UNIVERSITY OF CAMBRIDGE
Computer Laboratory

Computer Science Tripos
Part IB [P]
Part II General [C]
Diploma in Computer Science [D]

Compiler Construction

http://www.cl.cam.ac.uk/Teaching/2000/CompConstr

Alan Mycroft am@cl.cam.ac.uk

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Summary

This course starts with a brief survey of various features and concepts common to many programming languages, together with an outline of the kind of target code they require. The second part of the course covers the design of the various parts of a compiler.

The course is intended to study compilation of a range of languages and accordingly syntax for example constructs will be taken from various languages (with the intention that the particular choice of syntax is reasonably clear).

In terms of programming languages in which parts of compilers themselves are to be written, the preference varies between pseudo-code (as per the 'Data Structures and Algorithms' course) and language features (essentially) common to C/C++/Java. The language Standard ML (which the Diploma and Part II (general) students will see as part of the 'Functional Programming' course) is used when it significantly simplifies the code compared to C.

The following books contain material relevant to the course.

Compilers—Principles, Techniques, and Tools
A.V.Aho, R.Sethi and J.D.Ullman
Addison-Wesley (1986)
Ellis Horwood (1982)
Compiler Design in Java/C/ML (3 editions)
A.Appel
Cambridge University Press (1996)
Compiler Design
R.Wilhelm and D.Maurer
Addison Wesley (1995)
Introduction to Compiling Techniques
J.P.Bennett
McGraw-Hill (1990)
A Retargetable C Compiler: Design and Implementation
C.Frazer and D.Hanson
Benjamin Cummings (1995)
Compiler Construction
W.M.Waite and G.Goos
Springer-Verlag (1984)
High-Level Languages and Their Compilers
D.Watson
Addison Wesley (1989)

Acknowledgments

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Teaching and Learning Guide

[This is a Computer Laboratory mandated section of all lecture courses.]

The lectures largely follow the syllabus for the course which is as follows:

```
Survey of language constructs and their implementation.
  Expressions, applicative structure, lambda expressions. Environments
  and a simple lambda evaluator. Evaluation of function calls using
  static and dynamic chains, Dijkstra displays. Situations where a simple
  stack is inadequate. Objects and inheritance. Implementation of labels,
  jumps, arrays and exceptions. L-value and r-value; choices for argument
  passing and free-variable association. Dynamic and static binding.
  Dynamic and static types, polymorphism.

Survey of execution mechanisms.
  The spectrum of interpreters and compilers; compile-time and run-time.
  Structure of a simple compiler. Java virtual machine.

Lexical analysis and syntax analysis.
  Regular expressions and finite state machine implementations. Grammars,
  Chomsky classification of phrase structured grammars. Parsing
  algorithms: recursive descent, precedence and SLR(k). Syntax error
  recovery.

Simple type-checking.
  Type of an expression determined by type of subexpressions; inserting
  coercions.  Polymorphism.

Translation phase.
  Intermediate code design. Translation of commands, expressions and
  declarations. Translating variable references into access paths.

Code generation.
  Typical machine codes. Code generation from the parse tree and from
  intermediate code. Simple optimisation.

Compiler compilers.
  Summary of Lex and Yacc.

Runtime.
  Object modules and linkers. Resolving external references.
  Static and dynamic linking.  Debuggers, break points and
  single step execution. Profiling, portability.
```

A good source of exercises is the past 10 or 20 years' (sic) Tripos questions in that most of the basic concepts of block-structured languages and their compilation to stack-oriented code were developed in the 1960s. The course 'Optimising Compilation' in CST (part II) considers more sophisticated techniques for the later stages of compilation and the course 'Comparative Programming Languages' considers programming language concepts in rather more details.

Chapter 1

Introduction

1.1 Declarations, Expressions and Commands

A feature common to all computers is that they have memory and the fundamental ability to access and update the contents of arbitrary words of memory. Single computer instructions tend to cause simple changes to registers and memory. For example an instruction might have the effect of $x := e$ where x is a memory location or register and e is an expression constructed from memory locations and registers. The exact valid forms of such source expressions e and descriptions of destinations x depend strongly on the particular computer architecture.

