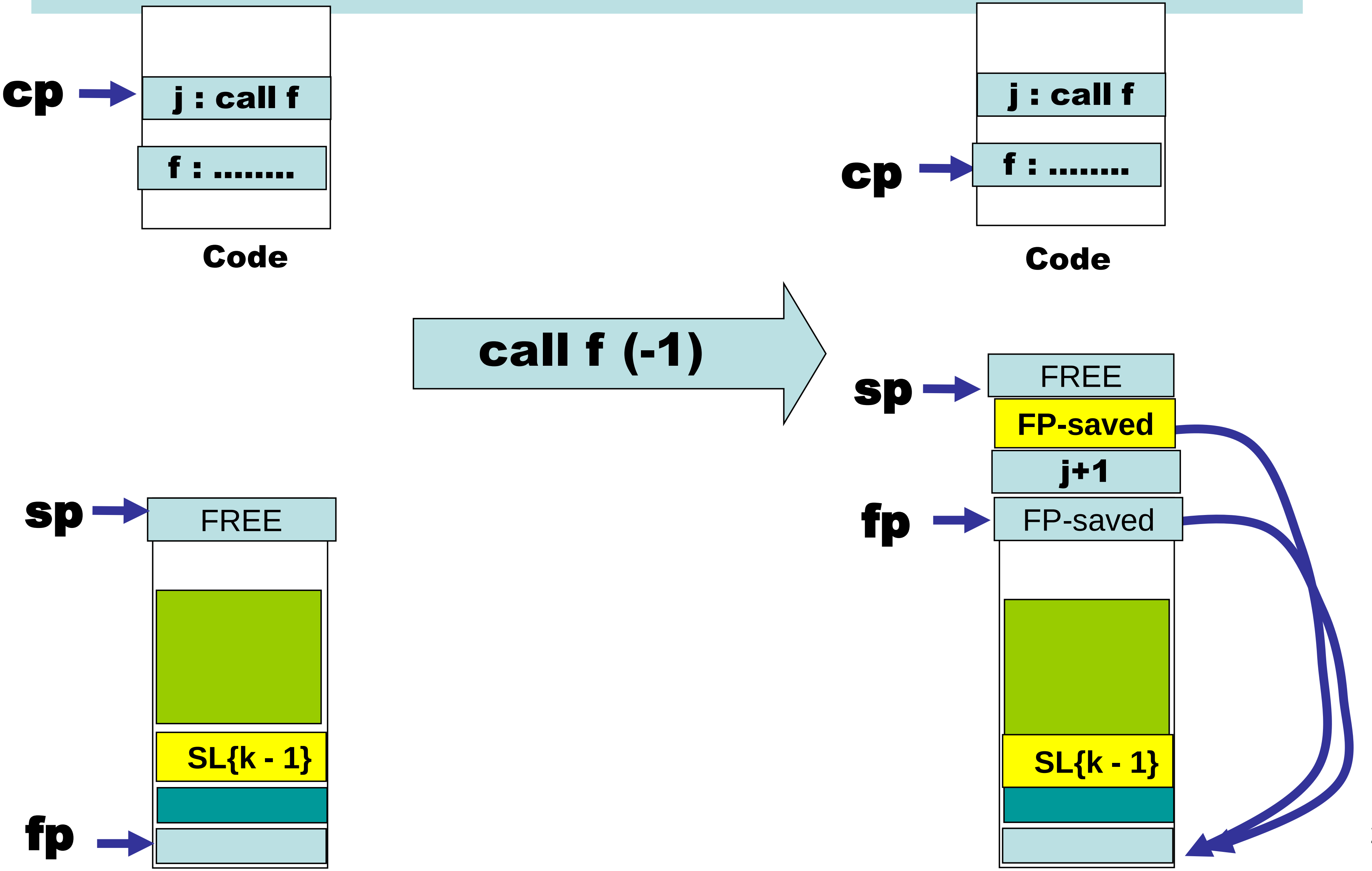


caller at depth k and callee at depth i < k

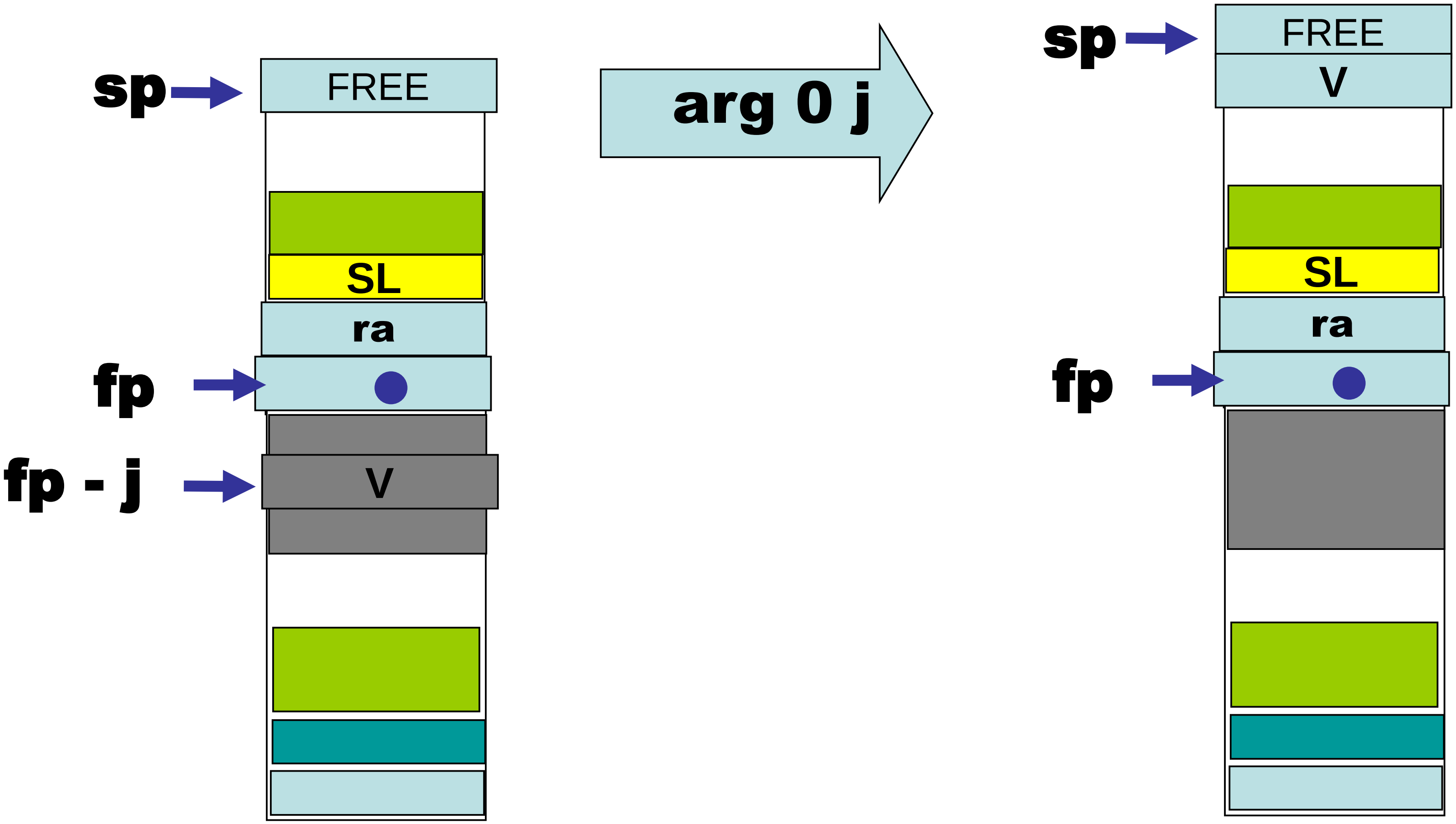



```
p := !(fp + 2);
for c = 1 to k - i
{
   p := !(p + 2);
}
SL{i-1} := p;
```

