

Foundations of Computer Science

Tree traversals

Dr. Robert Harle & Dr. Jeremy Yallop

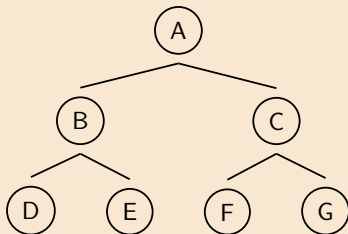
2020–2021

Tree traversal refers to visiting the nodes of each tree in a well-defined order.

preorder visits the label first (ABDECFG)

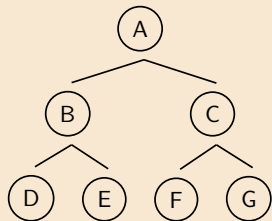
inorder visits the label midway (DBEAFCG)

postorder visits the label last (DEBFGCA)



preorder visits the label first (ABDECFG)

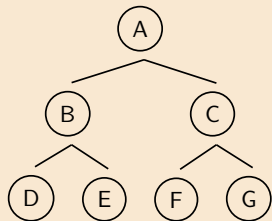
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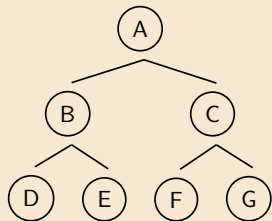
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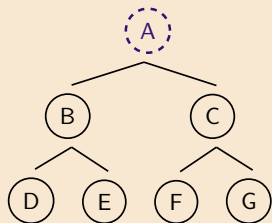
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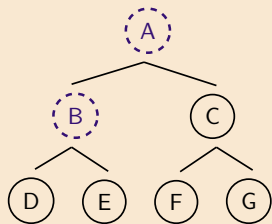
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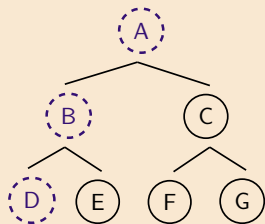
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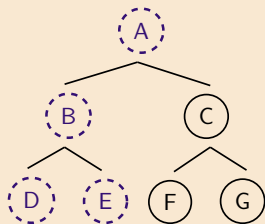
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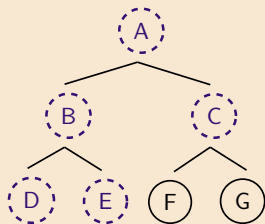
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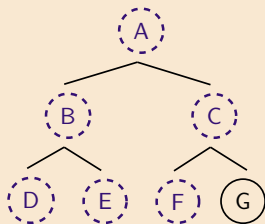
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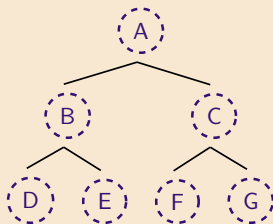
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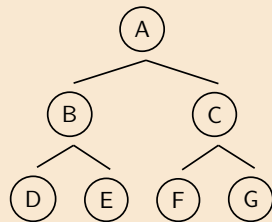
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inorder visits the label midway (DBEAFCG)

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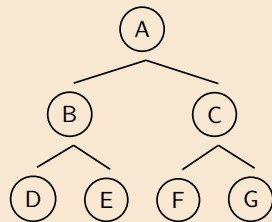
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Also imaginatively known as a treesort.

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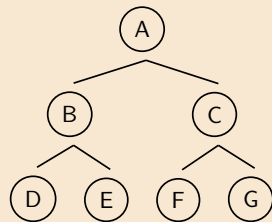


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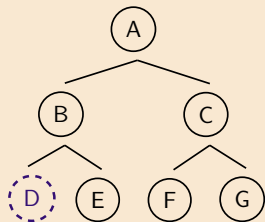
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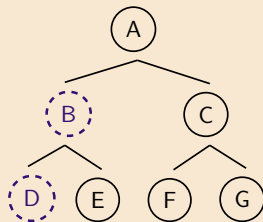


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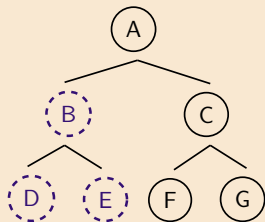


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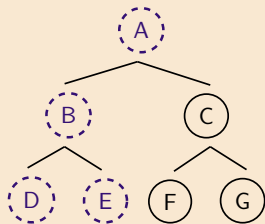


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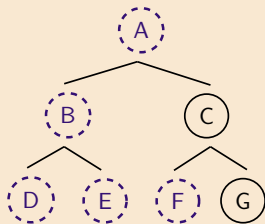


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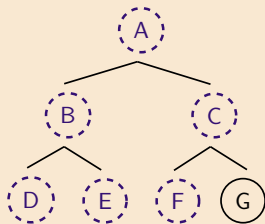


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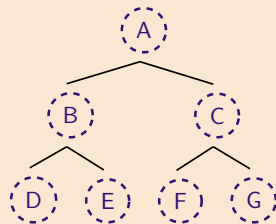


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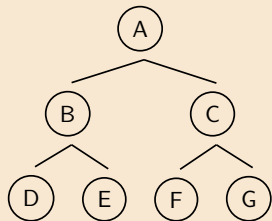


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postorder visits the label last (DEBFGCA)

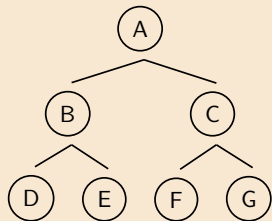
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In[3]: let rec postorder = function
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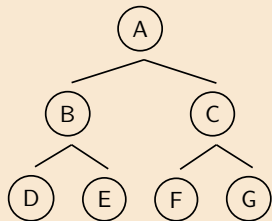
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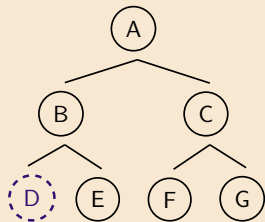
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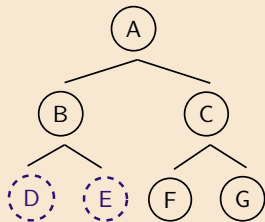
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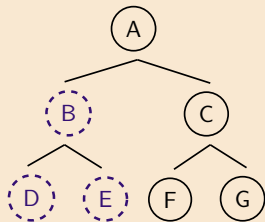
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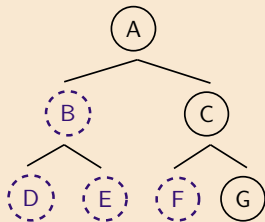
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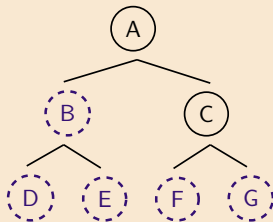
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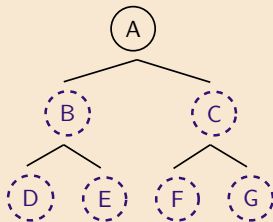
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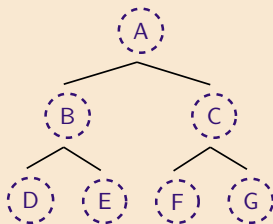
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preorder, inorder and postorder are **depth-first** traversal algorithms.

The other possibility is **breadth-first** by going across the levels of the tree.