Union axiom

Every collection of sets has a union.

\[ \bigcup \mathcal{F} \]

\[ x \in \bigcup \mathcal{F} \iff \exists X \in \mathcal{F}. x \in X \]
For non-empty $\mathcal{F}$ we also have

$$\bigcap \mathcal{F}$$

defined by

$$\forall x. \ x \in \bigcap \mathcal{F} \iff (\forall X \in \mathcal{F}. x \in X).$$
Disjoint unions

Definition 94  The disjoint union $A \uplus B$ of two sets $A$ and $B$ is the set

$$\ A \uplus B = (\{1\} \times A) \cup (\{2\} \times B) \ .$$

Thus,

$$\forall x. x \in (A \uplus B) \iff (\exists a \in A. x = (1, a)) \lor (\exists b \in B. x = (2, b)) \ .$$
Proposition 96  For all finite sets $A$ and $B$, 

$$A \cap B = \emptyset \implies \#(A \cup B) = \#A + \#B .$$

**Proof idea:**

Corollary 97  For all finite sets $A$ and $B$, 

$$\#(A \cup B) = \#A + \#B .$$