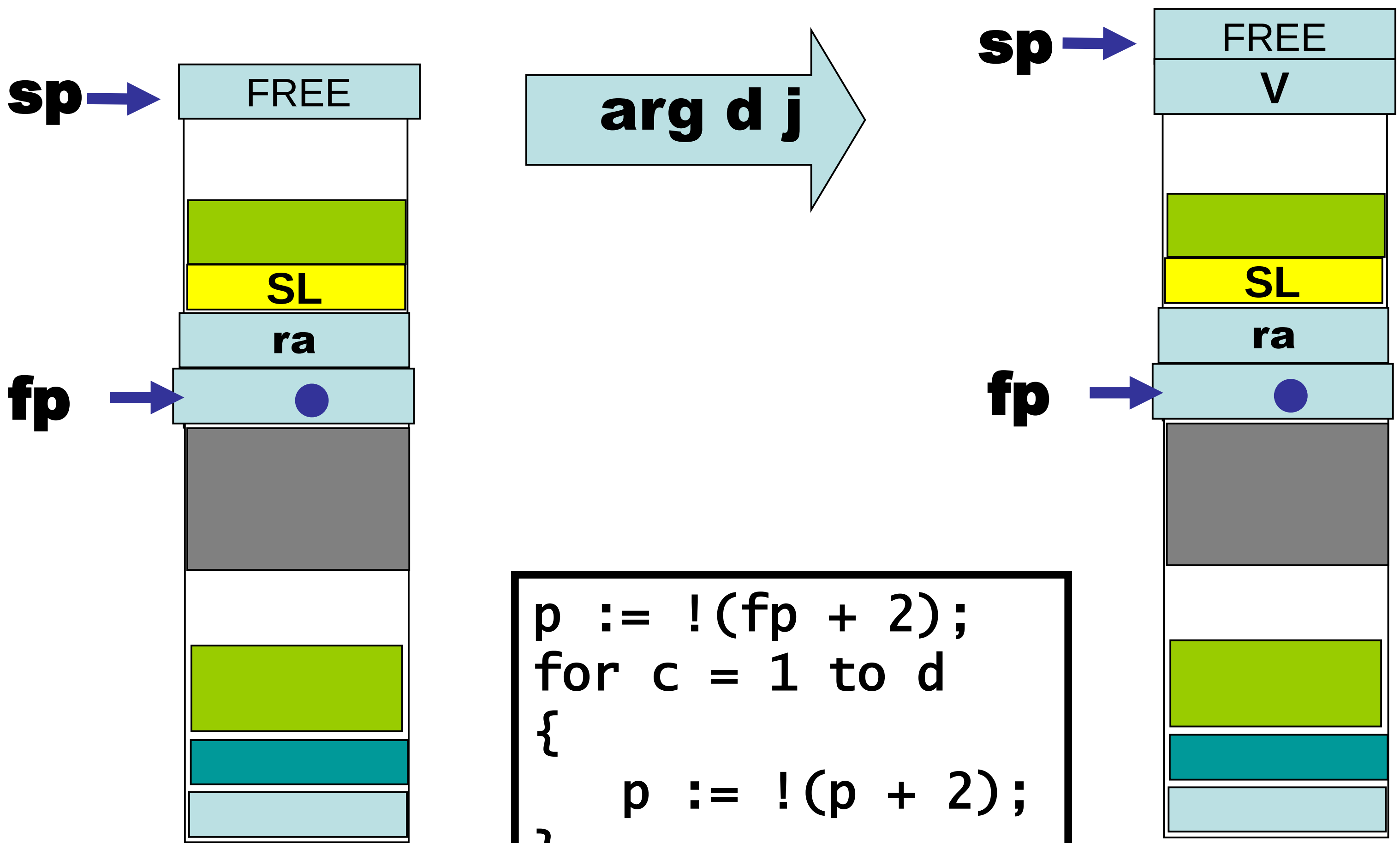
caller at depth k and callee at depth k + 1



Access to argument values at static distance 0



Access to argument values at static distance d, 0 < d




```
p := !(fp + 2);
for c = 1 to d
{
   p := !(p + 2);
}
v := !(p - j);
```

New Topic:
OOP Objects (single inheritance)

```
let start := 10

   class Vehicle extends Object {
      var position := start
      method move(int x) = {position := position + x}
   }
   class Car extends Vehicle {
      var passengers := 0
      method await(v : Vehicle) =
         if (v.position < position)
         then v.move(position - v.position)
         else self.move(10)
   }
   class Truck extends Vehicle {
      method move(int x) =
         if x <= 55 then position := position +x
   }
   var t := new Truck
   var c := new Car
   var v : Vehicle := c
in
   c.passengers := 2;
   c.move(60);
   v.move(70);
   c.await(t)
end
```

method override

subtyping allows a Truck or Car to be viewed and used as a Vehicle

Object Implementation?

- how do we access object fields?
  - both inherited fields and fields for the current object?
- how do we access method code?
  - if the current class does not define a particular method, where do we go to get the inherited method code?
  - how do we handle method override?
- How do we implement subtyping ("object polymorphism")?
  - If B is derived from A, then need to be able to treat a pointer to a B-object as if it were an A-object.

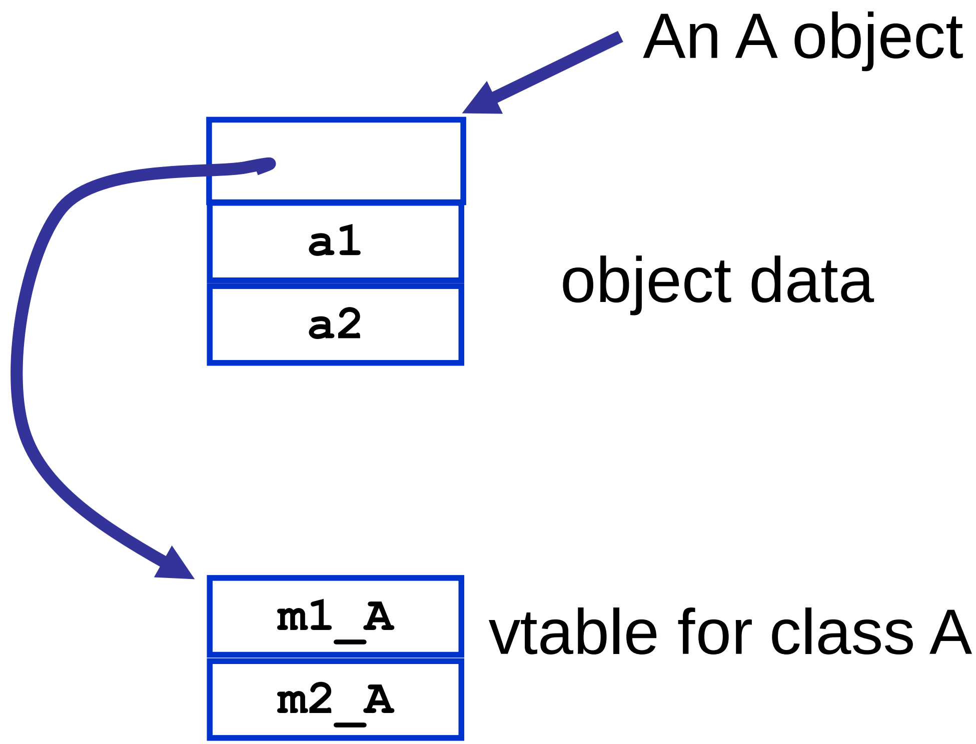
Another OO Feature

- Protection mechanisms
  - to encapsulate local state within an object, Java has “private” “protected” and “public” qualifiers
    - private methods/fields can’t be called/used outside of the class in which they are defined
  - This is really a scope/visibility issue! Front-end during semantic analysis (type checking and so on), the compiler maintains this information in the symbol table for each class and enforces visibility rules.