The has led to high-level languages providing an abstract notion of *expression* which can be constructed arbitrarily from its constituents (instead of being hardware restricted). Similarly, they introduce the notion of assignment *command* which updates memory. In passing we note that the forms which appear on the left-hand-side of an assignment are rather more restricted than the right, the words *Lvalue* and *Rvalue* are often used here ("given $x := e$ we evaluate x to give a location (Lvalue) and e to give a Rvalue and then store the Rvalue in the Lvalue"). Also, instead of using numeric descriptions of memory locations we give them symbolic names (e.g. `a,b,fred`). Finally most languages have a mechanism for giving a symbolic name to a *function* or *procedure*.

So to summarise, the majority of languages have *declarations* which introduce (and possibly initialise) new names, *expressions* which correspond to a calculation (often without effect on memory) and *commands* which update memory or cause other actions like I/O. Both expressions and commands may use the names introduced in declarations.

For the moment we will imagine that declarations introduce names which can contain values (such names are often called variables). Declarations can often also introduce type-names which name types or classes; this course concentrates on the former use.

1.2 Assignments, Lvalues and Rvalues

A feature common to all computers is that they have memory and the fundamental ability to access and update the contents of arbitrary words of memory. Correspondingly, a basic feature of most programming languages is the assignment command. For example,

```
x := 3;
x := y+1;
x := x+1;
```

In some languages the form of the assignment can be more general:

```
i := (a>b ? j : k);
v[i] := v[a>b ? j : k];
v[a>b ? j : k] := v[i];
(a>b ? j : k) := i;
```

In the above examples the notation `E ? E1 : E2` denotes a conditional expression, and `v[i]` is a subscripted expression.

It is common to describe the syntactic form of such assignments as

```
E1 := E2;
```

where `E1` and `E2` are both expressions, and to describe the meaning (semantics) of the command as the execution of three steps.

1. Evaluate E1 to give an address.
2. Evaluate E2 to give a value.
3. Update the addressed cell with the value.

To avoid the overtones and confusion that go with the terms *address* and *value* we will use the more neutral words *Lvalue* and *Rvalue* (first coined by C. Strachey).

An Lvalue (left hand value) is the address (or location) of an area of store capable of holding the Rvalue.

An Rvalue (right hand value) is a bit pattern used to represent an object (such as an integer, a floating point number, a function, etc.).

1.3 Definition of Variables

Most languages allow the programmer to declare variables. In C/Java for example, variables may be declared as follows:

```
float p = 3.4;
float q = p;
```

This causes a new storage cell to be found and associated with the name (or identifier) `p`. The contents of this cell is initially set to the `float` number `3.4`. Then a second new storage cell (identified by `q`) is initialised with the contents of `p` (clearly `3.4`). Here the defining operator `=` is said to *define by value*. One can also imagine language constructs permitting *definition by reference* (also called *aliasing*) where the defining expression is evaluated in left hand mode (or Lmode) and the Lvalue obtained associated with the identifier, e.g.

```
float r ≃ p;
```

Since `p` and `r` have the same Lvalue they share the same storage cell and so the assignments: `p := 1.63` and `r := 1.63` will have the same effect, whereas assignments to `q` happen without affecting `p` and `r`. In C++ definition this by reference is written

```
float &r = p;
```

whereas (for ML experts) in ML mutable storage cells are defined explicitly so the above example would be expressed:

```
val p = ref 3.4;
val q = ref (!p);
val r = p;
```

Observe that Lmode evaluation (like Rmode evaluation) in general requires some computation to be done. Consider the program:

```
int i = 2;
int x ≃ m[i, i];
...
i := 3;
...// here x still refers to m[2,2]
```

Note that `m[i,i]` is an expression which (like `i`) can be evaluated in both L and R modes. Some expressions have only Rvalues (e.g. `x+1`) and so may not be used, for instance, on the left hand side of an assignment.

1.4 Names

The term name is often used to mean different things and confusion frequently results. It is sometimes, for instance, unwittingly used as a synonym for Lvalue. In these notes we will always use the word name to refer to the identifiers coined by a programmer and used in a program. The only property we require of names is that, given two of them, it is possible to determine whether they are the same (i.e. same sequence of letters, digits, etc.) or not.

