CONJUNCTIONS

- How to prove them as goals.

&

- How to use them as assumptions.

Conjunction

Conjunctive statements are of the form

$$\boxed{P \textbf{ and } Q}$$

or, in other words,

$$\boxed{\text{both } P \text{ and also } Q \text{ hold}}$$

or, in symbols,

$$\boxed{P \wedge Q} \quad \text{or} \quad \boxed{P \,\&\, Q}$$

The proof strategy for conjunction:

To prove a goal of the form

$$P \wedge Q$$

first prove P and subsequently prove Q (or vice versa).

Proof pattern:

In order to prove

$$P \wedge Q$$

1. Write: Firstly, we prove P. and provide a proof of P.
2. Write: Secondly, we prove Q. and provide a proof of Q.

Scratch work:

Before using the strategy

Assumptions	Goal
	$P \wedge Q$
$\vdots$	

After using the strategy

Assumptions	Goal	Assumptions	Goal
	P		Q
$\vdots$		$\vdots$	

The use of conjunctions:

To use an assumption of the form $P \wedge Q$,
treat it as two separate assumptions: P and Q.

Theorem 20 *For every integer* n, *we have that* $6 \mid n$ *iff* $2 \mid n$ *and* $3 \mid n$.

PROOF: Let n be an integer.

<u>RTP</u>: $6 \mid n \iff (2 \mid n \wedge 3 \mid n)$

<u>RTP</u>: (1) $6 \mid n \implies (2 \mid n \wedge 3 \mid n)$

and (2) $(2 \mid n \wedge 3 \mid n) \implies 6 \mid n$

For <u>RTP</u> (1) $6 \mid n \Rightarrow (2 \mid n \wedge 3 \mid n)$

<u>Assume</u> $6 \mid n$, That is, $n = 6k$ for an integer k.

<u>RTP</u>: $2 \mid n \wedge 3 \mid n$

That is,

<u>RTP</u>: $2 \mid n$

We have

$n = 6k = 2(3k)$

Hence $2 \mid n$.

<u>RTP</u>: $3 \mid n$

We have

$n = 6k = 3(2k)$

Hence

$3 \mid n$

For <u>RTP</u> (2) $(2|n \wedge 3|n) \Rightarrow 6|n$

<u>Assume</u>: $2|n \wedge 3|n$ $(*)$

<u>RTP</u>: $6|n$

Using $(*)$, we have

(I) $n = 2a$ for an integer a

and (II) $n = 3b$ for an integer b

<u>RTP</u>: $n \overset{?}{=} 6c$ for an integer c

from (I) and (II).

From

(I) $n = 2a$ for an integer a

and (II) $n = 3b$ for an integer b

<u>RTP</u>: $n \stackrel{?}{=} 6c$ for an integer c

$$(\mathrm{I}) \Rightarrow 3n = 6a$$

$$(\mathrm{II}) \Rightarrow 2n = 6b$$

$$\Rightarrow n = 3n - 2n$$

$$= 6a - 6b$$

$$= 6(a-b)$$

$\boxtimes$

Can you either prove or disprove the following statements?

- For all integers n,
  $$30 \mid n \iff (2 \mid n \wedge 3 \mid n \wedge 5 \mid n).$$
- For all integers i, j, k,
  $$(i \cdot j) \mid k \iff (i \mid k \wedge j \mid k).$$

<u>NB</u>: To disprove a statement one needs to provide a counterexample to it.