Object representation

C++

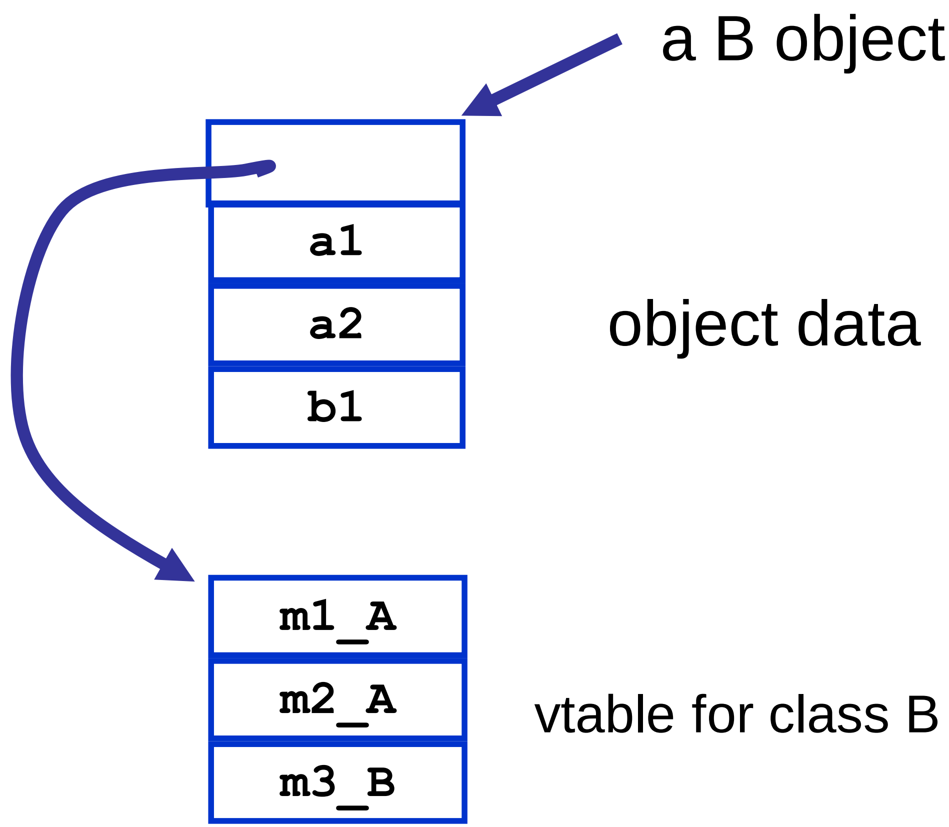
```
class A {
public:
   int a1, a2;

   virtial void m1(int i) {
      a1 = i;
   }
   virtual void m2(int i) {
      a2 = a1 + i;
   }
}
```



NB: a compiler typically generates methods with an extra argument representing the object (self) and used to access object data.

Inheritance (“pointer polymorphism”)

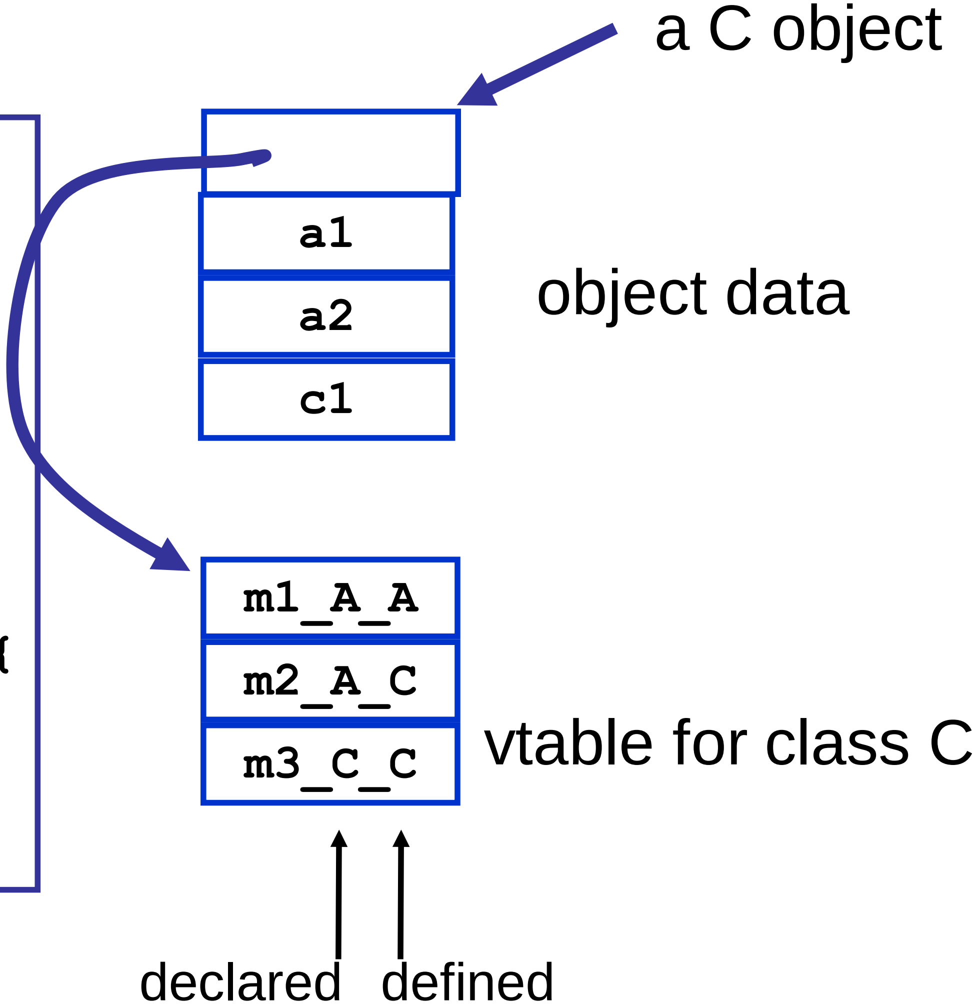
```
class B : public A {
public:
   int b1;

virtual void m3(void) {
      b1 = a1 + a2;
   }
}
```



Note that a pointer to a B object can be treated as if it were a pointer to an A object!

