

Complexity Theory

Lent 2003

Suggested Exercises 1

1. In the lecture, a proof was sketched showing a $\Omega(n \log n)$ lower bound on the complexity of the sorting problem. It was also stated that a similar analysis could be used to establish the same bound for the Travelling Salesman Problem. Give a detailed sketch of such an argument. Can you think of a way to improve the lower bound?

2. On page 10 of the notes, a number of functions are listed as being constructible. Show that this is the case by giving, for each one, a description of an appropriate Turing machine.

Prove that if f and g are constructible functions and $f(n) \geq n$, then so are $f(g)$, $f + g$, $f \cdot g$ and 2^f .

3. For any constructible function f , and any language $L \in \text{NTIME}(f(n))$, there is a nondeterministic machine M that accepts L and all of whose computations terminate in time $O(f(n))$ for all inputs of length n . Give a detailed argument for this statement, describing how M might be obtained from a machine accepting L in time $f(n)$.

4. Consider the language L in the alphabet $\{a, b\}$ given by $L = \{a^n b^n \mid n \in \mathbb{N}\}$. $L \notin \text{SPACE}(c)$ for any constant c . Why?