

Foundations of Computer Science

Enumerations and simple data types

Dr. Robert Harle & Dr. Jeremy Yallop

2020–2021

```
type vehicle =  
  Bike  
  | Motorbike  
  | ...
```

Custom types

```
exception NoChange
```

Exceptions

```
type t =  
  Null  
  | Join of t * t
```

Recursive types

This lecture introduces a powerful and distinctive feature of ML-style languages:

custom datatypes

With custom datatypes we can precisely describe the values used in our programs

```
In[1]: let number_of_wheels = function
      | "bike" -> 2
      | "motorbike" -> 2
      | "car" -> 4
      | "lorry" -> 18
Warning 8: this pattern-matching is not
exhaustive. Here is an example of a case
that is not matched: ""
```

```
Out[1]: val number_of_wheels : string -> int = <fun>
```

```
In[2]: number_of_wheels "bike"
```

```
Out[2]: ~ : int = 2
```

```
In[3]: number_of_wheels "motorbke"
```

```
Out: Exception: Match_failure ("//toplevel//", 1, 23).
```

How can we **make illegal states unrepresentable**?

```
In[1]: let number_of_wheels = function
```

```
    "bike" -> 2  
    | "motorbike" -> 2  
    | "car" -> 4  
    | "lorry" -> 18
```

```
Warning 8: this pattern-matching is not  
exhaustive. Here is an example of a case  
that is not matched: ""
```

```
Out[1]: val number_of_wheels : string -> int = <fun>
```

```
In[2]: number_of_wheels "bike"
```

```
Out[2]: ~ : int = 2
```

```
In[3]: number_of_wheels "motorbke"
```

```
Out: Exception: Match_failure ("//toplevel//", 1, 23).
```

How can we **make illegal states unrepresentable**?

```
In[1]: let number_of_wheels = function
```

```
    "bike" -> 2  
    | "motorbike" -> 2  
    | "car" -> 4  
    | "lorry" -> 18
```

```
Warning 8: this pattern-matching is not  
exhaustive. Here is an example of a case  
that is not matched: ""
```

```
Out[1]: val number_of_wheels : string -> int = <fun>
```

```
In[2]: number_of_wheels "bike"
```

```
Out[2]: ~ : int = 2
```

```
In[3]: number_of_wheels "motorbke"
```

```
Out: Exception: Match_failure ("//toplevel//", 1, 23).
```

How can we **make illegal states unrepresentable**?

```
In[1]: let number_of_wheels = function
```

```
    "bike" -> 2  
    | "motorbike" -> 2  
    | "car" -> 4  
    | "lorry" -> 18
```

```
Warning 8: this pattern-matching is not  
exhaustive. Here is an example of a case  
that is not matched: ""
```

```
Out[1]: val number_of_wheels : string -> int = <fun>
```

```
In[2]: number_of_wheels "bike"
```

```
Out[2]: ~ : int = 2
```

```
In[3]: number_of_wheels "motorbke"
```

```
Out: Exception: Match_failure ("//toplevel//", 1, 23).
```

How can we **make illegal states unrepresentable**?

```
In[1]: let number_of_wheels = function
```

```
    "bike" -> 2  
    | "motorbike" -> 2  
    | "car" -> 4  
    | "lorry" -> 18
```

```
Warning 8: this pattern-matching is not  
exhaustive. Here is an example of a case  
that is not matched: ""
```

```
Out[1]: val number_of_wheels : string -> int = <fun>
```

```
In[2]: number_of_wheels "bike"
```

```
Out[2]: ~ : int = 2
```

```
In[3]: number_of_wheels "motorbke"
```

```
Out: Exception: Match_failure ("//toplevel//", 1, 23).
```

How can we **make illegal states unrepresentable**?


```
In[1]: let number_of_wheels = function
```

```
    "bike" -> 2  
    | "motorbike" -> 2  
    | "car" -> 4  
    | "lorry" -> 18
```

```
Warning 8: this pattern-matching is not  
exhaustive. Here is an example of a case  
that is not matched: ""
```

```
Out[1]: val number_of_wheels : string -> int = <fun>
```

```
In[2]: number_of_wheels "bike"
```

```
Out[2]: - : int = 2
```

```
In[3]: number_of_wheels "motorbke"
```

```
Out: Exception: Match_failure ("//toplevel//", 1, 23).
```