It is the existence of *commands* which makes programming languages quite different from the rest of mathematics. We will first consider expressions in the absence of commands.

1.5 Expressions and Declarations

The characteristic feature of an expression is that it yields a value as a result. We have seen that it can be either an Lvalue or an Rvalue (depending on the context). However, without the assignment commands Lvalues are of no interest, and as in ordinary mathematics we shall, for the time being, only be concerned with Rvalues. (For the time being we will also assume that expressions do not contain operators with side effects such as `f(x++)` in C/Java.)

One consequence is that whether a variable is defined by copying or by aliasing is irrelevant for now—we return to this later.

Languages with only (side-effect-free) expressions and declarations are often called functional languages. We study them first because of their relative simplicity, but we take pains to ensure that each name is associated with a memory location so that later introducing an assignment operator is easy. The language ML, although it has non-functional parts, encourages a functional style (and the courses here on ML tend not to mention assignment).

A useful property of functional languages is *referential transparency* which means that the value of an expression only depends on the values of its subexpressions. This means that normal mathematical equivalences hold, e.g. $e + e = 2 * e$. However in the presence of side-effects this fails, e.g. in Java we have

$$\texttt{x++ + x++} \neq \texttt{2*(x++)}$$

One might fear than abandoning assignment leads to inexpressive languages. However, provided one keeps the idea of an initialised declaration (note initialisation differs conceptually from assignment) then functional languages are as powerful as other languages.

Because this part of the course is not concerned with any particular language, we will introduce an expression-based language of our own which captures ideas common in other languages. We will introduce an expression e to be one of the following forms:

- n, an integer;
- x, a name;
- $e_1 + e_2$, provided e_1 and e_2 are (smaller) expressions;
- $e_1 - e_2$, provided e_1 and e_2 are (smaller) expressions;
- $e_1 ? e_2 : e_3$, provided e_1, e_2 and e_3 are (smaller) expressions;
- $let\ x = e_1\ in\ e_2$, provided x is a name and e_1 and e_2 are (smaller) expressions. The phrase $x = e_1$ is seen as a *declaration*;
- $e_1 e_2$, (an application) provided e_1 and e_2 are (smaller) expressions;
- $\lambda x.e_1$, (a lambda-abstraction) provided e_1 is a (smaller) expression.

The first forms will be familiar to everyone. The final form $\lambda x.e_1$ is an (anonymous) function which takes an argument x and yields e_1. Thus the conventional function definition $f(x) = e$ would here be written $f = \lambda x.e$. We will later note that for many purposes $(\lambda x.e_2)e_1$ and $let\ x = e_1\ in\ e_2$ can be treated almost identically.

In examples, we will allow abstractions and applications to take multiple arguments and use additional operators from the above.

1.6 Environments

In order to evaluate `a+5+b/a` we need to know the values of `a` and `b`. We speak of evaluating an expression in an *environment* which provides the values of the names in the expression. One way to provide such an environment is by the above `let` form, e.g.

```
let a = 2+3/7 in a+3/a
let x = y+2/y in x+y+3/x
```

Yet another way is to use lambda calculus (described later):

```
(λa. a+3/a)(2+3/7)
(λx. x+y+3/x)(y+2/y)
```

(Note that in programming languages like ML, λ is often written `fn` and we will occasionally adopt this convention in these notes.) These two methods are exactly equivalent and have the same meaning. The name `y` in the second expression is not bound and its value must still be found in the environment in which the whole expression is to be evaluated. Variables of this sort are known as *free variables*. Variables with local definitions are known as *bound variables*.