Method overriding

```
class C : public A {
public:
    int c1;

    virtual void m3(void) {
        b1 = a1 + a2;
    }
    virtual void m2(int i) {
        a2 = c1 + i;
    }
}
```



Static vs. Dynamic

- which method to invoke on overloaded polymorphic types?

```
class C *c = ...;
class A *a = c;

a->m2(3);
```

???

```
m2_A_A(a, 3);
```

static

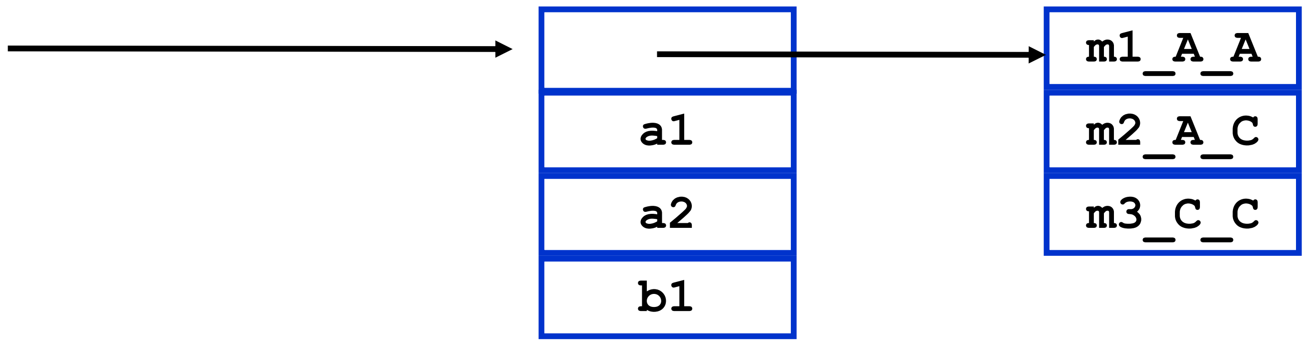
```
m2_A_C(a, 3);
```

dynamic

Dynamic dispatch implemented with vtables

A pointer to a class C object can be treated

as a pointer to a class A object




```
class C *c = ...;
class A *a = c;

a->m2(3);
```

```
*(a->vtable[1])(a, 3);
```

New Topic : Exceptions (informal description)

e handle f

If expression e evaluates "normally" to value v, then v is the result of the entire expression.

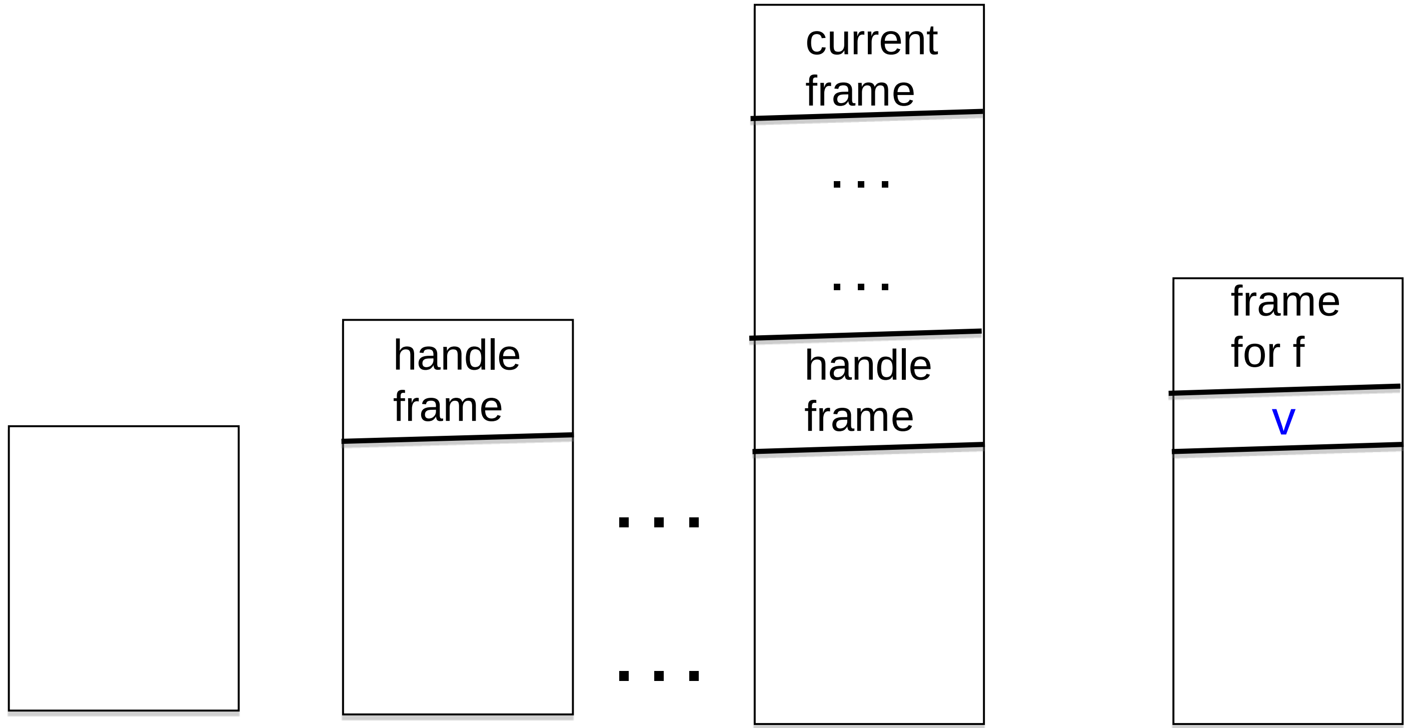
Otherwise, an exceptional value v' is "raised" in the evaluation of e, then result is (f v')

raise e

Evaluate expression e to value v, and then raise v as an exceptional value, which can only be "handled".

Implementation of exceptions may require a lot of language-specific consideration and care. Exceptions can interact in powerful and unexpected ways with other language features. Think of C++ and class destructors, for example.

Viewed from the call stack



Call stack just before evaluating code for

e handle f

Push a special frame for the handle

"raise v" is encountered while evaluating a function body associated with top-most frame

"Unwind" call stack. Depending on language, this may involve some "clean up" to free resources.

Possible pseudo-code implementation

e handle f

```
let fun _h27 () =
  build special “handle frame”
  save address of f in frame;
  … code for e …
  return value of e
in _h27 () end
```

raise e

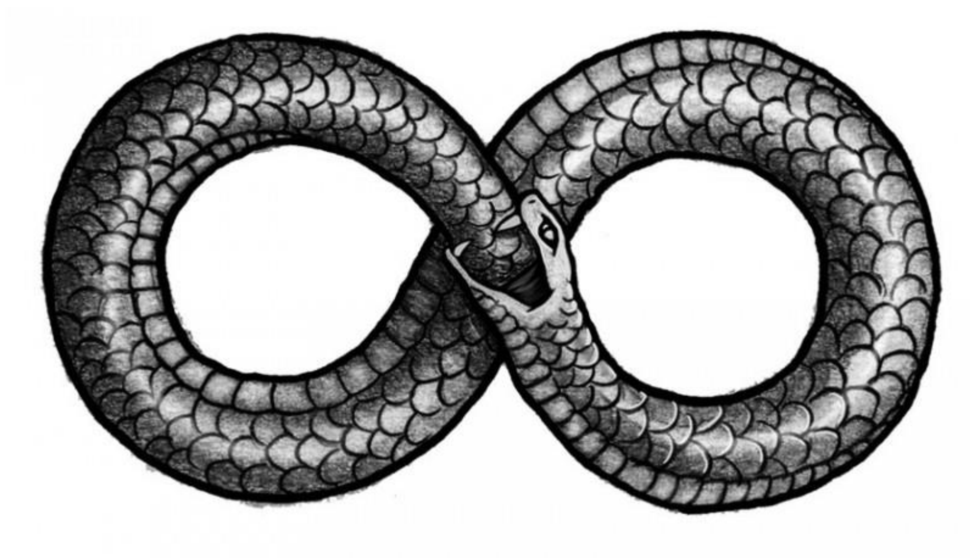
```
… code for e …
save v, the value of e;
unwind stack until first
fp found pointing at a handle frame;
Replace handle frame with frame
for call to (extracted) f using
v as argument.
```

