Foundations of Computer Science

Dictionaries and binary search trees

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A **dictionary** attaches values to identifiers (known as **keys**)

Define the **operations** we want over the dictionary:

lookup	find an item in the dictionary
update / **insert**	replace / store an item in the dictionary
delete	remove an item from the dictionary
empty	the null dictionary with no keys
Missing	exception for errors in lookup and delete

Simplest representation for a dictionary is an association list (a list of key/value pairs).

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Lookup is $O(n)$

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In[2]: let rec lookup = function
         | [], a -> raise Missing
         | (x, y) :: pairs, a -> if a = x then y
                                 else lookup (pairs, a)

Out[2]: val lookup : ('a * 'b) list * 'a -> 'b = <fun>
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Update is $O(1)$

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But what is the **space usage**?

Use binary trees as a more efficient representation to get a better lookup complexity.

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type 'a tree = Lf
             | Br of 'a * 'a tree * 'a tree
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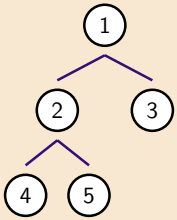

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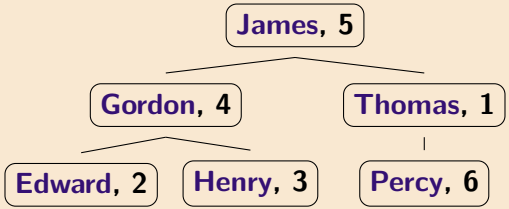

Each node holds a (key, value) with a total ordering for the keys

The **left** subtree holds **smaller** keys and the **right** subtree holds **larger** keys

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In[5]:  let rec lookup = function
            | Br ((a, x), t1, t2), b -> if b < a then lookup (t1, b)
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If **balanced** then lookup is $O(\log n)$

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Reconstruct the part of the structure that has changed; return the updated version.

OCaml shares the original structure; values pointing to the original remain unchanged.

This is also known as a **persistent data structure**

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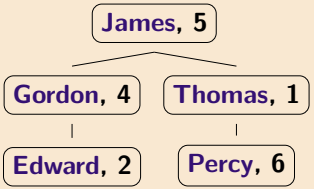

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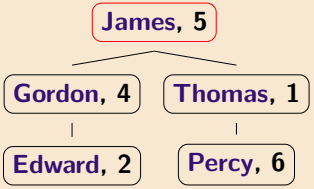

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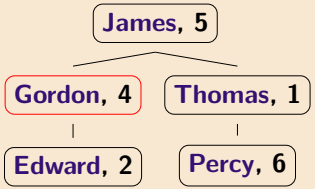

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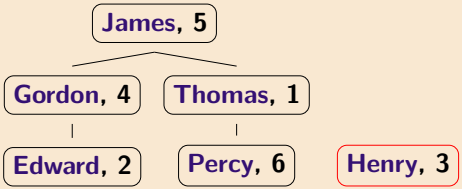

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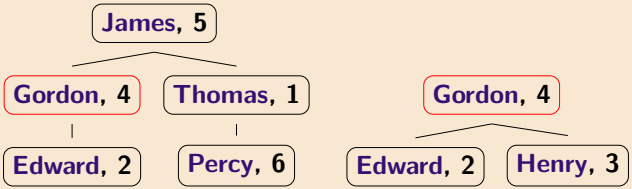

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