How can we **make illegal states unrepresentable**?

```
In[1]: let number_of_wheels = function
```

```
    "bike" -> 2
```

```
    | "motorbike" -> 2
```

```
    | "car" -> 4
```

```
    | "lorry" -> 18
```

```
Warning 8: this pattern-matching is not  
exhaustive. Here is an example of a case  
that is not matched: ""
```

```
Out[1]: val number_of_wheels : string -> int = <fun>
```

```
In[2]: number_of_wheels "bike"
```

```
Out[2]: - : int = 2
```

```
In[3]: number_of_wheels "motorbke"
```

```
Out: Exception: Match_failure ("//toplevel//", 1, 23).
```

How can we **make illegal states unrepresentable**?

```
In[1]: let number_of_wheels = function
```

```
    "bike" -> 2  
    | "motorbike" -> 2  
    | "car" -> 4  
    | "lorry" -> 18
```

```
Warning 8: this pattern-matching is not  
exhaustive. Here is an example of a case  
that is not matched: ""
```

```
Out[1]: val number_of_wheels : string -> int = <fun>
```

```
In[2]: number_of_wheels "bike"
```

```
Out[2]: - : int = 2
```

```
In[3]: number_of_wheels "motorbke"
```

```
Out: Exception: Match_failure ("//toplevel//", 1, 23).
```

How can we **make illegal states unrepresentable**?

```
type vehicle = Bike  
             | Motorbike  
             | Car  
             | Lorry
```

```
type vehicle = Bike  
              | Motorbike  
              | Car  
              | Lorry
```

We have declared a new type vehicle

```
type vehicle = Bike  
            | Motorbike  
            | Car  
            | Lorry
```

We have declared a new type vehicle

Instead of representing any string it can only contain the four constants defined.

```
type vehicle = Bike  
            | Motorbike  
            | Car  
            | Lorry
```

We have declared a new type vehicle

Instead of representing any string it can only contain the four constants defined.

These four constants become the **constructors** of the vehicle type

```
type vehicle = Bike  
              | Motorbike  
              | Car  
              | Lorry
```

We have declared a new type vehicle

Instead of representing any string it can only contain the four constants defined.

These four constants become the **constructors** of the vehicle type

The **representation** in memory is more efficient than using strings.


```
type vehicle = Bike  
                | Motorbike  
                | Car  
                | Lorry
```

We have declared a new type vehicle

Instead of representing any string it can only contain the four constants defined.

These four constants become the **constructors** of the vehicle type

The **representation** in memory is more efficient than using strings.

Adding new types of vehicles is straightforward by extending the definitions.

```
type vehicle = Bike  
              | Motorbike  
              | Car  
              | Lorry
```

We have declared a new type vehicle

Instead of representing any string it can only contain the four constants defined.

These four constants become the **constructors** of the vehicle type

The **representation** in memory is more efficient than using strings.

Adding new types of vehicles is straightforward by extending the definitions.

Different custom types cannot be intermixed, unlike strings or integers.

```
In[4]: let wheels = function
| Bike -> 2
| Motorbike -> 2
| Car -> 4
| Lorry -> 18
```

```
Out[4]: val wheels : vehicle -> int = <fun>
```

```
In[5]: let wheels = function
| "bike" -> 2
| "motorbike" -> 2
| "car" -> 4
| "lorry" -> 18
```

```
Out[5]: val wheels : string -> int = <fun>
```

```
In[4]: let wheels = function  
      | Bike -> 2  
      | Motorbike -> 2  
      | Car -> 4  
      | Lorry -> 18
```

```
Out[4]: val wheels : vehicle -> int = <fun>
```

```
In[5]: let wheels = function  
      | "bike" -> 2  
      | "motorbike" -> 2  
      | "car" -> 4  
      | "lorry" -> 18
```

```
Out[5]: val wheels : string -> int = <fun>
```

```
In[4]: let wheels = function  
      | Bike -> 2  
      | Motorbike -> 2  
      | Car -> 4  
      | Lorry -> 18
```

```
Out[4]: val wheels : vehicle -> int = <fun>
```

```
In[5]: let wheels = function  
      | "bike" -> 2  
      | "motorbike" -> 2  
      | "car" -> 4  
      | "lorry" -> 18
```

```
Out[5]: val wheels : string -> int = <fun>
```

```
In[4]: let wheels = function
      | Bike -> 2
      | Motorbike -> 2
      | Car -> 4
      | Lorry -> 18
```

```
Out[4]: val wheels : vehicle -> int = <fun>
```

```
In[5]: let wheels = function
      | "bike" -> 2
      | "motorbike" -> 2
      | "car" -> 4
      | "lorry" -> 18
```

```
Out[5]: val wheels : string -> int = <fun>
```

```
In[4]: let wheels = function  
      | Bike -> 2  
      | Motorbike -> 2  
      | Car -> 4  
      | Lorry -> 18
```

```
Out[4]: val wheels : vehicle -> int = <fun>
```

```
In[5]: let wheels = function  
      | "bike" -> 2  
      | "motorbike" -> 2  
      | "car" -> 4  
      | "lorry" -> 18
```

```
Out[5]: val wheels : string -> int = <fun>
```