Given an expression we can formally define its bound variables $BV(e)$ and its free variables $BV(e)$ inductively:

$$\begin{aligned}
BV(n) &= \{\} \\
BV(x) &= \{\} \\
BV(e_1 + e_2) &= BV(e_1) \cup BV(e_2) \\
BV(\lambda x.e) &= BV(e) \cup \{x\} \\
BV(\mathit{let}\ x = e_1\ \mathit{in}\ e_2) &= BV(e_1) \cup BV(e_2) \cup \{x\}
\end{aligned}$$

$$\begin{aligned}
FV(n) &= \{\} \\
FV(x) &= \{x\} \\
FV(e_1 + e_2) &= FV(e_1) \cup FV(e_2) \\
FV(\lambda x.e) &= FV(e) \setminus \{x\} \\
FV(\mathit{let}\ x = e_1\ \mathit{in}\ e_2) &= FV(e_1) \cup (FV(e_2) \setminus \{x\})
\end{aligned}$$

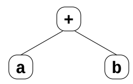
Note that the expected $BV(e) \cup FV(e) = \{\}$ does not always hold—consider

```
(let a = 2 in a)+a
```

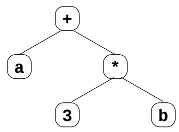
This has $FV(e) = \{\texttt{a}\}$ because of the final `a` (which is unbound) but also $BV(e) = \{\texttt{a}\}$ because of the `let` which binds `a` to 2 within the parentheses. Note also that this shows the idea of scope already exists such a simple language.

1.7 Applicative structure

Arithmetic expressions (and commands) can be represented as a tree structure which makes clear which operators act on which operands. For example, `a+b` can be represented as `+(a, b)` or diagrammatically as follows:



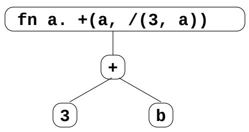
and `a+3*b` can be represented as `+(a, *(3,b))` or as



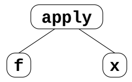
This structure is called the *applicative structure* of the expression. An alternative name is *syntax tree*. Applicative structures are difficult to read because of the deep nesting of brackets, but they help to emphasise the uniform way in which complicated expressions (and commands) can be built out of smaller ones. The rule for evaluation is simple and is as follows:

1. Evaluate the operands (in any order).
2. Apply the operator to the operand values.

One could regard a lambda abstraction as an operator of a complicated sort. For instance, the applicative structure of $\{\lambda$`a. a+3/a`$\}$`(3+b)` could be:



It is probably better to introduce a new basic operator `Apply` to deal with function applications, so that the structure for `f(x)` is:

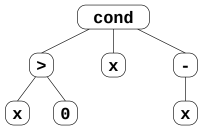


This maintains the convention that all applicative expression operators are constants of the system.

One apparent exception to the evaluation rule is the evaluation of the conditional expression. Consider

```
x>0 ? x : -x
```

One possible applicative form of this might be



where `Cond` is defined to have the rather special meaning of evaluating only one of the two alternatives depending on the result of the condition. This is necessary in `x=0 ? 1 : 1/x` for instance.

1.8 Lambda calculus

There are three main reasons for studying lambda calculus when considering programming languages. These are:

1. It is a useful notation for specifying the scope rules of identifiers in programming languages, and helps to demonstrate such notions as "holes in the scope of variables".

2. It helps one to understand what a function is, and what it means to pass a parameter.

3. There is a well established mathematical theory of lambda calculus.

The following are examples of lambda expressions:

```
λx. x
f(λg. g x)
f(x, y)
{λ(h,x). (h x) y} ((λt. t),(λr. r))
(λf. (λg. g g) (λg. (f g) g))
```

To enhance readability, certain identifiers (e.g. those spelt with non-alphanumeric characters) may be used as infixed dyadic operators. For example, `x+y*z` may be written to denote `+(x,*(y,z))`.

In an abstraction $\lambda x.e$, the identifier x is called the *bound variable* and the expression e is called the *body*.

Evaluation of lambda expressions is by means of two simple rewrite rules.

- **α-conversion.** The expression $(\lambda \mathtt{x}.e)$ can be replaced by $(\lambda \mathtt{y}.e')$ where `y` is any identifier that does not occur in e and where e' is e rewritten with all occurrences of the identifier `x` replaced by the identifier `y`.

- **β-reduction.** The expression $(\lambda \mathtt{x}.e_1)e_2$ can be replaced by a copy of e_1 with all occurrences of the bound variable `x` replaced by the argument e_2, provided that

  1. e_1 contains no (inner) bound occurrences of identifier `x`, and
  2. e_2 contains no free variables that are bound in e_1.

