Foundations of Computer Science

Lazy Lists: consuming and joining sequences

Dr. Robert Harle & Dr. Jeremy Yallop

2020–2021

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         | Nil
         | Cons of 'a * (unit -> 'a seq)

Out[1]: type 'a seq = Nil | Cons of 'a * (unit -> 'a seq)
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Get the first n elements as a list:

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In[2]: let rec get n s =
         match n, s with
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Out[2]: val get : int -> 'a seq -> 'a list = <fun>
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Consumer ← Producer

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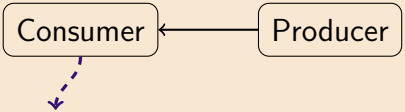


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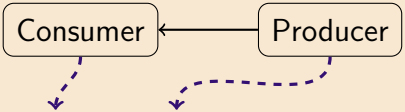

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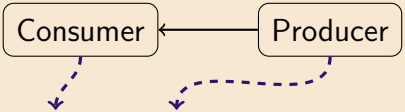


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get 2 (from 6)  ⇒  get 2 (Cons (6, fun () -> from (6 + 1)))
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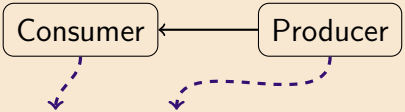


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get 2 (from 6)  ⇒  get 2 (Cons (6, fun () -> from (6 + 1)))
                ⇒  6 :: get 1 (from (6 + 1))
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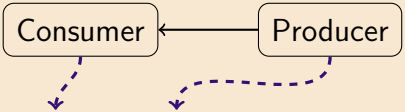


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get 2 (from 6)  ⇒  get 2 (Cons (6, fun () -> from (6 + 1)))
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                ⇒  6 :: get 1 (Cons (7, fun () -> from (7 + 1)))
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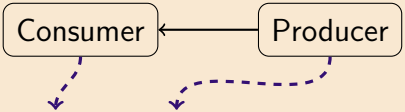


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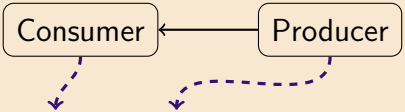


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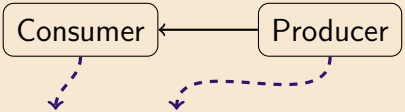


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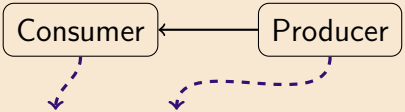


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                ⇒  6 :: 7 :: []
                ⇒  [6; 7]
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Joining Two Sequences

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In[3]:  let rec appendq xq yq =
          match xq with
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          | Cons (x, xf) -> Cons (x, fun () -> appendq (xf ()) yq)

Out[3]: val appendq : 'a seq -> 'a seq -> 'a seq = <fun>

In[4]:  get 5 (appendq (from 0) (from 100))

Out[4]: - : int list = [0; 1; 2; 3; 4]
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A *fair* alternative:

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In[5]:  let rec interleave xq yq =
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