Important mathematical jargon: Sets

Very roughly, sets are the mathematicians’ data structures. Informally, we will consider a set as a (well-defined, unordered) collection of mathematical objects, called the elements (or members) of the set.

Examples

\[ \mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C} \]
Example

$n$ is a natural number

$n \in \mathbb{N}$

Set membership

The symbol ‘$\in$’ known as the *set membership* predicate is central to the theory of sets, and its purpose is to build statements of the form

$$x \in A$$

that are true whenever it is the case that the object $x$ is an element of the set $A$, and false otherwise.
| The set                  | of even primes of booleans $[-2..3]$ | is | \{2\} | \{true, false\} | \{-2, -1, 0, 1, 2, 3\} | \{0, -2, 1, 2, -1, 3\} |
Examples

- The set of even primes is \( \{ x \in \mathbb{N} : 2 \mid x \land x \text{ is prime} \} \)

Set comprehension

The basic idea behind set comprehension is to define a set by means of a property that precisely characterises all the elements of the set.

Notations:

- \( \{ x \in A \mid P(x) \} \), \( \{ x \in A : P(x) \} \)

- The set \([-2..3]\) is \( \{ x \in \mathbb{Z} \mid x = -2 \lor x = -1 \lor x = 0 \lor x = 1 \lor x = 2 \lor x = 3 \} \)
Set Equality

Two sets are equal precisely when they have the same elements.

Example:

- \( \{ x \in \mathbb{N} : 2 \mid x \land x \text{ is prime} \} = \{ 2 \} \)
- For a positive integer \( m \), \( \{ x \in \mathbb{Z} : m \mid x \} = \{ x \in \mathbb{Z} \mid x \equiv 0 \pmod{m} \} \)
- \( \{ d \in \mathbb{N} : d \mid 0 \} = \mathbb{N} \)
Equivalent predicates specify equal sets

\[ \{ x \in A \mid P(x) \} = \{ x \in A \mid Q(x) \} \]

iff

\[ \forall x \in A. \; P(x) \iff Q(x) \]

Example: For a positive integer \( m \),

\[ \{ x \in \mathbb{Z}_m \mid x \text{ has a reciprocal in } \mathbb{Z}_m \} \]

= \[ \{ x \in \mathbb{Z}_m \mid 1 \text{ is an integer linear combination of } m \text{ and } x \} \]