The α-conversion rule is used to remove conflicts that prevent β-reductions from being applicable.

To enhance readability in these notes, lists of bound variables enclosed in parentheses will be allowed after λ instead of a single variable and lists of arguments will be allowed in applications; this follows the ML practice.

A few example evaluations are given below.

$$
\begin{array}{rcl}
\{\lambda \mathtt{t}.\mathtt{t} + 16\}(4) & = & 20 \\
\{\lambda(\mathtt{a}, \mathtt{b}).\mathtt{a} + \mathtt{b}\}(5, 6) & = & 11 \\
\{\lambda \mathtt{f}.\{(\lambda \mathtt{x}.\mathtt{f}(\mathtt{x} + 1))(3)\}\}(\lambda \mathtt{y}.\mathtt{y} * 2) & = & (\lambda \mathtt{x}.\{\lambda \mathtt{y}.\mathtt{y} * 2\}(\mathtt{x} + 1))(3) \\
& = & \{\lambda \mathtt{y}.\mathtt{y} * 2\}(4) \\
& = & 8
\end{array}
$$

An important property of a lambda expression is that one can determine from the text of the expression (i.e. without having to evaluate it) to which bound variable each occurrence of a name is bound (unless it is free). A suitable algorithm is as follows:

Given an occurrence of the name x

1. Find the smallest textually enclosing lambda (or let) expression.
2. Compare x with the bound variables names, if there is a match we have finished, otherwise repeat from (1) to try the lambda expression one level further out.

1.9 The correspondence between programming languages and lambda calculus

Here we show that the `let` notation, both for introducing definitions of simple names and functions, can be eliminated in favour of λ.

Consider the following expression:

```
{ let f(y) = y*2
  in let x = 3
  in f(x+1)
}
```

In this `f` is a function with one bound variable `y` whose body is the expression `y*2`. We might write: `let f = (λy. y*2)`. Similarly `x` is like a bound variable of a lambda expression whose body is `f(x+1)` and whose argument is `3`. Thus we could re-write the whole expression as:

```
{ let f = λy. y*2
  in (λx. f(x+1)) (3)
}
```

which further can be re-written as:

```
{λf. (λx. f(x+1)) (3)} (λy. y*2)
```

Hence, the lambda notation can completely express both simple variable and function definitions. Indeed it can usefully be seen as a machine code in its own right (there was even a machine built at Cambridge some years back which used essentially λ-calculus as its machine code!). Just as we chose to prefer higher level notation than (say) Pentium machine code, one prefers the more usual `let` and function forms rather than the rebarbarative λ form for real programming—its real benefit is that of understanding concepts like scoping.

1.10 A short interlude on recursion

When a function is defined in terms of itself as in the following C/Java definition

```
int scantree(Tree *x) { ...
                         ... scantree(x->left) ...
                         ...
                         ... scantree(x->right) ...
                         ...
                       }
```

It is said to be *defined recursively*. If several functions are defined in terms of themselves they are said to be *mutually recursive*. Suppose there is a call `scantree(sometree)` to the function given above, then while this call is being evaluated it may happen that the call `scantree(x->left)` is executed. While this second call is active there are two *activations* of `scantree` in existence at once. The second call is said to be a *recursive call* of `scantree`. Note that therefore there will be two distinct variables called `x` (holding different values) in such circumstances; we therefore need to use fresh storage for variable `x` at each call to `scantree`. Below we see how to use a stack for this purpose.

Notice that it is possible to call a function recursively without defining it recursively.

```
let f(g,n) = { ...
               ... g(g,n-1)
               ...
             }
in f(f, 5)
```

Here the call `g(g,n-1)` is a recursive call of `f`.
[Exercise: complete the body of `f` so that the call yields $5! = 120$.]

