Euclid’s infinitude of primes

Theorem 80  The set of primes is infinite.

Proof: We proceed by contradiction. Assume that the set of primes is finite. We may then define

\[ c = \prod (p_1, \ldots, p_e) + 1 \]

the sequence of all primes

As \( c > p_i \) for all \( 1 \leq i \leq e \), it is not a prime and so by the Fundamental Theorem of Arithmetic, it is a product of primes. Let \( p \) be one of its prime factors. Then, \( plc \) and as \( plc-1 \) it follows that \( plc-1 \) a contradiction.