Animation II
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Character Animation

• Rigging
  – Attaching a skeleton to a model
  – Skeleton is key-framed to animate the model
Character Animation

• Rigging
  – Attach the bones to the model
  – Weights indicate how much a vertex is effected by a bone
Character Animation

• Rigging
  – Attach the bones to the model
Character Animation

• Rigging
  – Attach the bones to the model

\[ T(x) = \text{avg}(T_1, T_2, w_1, w_2) \]
Character Animation

• Rigging
  – How to blend (average) transformations

Linear Blend Skinning

Represent $T_i$ with $T_i$ in homogenous coordinates

$$T(x) = w_1(x)T_1 + w_2(x)T_2$$

$$x' = T(x)x$$

$$T(x) = \text{avg}(T_1, T_2, w_1, w_2)$$
Blended Rigid Transformations

• How to blend (average) transformations

Linear Blend Skinning

\[ w_1 T_1 + w_2 T_2 \]

\[ w_1 t_1 + w_2 t_2 \]
Translation

\[ w_1 R_1 + w_2 R_2 \]
Rotation

Not a valid rotation matrix!
Blended Rigid Transformations

• How to blend (average) transformations

Valid rotation matrix
\[ R^T = R^{-1} \]
\[ \det(R) = 1 \]

Linear blending
\[ (w_1 R_1 + w_2 R_2)^T \]
\[ = (w_1 R_1^T + w_2 R_2^T) \]
\[ \neq (w_1 R + w_2 R)^{-1} \]
Blended Rigid Transformations

- How to blend (average) transformations

Linear Blend Skinning: problems
Blended Rigid Transformations

• How to blend transformations

Manifold of rigid transformations

$$w_1 R_1 + w_2 R_2$$

Shortest path on the manifold
Rigid Transformations

- Manifold of rotations – SO (3)
  
  Valid rotation matrix
  
  \[ R^T = R^{-1} \]
  
  \[ \det(R) = 1 \]

- Manifold of rigid transformations – SE (3)

  \[ R^T = R^{-1} \]
  
  \[ \det(R) = 1 \]

  \[ T = \begin{pmatrix} R & t \\ 0 & 0 & 0 & 1 \end{pmatrix} \]
Rigid Transformations

• Matrices not convenient for blending
• Alternative representation: dual quaternions
Rigid Transformations

- Representing rigid transformations

Rotations with quaternions

Rigid motions with dual quaternions
Rotations

• Representing rotations with quaternions

\[ q = \cos \left( \frac{\theta}{2} \right) + s \sin \left( \frac{\theta}{2} \right) \]
Rotations

• Quaternions

\[ q = \cos \left( \frac{\theta}{2} \right) + s \sin \left( \frac{\theta}{2} \right) \]

\[ s = s_i i + s_j j + s_k k \]

\[ s_i^2 + s_j^2 + s_k^2 = 1 \]

\[ i^2 = j^2 = k^2 = ijk = -1 \]
Rotations

• Operations on quaternions

\[ q = \cos \left( \frac{\theta}{2} \right) + s \sin \left( \frac{\theta}{2} \right) \]

Conjugate

\[ q^* = \cos \left( \frac{\theta}{2} \right) - s \sin \left( \frac{\theta}{2} \right) = \cos \left( -\frac{\theta}{2} \right) + s \sin \left( -\frac{\theta}{2} \right) \]

Inverse (for unit quaternions)

\[ q^{-1} = q^* \]
Rotations

• Operations on quaternions

Multiplication

$$q_1 q_2 = (a_1 + b_1 i + c_1 j + d_1 k)(a_2 + b_2 i + c_2 j + d_2 k)$$

Norm

$$\|q\|^2 = qq^* = \cos^2 \left( \frac{\theta}{2} \right) + \|s\|^2 \sin^2 \left( \frac{\theta}{2} \right) = 1$$
Rotations