See 2019 Paper 4 Question 4

New topic : Bootstrapping a compiler

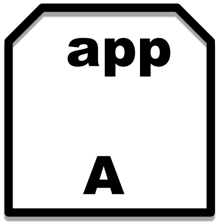
- Compilers compiling themselves!
- Read Chapter 13 Of
  - Basics of Compiler Design
  - by Torben Mogensen

    http://www.diku.dk/hjemmesider/ansatte/torbenm/Basics/

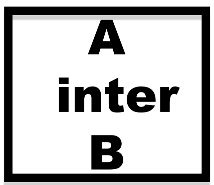


http://mythologian.net/ouroboros-symbol-of-infinity/

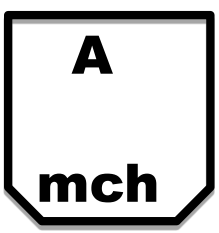
Bootstrapping. We need some notation . . .



An application called **app** written in language **A**

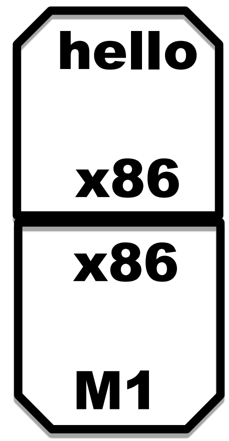


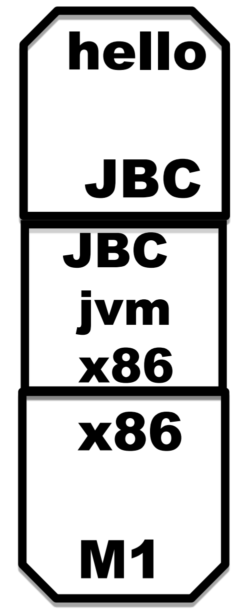
An interpreter or VM for language **A** Written in language **B**



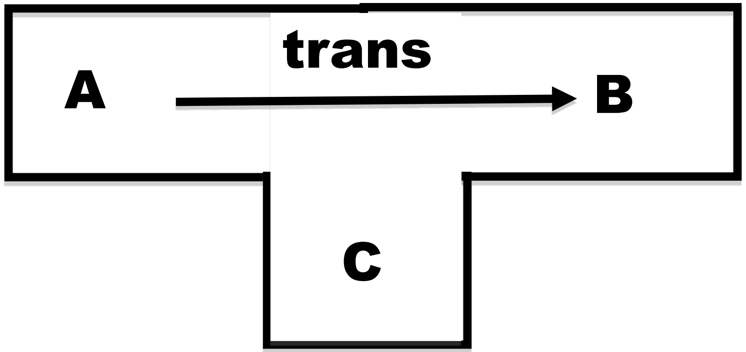
A machine called **mch** running language **A** natively.

