#### Important mathematical jargon: Sets

Very roughly, sets are the mathematicians' data structures. Informally, we will consider a <u>set</u> as a (well-defined, unordered) collection of mathematical objects, called the <u>elements</u> (or <u>members</u>) of the set.

Examples IN, 7t, Q, R, C

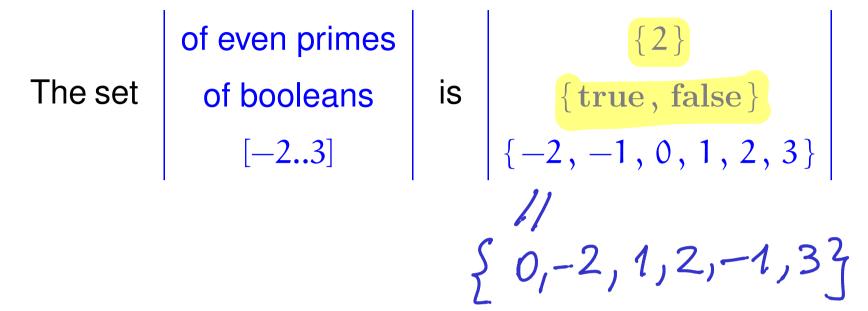
## Example N is a natural number is equivalently N E M Set membership

The symbol ' $\in$ ' known as the *set membership* predicate is central to the theory of sets, and its purpose is to build statements of the form

#### $\mathbf{x} \in \mathbf{A}$

that are true whenever it is the case that the object x is an element of the set A, and false otherwise.

### Defining sets



# Examples The set of even primes 15 £x ∈ N: 21x ∧ x 15 prime }

#### Set comprehension

The basic idea behind set comprehension is to define a set by means of a property that precisely characterises all the elements of the set.

Notations:

{ $x \in A \mid P(x)$ }, { $x \in A : P(x)$ } The set [-2..3] ij { $x \in 72$  |  $x = -2 \lor x = -1 \lor x = 0 \lor x = 1 \lor x = 2 \lor x = 3$ } -178 -

Set Equality. Two sets are equal precisely when they have the same elements Ezample: •  $f x \in \mathbb{N}$ :  $2|x \wedge x|s$  prime f = f 2 f• For a positive integer m,  $\{x \in \mathbb{Z} : m \mid x \} = \{x \in \mathbb{Z} \mid x \equiv 0 (n v d m)\}$ 

Equivalent predicates specify equal sets  $\begin{aligned} & \int \mathcal{E}_{x \in A} | P(x)^2 = \int z \in A | Q(x)^2 \\ & \forall x \in A . P(x) \rightleftharpoons Q(x) \end{aligned}$ Example: For a positive integer m, = {x \in Zm | x hes a reciprocal in Zm } = {x \in Zm | 1 is on integer hineor combination } of m and x