- Operations on quaternions

\[ q = \cos \left( \frac{\theta}{2} \right) + s \sin \left( \frac{\theta}{2} \right) \]

Power

\[ q^t = e^{t \log q} \]

\[ \log q = \frac{\theta}{2} s \quad e^q = \cos ||q|| + \frac{q}{||q||} \sin ||q|| \]
Rotations

• Operations on quaternions

\[ q = \cos \left( \frac{\theta}{2} \right) + s \sin \left( \frac{\theta}{2} \right) \]

Applying to location vectors

\[ \mathbf{v} = v_i \mathbf{i} + v_j \mathbf{j} + v_k \mathbf{k} \]

\[ \mathbf{v}' = q \mathbf{v} q^* \]
Rotations

• Blending quaternions

\[ s = s_1 = s_2 \]

interpolate(\(q_1, q_2, t\))

\[ \theta(t) = (1 - t)\theta_1 + t\theta_2 \]

\[ q(t) = \cos\left(\frac{\theta(t)}{2}\right) + s\sin\left(\frac{\theta(t)}{2}\right) \]
Rotations

• Blending quaternions
  – In general, $s_1 \neq s_2$
  – Spherical blending
    \[(q_2q_1^*)^t q_1\]
  – More than two rotations?
Rotations

• Blending quaternions

\[ q_1 \cdots q_n \quad \omega_1 \cdots \omega_n \]

– Good approximation:

\[ b = \sum_{i=1}^{n} \omega_i q_i \]
Rigid Transformations

- Rotation & translation
- Dual numbers

\[ \hat{x} = x_0 + \varepsilon x_\varepsilon \quad \varepsilon^2 = 0 \]

E.g. multiplication

\[ (a_0 + \varepsilon a_\varepsilon)(b_0 + \varepsilon b_\varepsilon) \]
\[ = a_0b_0 + \varepsilon(a_0b_\varepsilon + a_\varepsilon b_0) \]
Rigid Transformations

• Dual quaternions
  – Replace numbers in quaternions with dual numbers
  \[
  \hat{q} = \cos \left( \frac{\hat{\theta}}{2} \right) + \hat{s} \sin \left( \frac{\hat{\theta}}{2} \right)
  \]
  – Almost all operations & notations are the same
  – In particular:
  \[
  \hat{b} = \sum_{i=1}^{n} w_i \hat{q}_i
  \]
Rigid Transformations

• Representing rigid transformations

- Quaternions: 4 numbers
- Dual quaternions: 8 numbers
Blended Rigid Transformations

• Properties

\[ \hat{b} = \sum_{i=1}^{n} w_i \hat{q}_i \]

1. Generates valid transformations
   - Only if normalized!

\[ \hat{b} = \frac{\sum_{i=1}^{n} w_i \hat{q}_i}{\| \sum_{i=1}^{n} w_i \hat{q}_i \|} \]
Blended Rigid Transformations

- Properties

\[ \hat{b} = \frac{\sum_{i=1}^{n} w_i \hat{q}_i}{\left\| \sum_{i=1}^{n} w_i \hat{q}_i \right\|} \]

2. Coordinate invariance
Blended Rigid Transformations

• Properties

\[ \hat{b} = \frac{\sum_{i=1}^{n} w_i \hat{q}_i}{\left\| \sum_{i=1}^{n} w_i \hat{q}_i \right\|} \]

3. Shortest path on SE (3)

Shortest path on the manifold
Blended Rigid Transformations

- Challenges
  - Blending transformations – dual quaternions
  - Weights $w_i(x)$
    - Shape adaptive
    - Intuitive deformations
    - Smooth deformations
Blended Rigid Transformations

- Weights – desired properties
  - Partition of unity
    \[ \sum_{i=1}^{n} w_i(x) = 1 \]
  - Smoothness
Blended Rigid Transformations

• Weights – desired properties
  – Shape-awareness

Shape-aware weights

Shape-unaware weights