Simple Examples



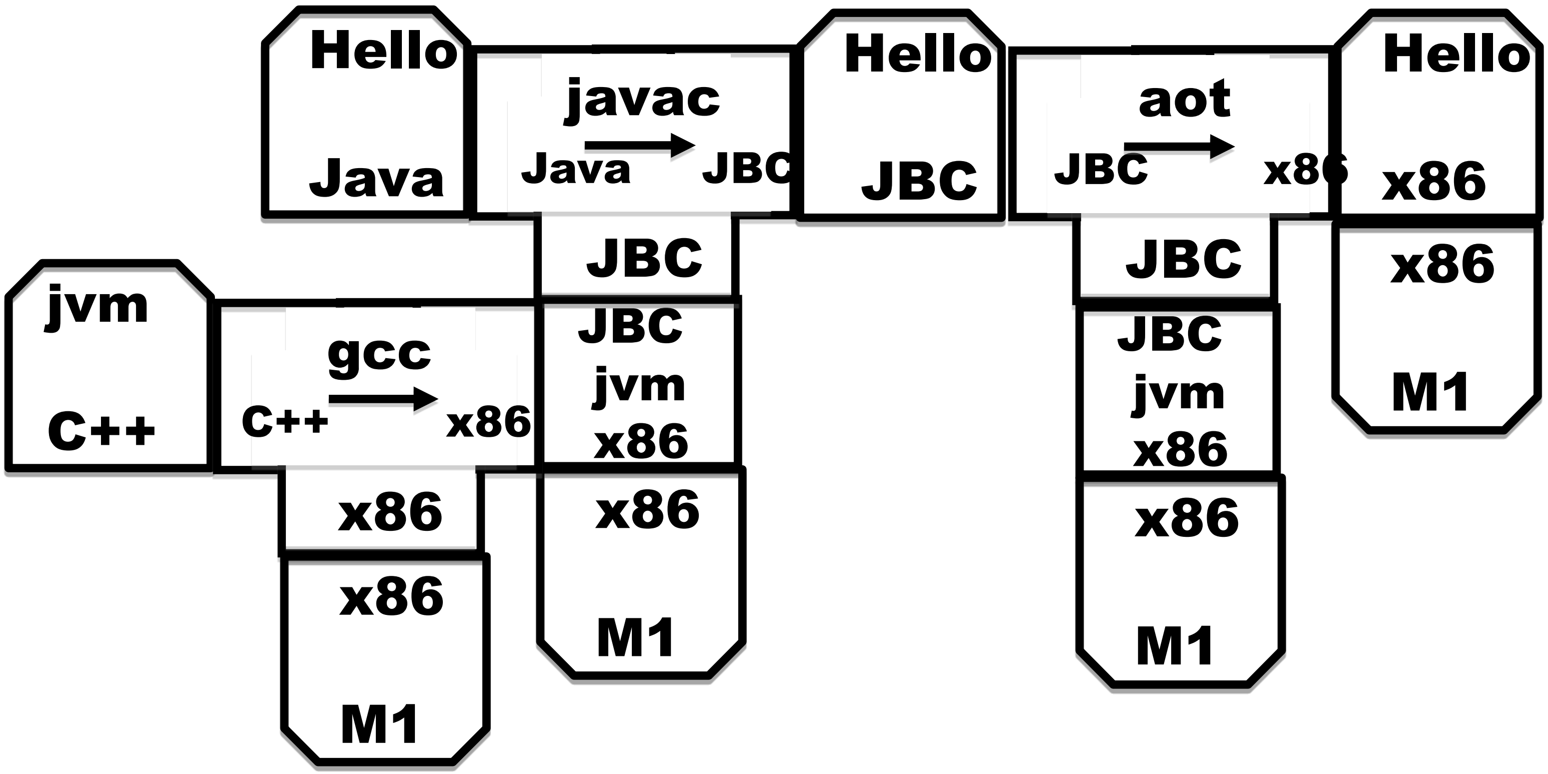


Tombstones



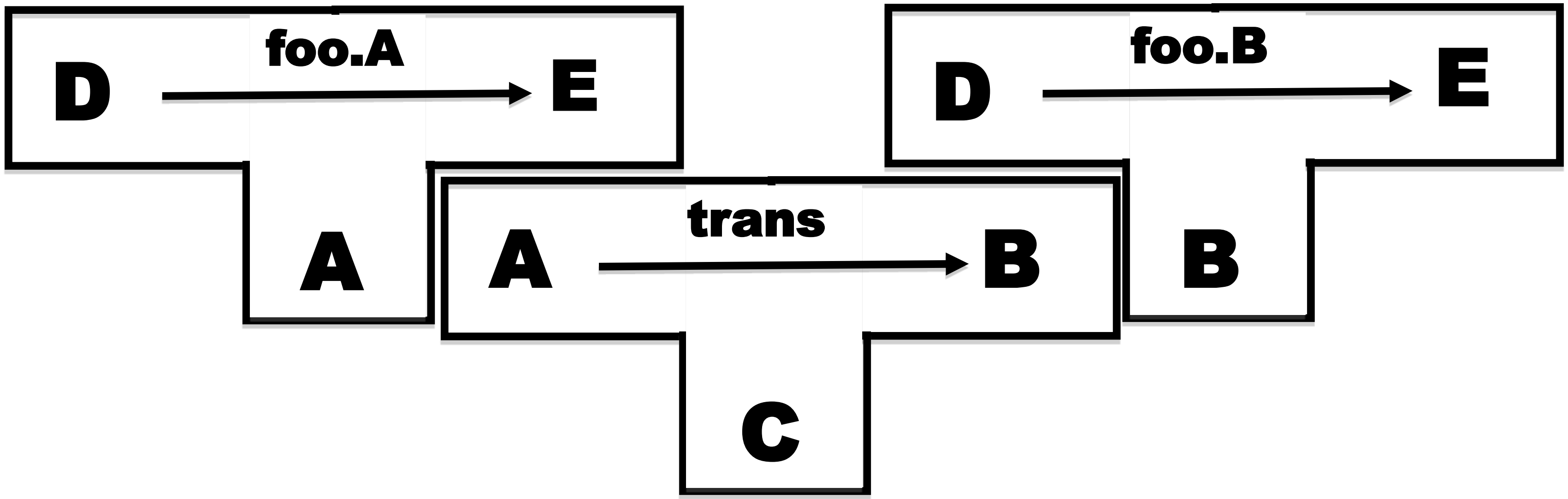
This is an application called **trans** that translates programs in language **A** into programs in language **B**, and it is written in language **C**.

Ahead-of-time compilation



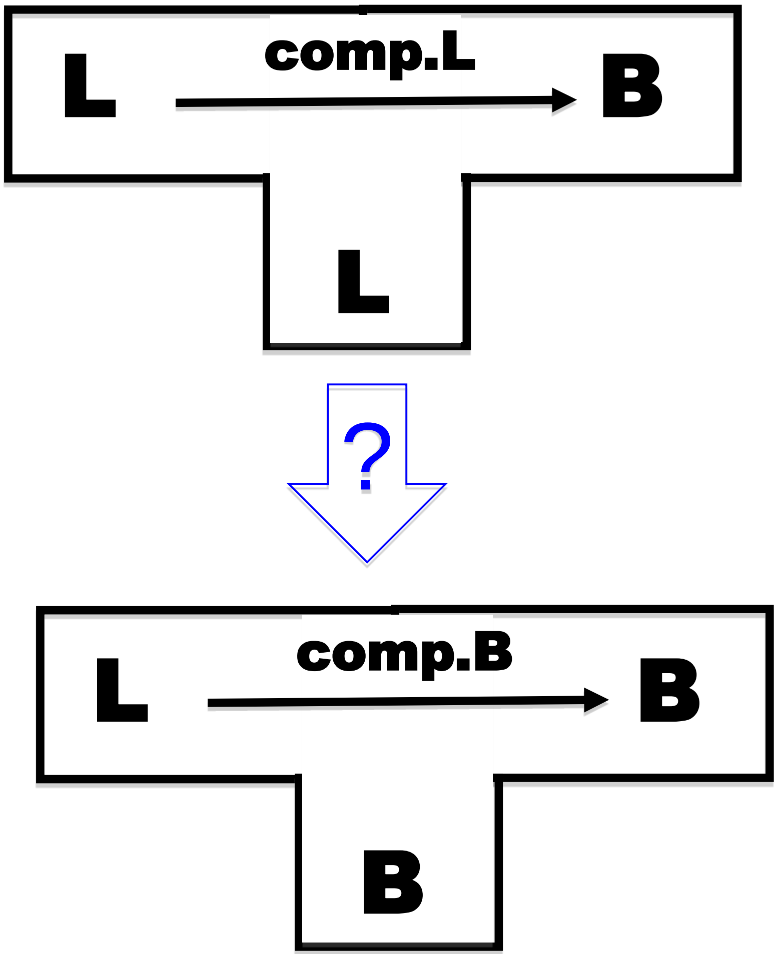
Thanks to David Greaves
for the example.

Of course translators can be translated



Translator **foo.B** is produced as output from **trans** when given **foo.A** as input.

Our seemingly impossible task



We have just invented a really great new language **L** (in fact we claim that "**L** is far superior to C++"). To prove how great **L** is we write a compiler for **L** in **L** (of course!). This compiler produces machine code **B** for a widely used instruction set (say **B** = x86).

Furthermore, we want to compile our compiler so that it can run on a machine running **B.**

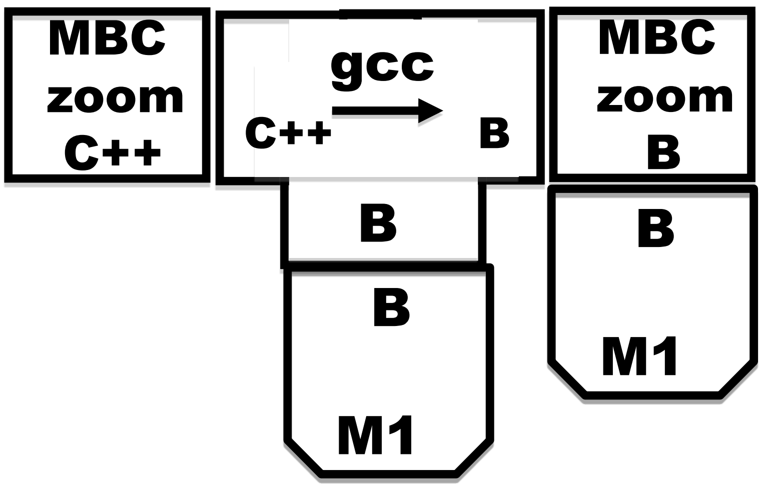
Our compiler is written in L!

How can we compiler our compiler?

There are many many ways we could go about this task. The following slides simply sketch out one plausible route to fame and fortune.