Adding new vehicle types is straightforward: extend the definitions and **fix warnings**.

```
In[6]: let wheels = function
      | Bike -> 2
      | Motorbike -> 2
      | Car -> 4
      | Lorry -> 18
```

```
Out[6]: val wheels : vehicle -> int = <fun>
```

```
In[7]: let wheels = function
      | Bike -> 2
      | Motorbike -> 2
      | Car -> 4
Warning 8: this pattern-matching is not exhaustive.
Here is an example of a case that is not matched:
Lorry
```

```
Out[7]: val wheels : vehicle -> int = <fun>
```


Adding new vehicle types is straightforward: extend the definitions and **fix warnings**.

```
In[6]: let wheels = function
      | Bike -> 2
      | Motorbike -> 2
      | Car -> 4
      | Lorry -> 18
```

```
Out[6]: val wheels : vehicle -> int = <fun>
```

```
In[7]: let wheels = function
      | Bike -> 2
      | Motorbike -> 2
      | Car -> 4
Warning 8: this pattern-matching is not exhaustive.
Here is an example of a case that is not matched:
Lorry
```

```
Out[7]: val wheels : vehicle -> int = <fun>
```

Adding new vehicle types is straightforward: extend the definitions and **fix warnings**.

```
In[6]: let wheels = function
      | Bike -> 2
      | Motorbike -> 2
      | Car -> 4
      | Lorry -> 18
```

```
Out[6]: val wheels : vehicle -> int = <fun>
```

```
In[7]: let wheels = function
      | Bike -> 2
      | Motorbike -> 2
      | Car -> 4
Warning 8: this pattern-matching is not exhaustive.
Here is an example of a case that is not matched:
Lorry
```

```
Out[7]: val wheels : vehicle -> int = <fun>
```

Declaring functions on vehicles

Adding new vehicle types is straightforward: extend the definitions and **fix warnings**.

```
In[6]: let wheels = function
      | Bike -> 2
      | Motorbike -> 2
      | Car -> 4
      | Lorry -> 18
```

```
Out[6]: val wheels : vehicle -> int = <fun>
```

```
In[7]: let wheels = function
      | Bike -> 2
      | Motorbike -> 2
      | Car -> 4
```

```
Warning 8: this pattern-matching is not exhaustive.
Here is an example of a case that is not matched:
Lorry
```

```
Out[7]: val wheels : vehicle -> int = <fun>
```

Declaring functions on vehicles

Adding new vehicle types is straightforward: extend the definitions and **fix warnings**.

```
In[6]: let wheels = function
      | Bike -> 2
      | Motorbike -> 2
      | Car -> 4
      | Lorry -> 18
```

```
Out[6]: val wheels : vehicle -> int = <fun>
```

```
In[7]: let wheels = function
      | Bike -> 2
      | Motorbike -> 2
      | Car -> 4
```

Warning 8: this pattern-matching is not exhaustive.
Here is an example of a case that is not matched:
Lorry

```
Out[7]: val wheels : vehicle -> int = <fun>
```

Adding new vehicle types is straightforward: extend the definitions and **fix warnings**.

```
In[6]: let wheels = function
      | Bike -> 2
      | Motorbike -> 2
      | Car -> 4
      | Lorry -> 18
```

```
Out[6]: val wheels : vehicle -> int = <fun>
```

```
In[7]: let wheels = function
      | Bike -> 2
      | Motorbike -> 2
      | Car -> 4
```

Warning 8: this pattern-matching is not exhaustive.
Here is an example of a case that is not matched:
Lorry

```
Out[7]: val wheels : vehicle -> int = <fun>
```

Declaring functions on vehicles

OCaml generalises the notion of enumeration types to allow data to be stored alongside each variant.

```
type vehicle = Bike  
              | Motorbike of int  
              | Car of bool  
              | Lorry of int
```

```
In[8]: Bike
```

```
Out[8]: ~ : vehicle = Bike
```

```
In[9]: Motorbike 25
```

```
Out[9]: ~ : vehicle = Motorbike 25
```

```
In[10]: Car true
```

```
Out[10]: ~ : vehicle = Car true
```

