Important mathematical jargon: Sets

Very roughly, sets are the mathematicians' data structures.

Informally, we will consider a ***set*** as a (well-defined, unordered) collection of mathematical objects, called the ***elements*** (or ***members***) of the set.

Examples

$\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$

Example

n is a natural number

is equivalently

$n \in \mathbb{N}$

Set membership

The symbol '$\in$' known as the *set membership* predicate is central to the theory of sets, and its purpose is to build statements of the form

$$x \in A$$

that are true whenever it is the case that the object x is an element of the set A, and false otherwise.

Defining sets

The set	of even primes	is	$\{2\}$
	of booleans		$\{\mathbf{true}, \mathbf{false}\}$
	$[-2..3]$		$\{-2, -1, 0, 1, 2, 3\}$

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$\{0, -2, 1, 2, -1, 3\}$

Examples

- The set of even primes is $\{x \in \mathbb{N} : 2 | x \wedge x \text{ is prime}\}$

Set comprehension

The basic idea behind set comprehension is to define a set by means of a property that precisely characterises all the elements of the set.

Notations:

$$\{x \in A \mid P(x)\} \quad , \quad \{x \in A : P(x)\}$$

- The set $[-2..3]$ is $\{x \in \mathbb{Z} \mid x = -2 \vee x = -1 \vee x = 0 \vee x = 1 \vee x = 2 \vee x = 3\}$

Set Equality

Two sets are equal precisely when they have the same elements

Example:

- $\{x \in \mathbb{N} : 2 | x \wedge x \text{ is prime}\} = \{2\}$
- For a positive integer m,
  $\{x \in \mathbb{Z} : m | x\} = \{x \in \mathbb{Z} \mid x \equiv 0 \pmod{m}\}$
- $\{d \in \mathbb{N} : d | 0\} = \mathbb{N}$

Equivalent predicates specify equal sets

$$\{x \in A \mid P(x)\} = \{x \in A \mid Q(x)\}$$

iff

$$\forall x \in A.\ P(x) \Leftrightarrow Q(x)$$

Example: For a positive integer m,

$$\{x \in \mathbb{Z}_m \mid x \text{ has a reciprocal in } \mathbb{Z}_m\}$$

$=$

$$\{x \in \mathbb{Z}_m \mid 1 \text{ is an integer linear combination of } m \text{ and } x\}$$