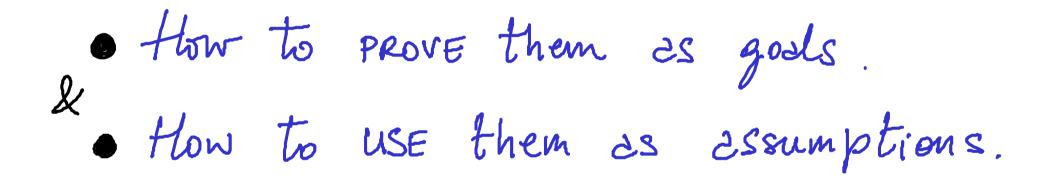
CONJUNCTIONS



Conjunction

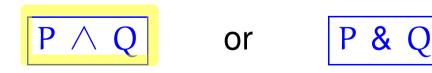
Conjunctive statements are of the form

P and Q

or, in other words,

both P and also Q hold

or, in symbols,



The proof strategy for conjunction:

To prove a goal of the form

$P\,\wedge\,Q$

first prove P and subsequently prove Q (or vice versa).

Proof pattern:

In order to prove

$P \wedge Q$

- 1. Write: Firstly, we prove P. and provide a proof of P.
- 2. Write: Secondly, we prove Q. and provide a proof of Q.

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The use of conjunctions:

To use an assumption of the form $P \land Q$, treat it as two separate assumptions: P and Q. **Theorem 20** For every integer n, we have that $6 \mid n$ iff $2 \mid n$ and $3 \mid n$.

PROOF: Let n be an integer. $RTP: G[n \rightleftharpoons) (2[n \land 3[n])$ $RTP: (1) G[n \Longrightarrow) (2[n \land 3[n])$ and $(2) (2[n \land 3[n]) \Longrightarrow G[n])$

For RTP(1) $G(n =)(2|n \land 3|n)$ Assume 6/n, That is, n=6k for on integerk. $RTP: 2ln \wedge 3ln$ That is, $RTP: 2 \ln$ We have n = 6k = 2(3k) $RTP: 3 \ln$ We have n = 6k = 3(2k)Hence 3/n Hence 21n.

For $RTP(2)(2\ln \sqrt{3}\ln) \Longrightarrow 6\ln$ Assume: 21n ~ 31n (*) RTP: 61n Using (*), we have (I) n=20 for on integer a and (II) n=35 for an integer b RTP: n = 6c for an integer c from (I) and (II).

(I) n = 2a for an integer a and (II) n = 35 for an integer b From $RTP: n \stackrel{?}{=} 6c$ for an integer c $(I) \implies 3n = 6a$ $(II) \implies 2n = 65$ \implies n = 3n - 2n= 6a - 6b= 6(a-b)



Con you either prove or disprove The following statements? · For all integers n, $30|n \iff (2|n \land 3|n \land 5|n).$ · For all integers i,j,k, $(i\cdot j)|k \iff (i|k \land j|k)$. NB: To disprore à statement one needs to provide à counter example to it.