Declaring functions on vehicles

OCaml generalises the notion of enumeration types to allow data to be stored alongside each variant.

```
type vehicle = Bike  
              | Motorbike of int  
              | Car of bool  
              | Lorry of int
```

```
In[8]: Bike
```

```
Out[8]: ~ : vehicle = Bike
```

```
In[9]: Motorbike 25
```

```
Out[9]: ~ : vehicle = Motorbike 25
```

```
In[10]: Car true
```

```
Out[10]: ~ : vehicle = Car true
```

Declaring functions on vehicles

OCaml generalises the notion of enumeration types to allow data to be stored alongside each variant.

```
type vehicle = Bike  
              | Motorbike of int  
              | Car of bool  
              | Lorry of int
```

```
In[8]: Bike
```

```
Out[8]: - : vehicle = Bike
```

```
In[9]: Motorbike 25
```

```
Out[9]: - : vehicle = Motorbike 25
```

```
In[10]: Car true
```

```
Out[10]: - : vehicle = Car true
```


Declaring functions on vehicles

OCaml generalises the notion of enumeration types to allow data to be stored alongside each variant.

```
type vehicle = Bike  
              | Motorbike of int  
              | Car of bool  
              | Lorry of int
```

```
In[8]: Bike
```

```
Out[8]: - : vehicle = Bike
```

```
In[9]: Motorbike 25
```

```
Out[9]: - : vehicle = Motorbike 25
```

```
In[10]: Car true
```

```
Out[10]: - : vehicle = Car true
```

Declaring functions on vehicles

OCaml generalises the notion of enumeration types to allow data to be stored alongside each variant.

```
type vehicle = Bike  
              | Motorbike of int  
              | Car of bool  
              | Lorry of int
```

```
In[8]: Bike
```

```
Out[8]: - : vehicle = Bike
```

```
In[9]: Motorbike 25
```

```
Out[9]: - : vehicle = Motorbike 25
```

```
In[10]: Car true
```

```
Out[10]: - : vehicle = Car true
```

Declaring functions on vehicles

OCaml generalises the notion of enumeration types to allow data to be stored alongside each variant.

```
type vehicle = Bike  
              | Motorbike of int  
              | Car of bool  
              | Lorry of int
```

```
In[8]: Bike
```

```
Out[8]: - : vehicle = Bike
```

```
In[9]: Motorbike 25
```

```
Out[9]: - : vehicle = Motorbike 25
```

```
In[10]: Car true
```

```
Out[10]: - : vehicle = Car true
```

Declaring functions on vehicles

OCaml generalises the notion of enumeration types to allow data to be stored alongside each variant.

```
type vehicle = Bike  
              | Motorbike of int  
              | Car of bool  
              | Lorry of int
```

```
In[8]: Bike
```

```
Out[8]: - : vehicle = Bike
```

```
In[9]: Motorbike 25
```

```
Out[9]: - : vehicle = Motorbike 25
```

```
In[10]: Car true
```

```
Out[10]: - : vehicle = Car true
```

Declaring functions on vehicles

```
type vehicle =  
| Bike  
| Motorbike of int    (* engine size in CCs *)  
| Car of bool    (* true if a Reliant Robin *)  
| Lorry of int    (* number of wheels *)
```

Even though the constructors have different data, they are all of type vehicle when wrapped by the constructor.

```
In[11]: [ Bike; Car true; Motorbike 450 ]  
Out[11]: ~ : vehicle list
```

```
type vehicle =  
| Bike  
| Motorbike of int  (* engine size in CCs *)  
| Car of bool  (* true if a Reliant Robin *)  
| Lorry of int  (* number of wheels *)
```

Even though the constructors have different data, they are all of type vehicle when wrapped by the constructor.

```
In[11]: [ Bike; Car true; Motorbike 450 ]
```

```
Out[11]: - : vehicle list
```

```
type vehicle =  
| Bike  
| Motorbike of int    (* engine size in CCs *)  
| Car of bool    (* true if a Reliant Robin *)  
| Lorry of int    (* number of wheels *)
```

Even though the constructors have different data, they are all of type vehicle when wrapped by the constructor.

```
In[11]: [ Bike; Car true; Motorbike 450 ]  
Out[11]: - : vehicle list
```

```
let wheels = function  
| Bike -> 2  
| Motorbike _ -> 2  
| Car robin -> if robin then 3 else 4  
| Lorry w -> w
```

A Bike has two wheels.

A Motorbike has two wheels.

A Reliant Robin has three wheels; all other cars have four.

A Lorry has the number of wheels stored with its constructor.