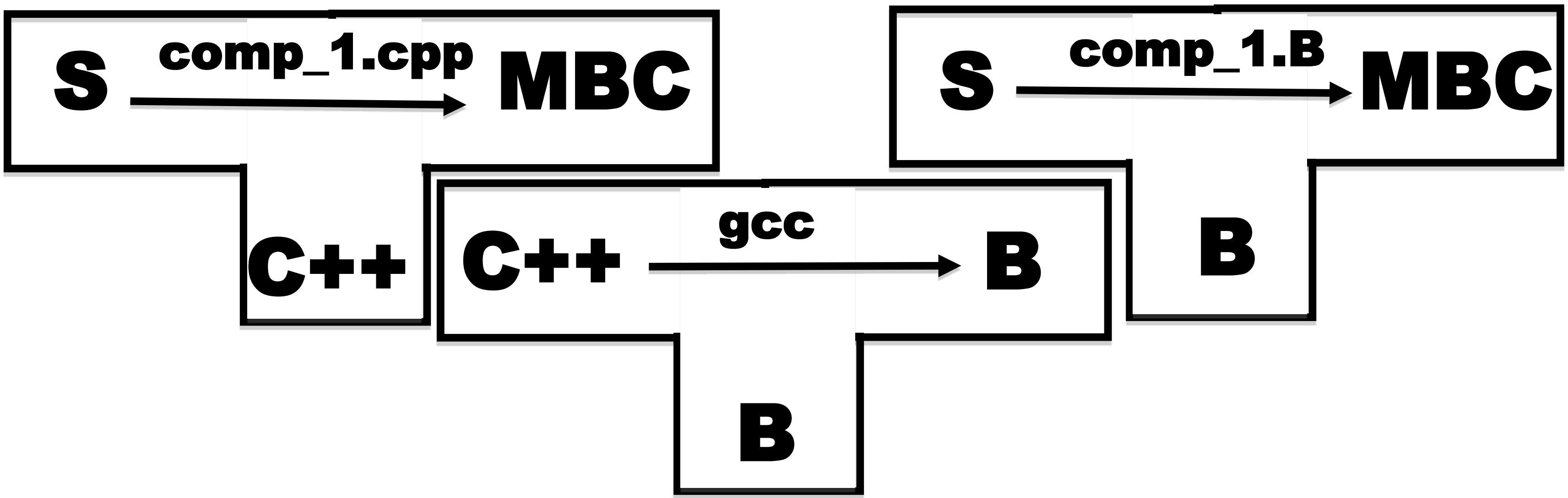
Step 1
Write a small interpreter (VM) for a small language of byte codes

MBC = My Byte Codes



The **zoom** machine!

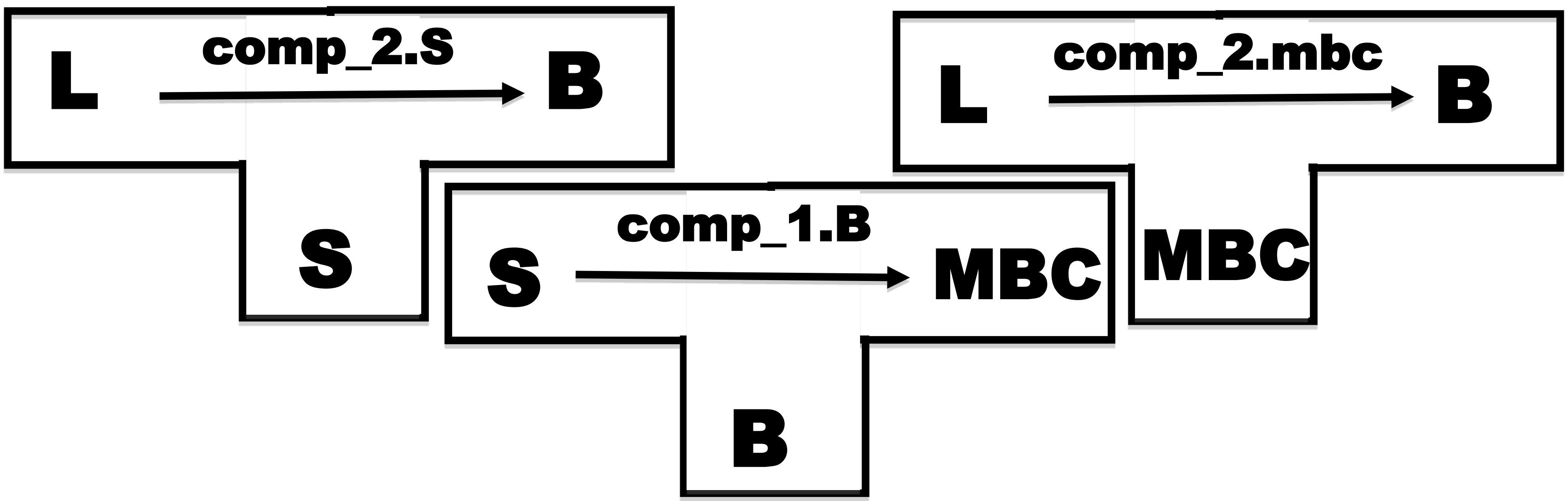
Step 2
Pick a small subset S of L and write a translator from S to MBC



Write **comp_1.cpp** by hand. (It sure would be nice if we could hide the fact that this is written is C++.)

Compiler **comp_1.B** is produced
as output from **gcc** when **comp_1.cpp** is given as input.

