2005 Paper 7 Question 8 / Paper 8 Question 10

(Computer Science Tripos Part II)

Information Theory and Coding (MGK)

(*a*) Give a bit-string representation of the number 13 in

(*i*) unary code for non-negative integers; [1 mark]

(*ii*) Golomb code for non-negative integers with parameter $b = 3$; [2 marks]

(*iii*) Elias gamma code for positive integers. [2 marks]

(*b*) Briefly explain

(*i*) how a signal amplitude of 10 V is expressed in dBμV; [1 mark]

(*ii*) the YCrCb coordinate system. [4 marks]

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Information Theory and Coding – Solution notes

(*a*)

(*i*) $11111111111110 = 1^{13}0$
The unary code word for 13 is simply 13 ones, followed by a final zero.

(*ii*) $1111010 = 1^4 0\ 10$
We first divide $n = 13$ by $b = 3$ and obtain the representation $n = q \times b + r = 4 \times 3 + 1$ with remainder $r = 1$. We then encode $q = 4$ as the unary code word "11110". To this we need to attach an encoding of $r = 1$. Since r could have a value in the range $\{0, \ldots, b-1\} = \{0, 1, 2\}$, we first use all $\lfloor \log_2 b \rfloor = 1$-bit words that have a leading zero (here only "0" for $r = 0$), before encoding the remaining possible values of r using $\lceil \log_2 b \rceil = 2$-bit values that have a leading one (here "10" for $r = 1$ and "11" for $r = 2$).

(*iii*) $1110101 = 1^3 0\ 101$
We first determine the length indicator $m = \lfloor \log_2 13 \rfloor = 3$ (because $2^3 \leq 13 < 2^4$) and encode it using the unary code word "1110", followed by the binary representation of 13 (1101_2) with the leading one removed: "101".

[This question relates to variable-length codes.]

(*b*)

(*i*) $10\ \text{V} = 10^7\ \mu\text{V} = (20 \times 7)\ \text{dB}\mu\text{V} = 140\ \text{dB}\mu\text{V}$

(*ii*) Human colour vision splits the red/green/blue input signal into separate luminosity and colour channels. Compression algorithms can achieve a simple approximation of this by taking a linear combination of about 30% red, 60% green, and 10% blue as the luminance signal $Y = 0.3R + 0.6G + 0.1B$ (the exact coefficients differ between standards and do not matter here). The remaining colour information can be preserved, without adding redundancy, in the form of the difference signals $R - Y$ and $B - Y$. These are usually encoded scaled as $Cb = (B - Y)/2 + 0.5$ and $Cr = (R - Y)/1.6 + 0.5$, such that the colour cube remains, after this "rotation", entirely within the encoded unit cube, assuming that the original RGB values were all in the interval $[0, 1]$.

[This question relates to (*i*) the section on perceptual scales, (*ii*) the section on colour coordinates.]