Union axiom

$\boxed{\text{Every collection of sets has a union.}}$

$$\bigcup \mathcal{F}$$

$$x \in \bigcup \mathcal{F} \iff \exists X \in \mathcal{F}.\, x \in X$$

For *non-empty* $\mathcal{F}$ we also have

$$\bigcap \mathcal{F}$$

defined by

$$\forall x.\ x \in \bigcap \mathcal{F} \iff \left(\forall X \in \mathcal{F}.\, x \in X \right) \quad .$$

Disjoint unions

Definition 94 *The* disjoint union $A \uplus B$ *of two sets* A *and* B *is the set*

$$A \uplus B \;=\; \big(\{1\} \times A\big) \cup \big(\{2\} \times B\big) \quad .$$

NB: $\big(\{1\}\times A\big) \cap \big(\{2\}\times B\big) = \emptyset$

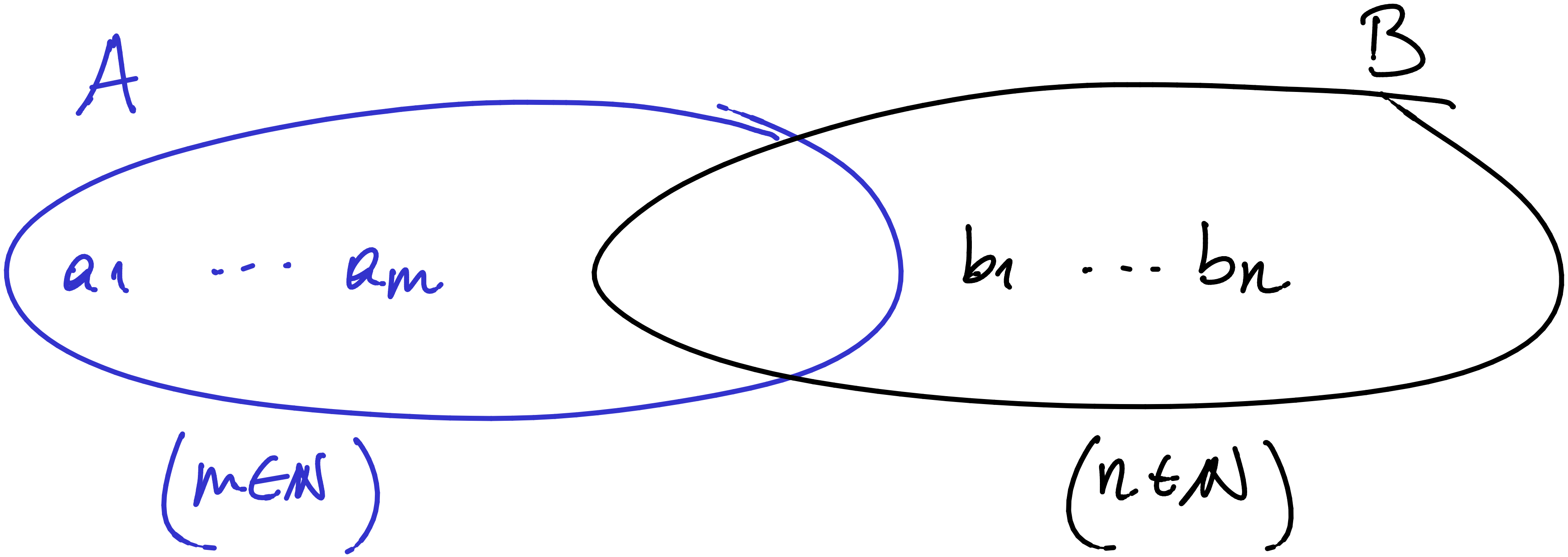
Thus,

$$\forall\, x.\, x \in (A \uplus B) \iff \big(\exists a \in A.\, x = (1, a)\big) \vee \big(\exists b \in B.\, x = (2, b)\big)\,.$$

Proposition 96 *For all finite sets* A *and* B,

$$A \cap B = \emptyset \implies \#(A \cup B) = \#A + \#B \quad .$$

PROOF IDEA:



Corollary 97 *For all finite sets* A *and* B,

$$\#(A \uplus B) = \#A + \#B \quad .$$