

Euclid's infinitude of primes

Theorem 80 *The set of primes is infinite.*

PROOF: We proceed by contradiction. Assume that the set of primes is finite. We may then define

$$c = \prod (p_1, \dots, p_l) + 1$$

the sequence of all primes

As $c > p_i$ for all $1 \leq i \leq l$; it is not a prime and so by the Fundamental Theorem of Arithmetic it is a product of primes. Let p be one of its prime factors. Then, $p | c$ and as $p | c - 1$ it follows that $p | 1$; a contradiction. 