

14: Clique Finding

Machine Learning and Real-world Data (MLRD)

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Last session: betweenness centrality

- You implemented betweenness centrality.
- This let you find “gatekeeper” nodes in the Facebook network.
- We will now turn to the task of finding **clusters** in networks.

Clustering and Classification

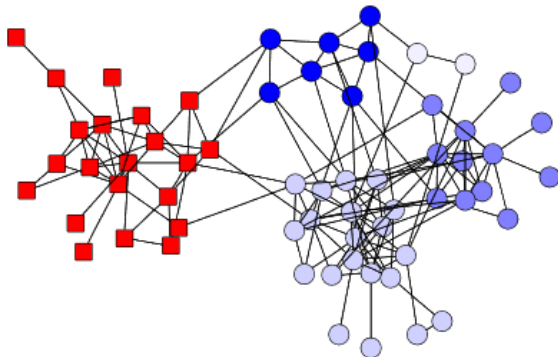
- **Clustering**: automatically grouping data according to some notion of closeness or similarity.
- **Classification** (e.g., sentiment classification): assigning data items to predefined classes.
- Clustering: groupings can emerge from data, **unsupervised**.
- Clustering for documents, images etc: anything where there's a notion of similarity between items.
- Most famous technique for hard clustering is **k-means**: very general (also variant for graphs).
- Also soft clustering: clusters have graded membership

Agglomerative vs. divisive clustering

- **agglomerative clustering** works bottom-up.
- **divisive clustering** works top-down, by splitting.
- Newman-Girvan method — a form of divisive clustering.
- Criterion for breaking links is edge betweenness centrality.

Dolphin data: different clustering layers

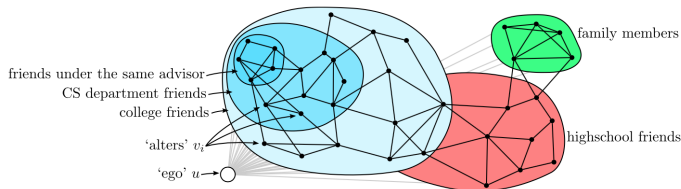
- squares vs circles: first split
- shades of blue: further splits



Facebook circles dataset: McAuley and Leskovec (2012)

- Profile and network data from 10 Facebook [ego-networks](#).
- An ego network is a network emanating from one person.
- Circles are defined as Facebook friends in a particular social group.
- Gold-standard circles are manually identified by the egos themselves.

Facebook Circles task



- Complete network consists of 4,039 nodes in 193 circles.
- Average: 19 circles per ego, each circle with average of 22 **alters**.
- You will cluster only a small network derived from one ego.

Doing the full Facebook Circles task

25% of circles are contained completely within another circle

50% overlap with another circle

25% have no members in common with any other circle

Requires more sophisticated methods than Newman-Girvan:

- Nodes may be in multiple circles, so we need soft clustering.
- Use sociological/demographic data from outside the network data.

Evaluating simple clustering

- Assume data sets with gold standard or ground truth clusters.
- But: unlike classification, we don't have labels for clusters, number of clusters found may not equal true classes.
- **purity**: assign label corresponding to majority class found in each cluster, then count correct assignments, divide by total elements (cf accuracy).
`http://nlp.stanford.edu/IR-book/html/htmledition/evaluation-of-clustering-1.html`
- But best evaluation (if possible) is **extrinsic**: use the system to do a task and evaluate that.

Newman-Girvan method

while number of connected subgraphs $<$ specified number of clusters (and there are still edges):

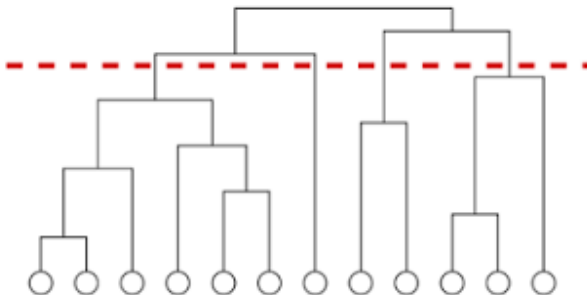
- 1 calculate edge betweenness for every edge in the graph
- 2 remove edge(s) with highest betweenness
- 3 recalculate number of connected components

Note:

- Treatment of tied edges: either remove all (today) or choose one randomly.

Newman-Girvan Method: Stopping Criterion

- The image below is called a **dendrogram**.
- Either: stop at prespecified level (tick).
- Or: complete process and choose best level by 'modularity' (Newman, 2004; starred tick).



Edge betweenness centrality

- Previously: $\sigma(s, t|v)$ — the number of shortest paths between s and t going through node v .
- Now: $\sigma(s, t|e)$ — the number of shortest paths between s and t going through edge e .
- Algorithm only changes in the bottom-up (accumulation) phase: $\delta(v)$ much as before, but $c_B[(v, w)]$ is now

Edge Betweenness (Brandes 2008)

```
└─  
  ▼ accumulation // — back-propagation of dependencies  
  for  $v \in V$  do  $\delta[v] \leftarrow 0$   
  while  $S$  not empty do  
    pop  $w \leftarrow S$   
    for  $v \in \text{Pred}[w]$  do  $\delta[v] \leftarrow \delta[v] + \frac{\sigma[v]}{\sigma[w]} \cdot (1 + \delta[w])$   
    if  $w \neq s$  then  $c_B[w] \leftarrow c_B[w] + \delta[w]$ 
```

Edge betweenness

output: betweenness $c_B[q]$ for $q \in V \cup E$ (initialized to 0)

```
▼ accumulation  
for  $v \in V$  do  $\delta[v] \leftarrow 0$   
while  $S$  not empty do  
  pop  $w \leftarrow S$   
  for  $v \in \text{Pred}[w]$  do  
     $c \leftarrow \frac{\sigma[v]}{\sigma[w]} \cdot (1 + \delta[w])$   
     $c_B[(v, w)] \leftarrow c_B[(v, w)] + c$   
     $\delta[v] \leftarrow \delta[v] + c$   
  if  $w \neq s$  then  $c_B[w] \leftarrow c_B[w] + \delta[w]$ 
```

ignore last line

Final Task

Task 12:

- Determine connected components
- Change code for betweenness centrality (from node to edge)
- Implement the Newman-Girvan method to discover clusters in the network provided.

Code for determining connected components

- Today's graph is disconnected: there are five **connected components**.
- Finding connected components: depth-first search, start at an arbitrary node and mark the other nodes you reach.
- Repeat with unvisited nodes, until all are visited.
- Implementation hint: depth-first, so use recursion (the program stack stores the search state).

End of Course

- Thanks for your attention
- Please fill in the evaluation questionnaire (we actually read these carefully)
- Two catch-up sessions to follow – everybody must get every tick
- Last-chance session in Easter term TBA
- A pen will be waiting for you upon successful completion; pick up at student reception once full access is restored.