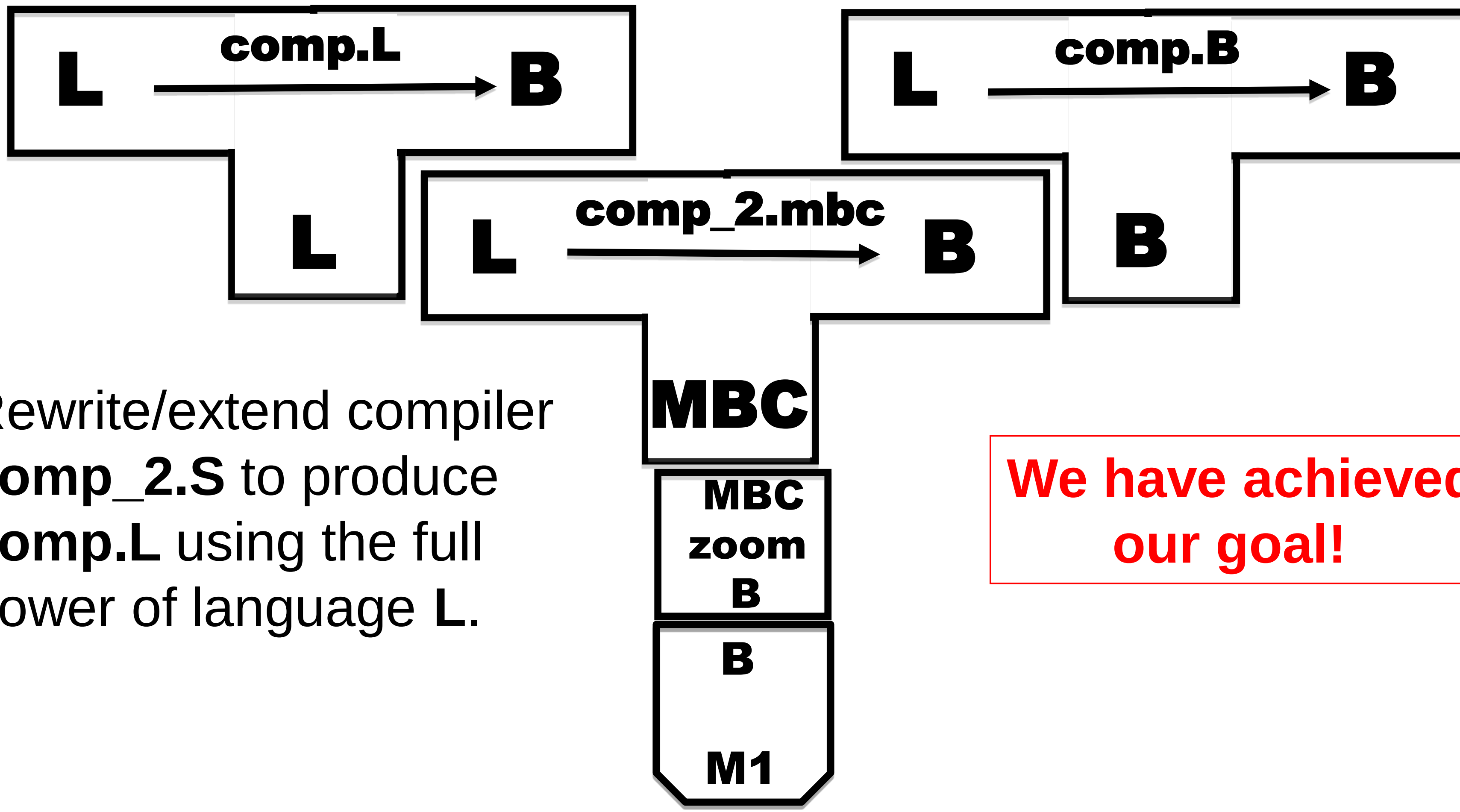
Step 3
Write a compiler for L in S



Write a compiler **comp_2.S** for the full language **L**, but written only in the sub-language **S**.

Compile **comp_2.S** using **comp_1.B** to produce **comp_2.mbc**

Step 4
Write a compiler for L in L, and then compile it!

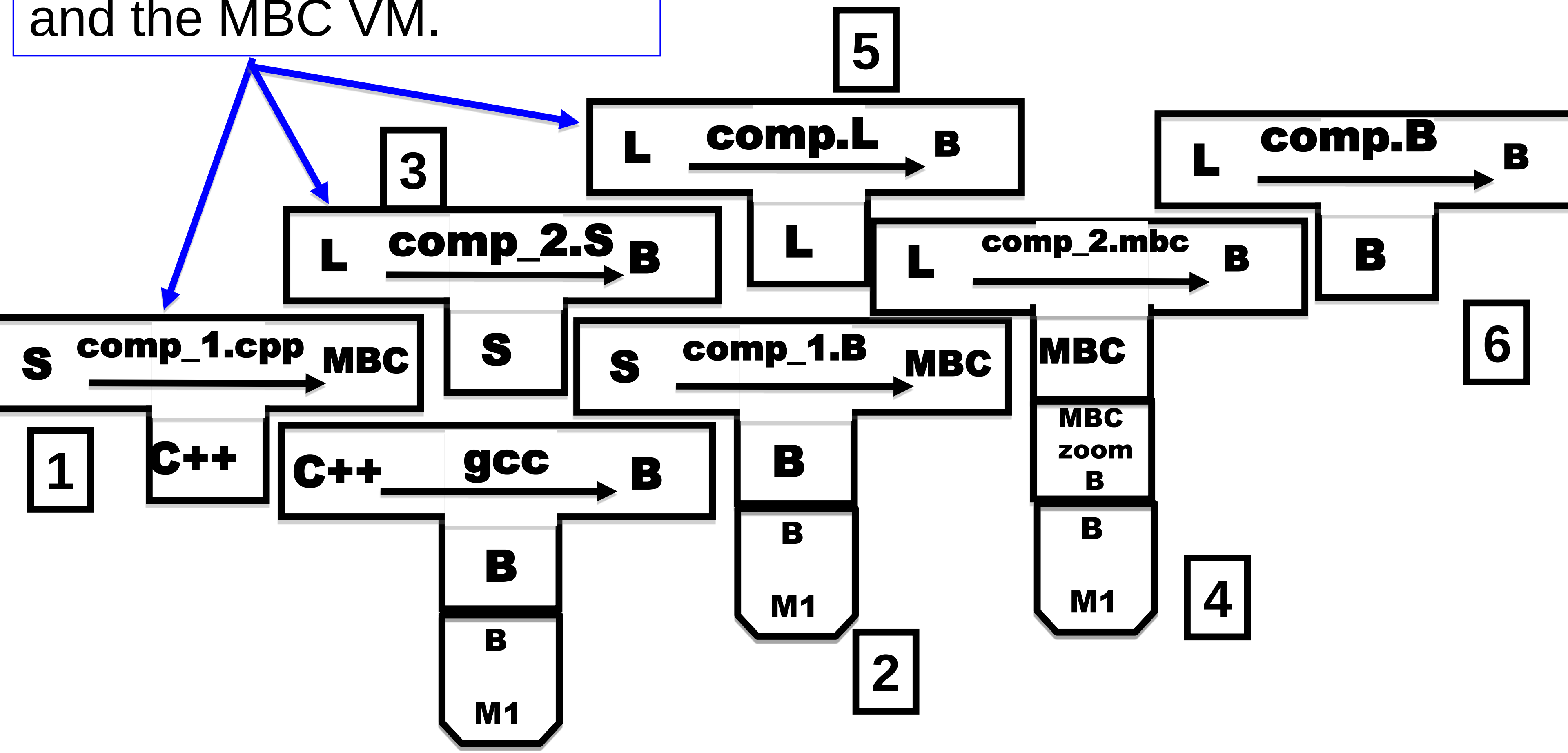


Rewrite/extend compiler **comp_2.S** to produce **comp.L** using the full power of language **L**.

We have achieved our goal!

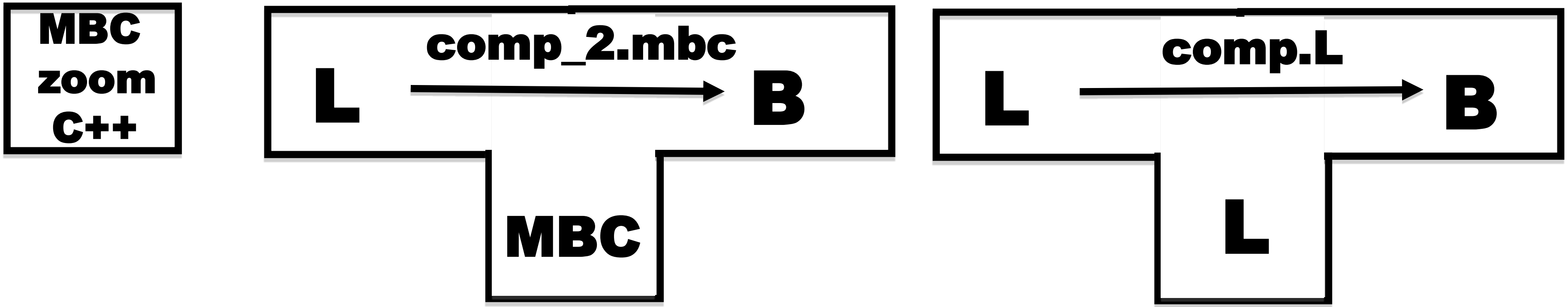
Putting it all together

We wrote these compilers and the MBC VM.



Step 5 : Cover our tracks and leave the world mystified and amazed!

Our **L** compiler download site contains <u>only three</u> components:



comp_2.mbc is a just file of **bytes**. We give it the mysterious name such as **mr-e**

Shhhh! Don't tell anyone that we wrote the first compiler in C++

Our instructions:

1. Use **gcc** to compile the **zoom** interpreter
2. Use **zoom** to run **mr-e** with input **comp.L** to output the compiler **comp.B**. MAGIC!

Another example (Mogensen, Page 285)

Solving a different problem.

You have:

(1) An ML compiler on ARM. Who knows where it came from.

(2) An ML compiler written in ML, generating x86 code.

You want:

An ML compiler generating x86 and running on an x86 platform.