1.11 The need for the word `rec`

Consider the following expression:

```
{ let f(n) = n=0 ? 1 : n*f(n-1)
  in f(4)
}
```

The corresponding lambda expression is

```
(λf. f(4)) (λn. n=0 ? 1 : n*f(n-1) )
```

We observe that the scope of `f` is `f(4)`, and that the `f` in `f(n-1)` is unbound and certainly different from the `f` in `f(4)`. However, here the programmer presumably was trying to define the recursive factorial function and so meant the `f` on the right hand side to be the same as the `f` he was defining. To indicate that the scope of x in (`let` x=e `in` e') extends to include both e as well as e' the keyword `rec` is normally used.

```
let rec f(n) = n=0 ? 1 : n*f(n-1)
```

which can be more primitively written as

```
let rec f = λn. n=0 ? 1 : n*f(n-1)
```

The linguistic effect of `rec` is to extend the the scope of the defined name to include the right hand side of the definition.

In ML, all `fun`-based definitions are assumed to be recursive, and so the definition

```
fun f(x) = e;
```

is first simplified (de-sugared) by the most ML systems to

```
val rec f = fn x => e;
```

1.12 The Y operator

At first sight, the `rec` construction seems to have no lambda calculus equivalent; however, postulating a new operator `Y` enables a solution to be found. Consider

```
let H = λf. λn. n=0 ? 1 : n*f(n-1)
```

`f` is now bound, but `H` is not the factorial function, for

```
H(λx. x)(6)  =  {λn. n=0 ? 1 : n*(λx. x)(n-1)} (6)
             =  {λn. n=0 ? 1 : n*(n-1)} (6)
             =  6*5
             =  30
```

However, if `g` were the factorial function, then

```
H(g)  =  λn. n=0 ? 1 : n*g(n-1)
      =  the factorial function
      =  g
```

thus `H(g)=g` if `g` is the factorial function. It therefore seems plausible that, if we can find a `g` for which `H(g)=g`, then the `g` we found would be the factorial function. Given an function, Φ say, any value v such that $\Phi(v) = v$ called a *fixed point* of Φ. (Observe that 1 and 2 are both fixed points of the ordinary function on reals given by $\phi(x) = x^2 - 2x + 2$, but note that our Φ will typically map functions to functions.)

In the same sense that the fixed points of a quadratic $ax^2 + bx + c$ can be found by a formula (this can be seen as a function which operates on ϕ and gives us x)

$$x = \frac{-(b-1) \pm \sqrt{(b-1)^2 - 4ac}}{2a}$$

we could *hope* for a function Y which returns the fixed point of its argument, e.g. $Y(\texttt{H}) =$ factorial or more generally

$$Y\Phi = \text{some value } f \text{ such that } \Phi f = f.$$

Put more simply we want

$$Y\Phi = \Phi(Y\Phi).$$

If this can be done, then we can rewrite any recursive function simply using Y, e.g. writing

```
let rec f = λn. n=0 ? 1 : n*f(n-1)
```

as

```
let f = Y( λf. λn. n=0 ? 1 : n*f(n-1) )
```

and we can evaluate a call of `f` knowing no more about Y than the property $Y\Phi = \Phi(Y\Phi)$. For example, with `H = λf. λn. n=0 ? 1 : n*f(n-1)`

```
f(3)  =  Y(H) (3)                             [definition of f]
      =  H(Y(H)) (3)                          [property of Y]
      =  {λn. n=0 ? 1 : n*(Y(H)(n-1))} (3)    [lambda reduction]
      =  3 * Y(H)(2)
      =  3 * 2 * Y(H)(1)                      [similarly]
      =  3 * 2 * 1 * Y(H)(0)                  [similarly]
      =  3 * 2 * 1 * 1                        [similarly]
```

It is somewhat remarkable at first that such a Y exists at all and moreover that it can be written just using λ and *apply*. One can write[1]

$$Y = \lambda f.\ (\lambda g.\ (f(\lambda a.\ (gg)a)))(\lambda g.\ (f(\lambda a.\ (gg)a))).$$

(Please note that learning this lambda-definition for Y is *not* examinable for this course!) For those entertained by the "Computation Theory" course, this (and a bit more argument) means that the lambda-calculus is "Turing powerful".

Note that the definition of Y given here will only find fixed points of functions, like H above of type $(int \to int) \to (int \to int)$ and in doing so yield a function of type $int \to int$. It will not find the numeric fixed points of

$$\lambda x.\ x^2 - 2x + 2.$$

(Why?)

Finally, an alternative implementation of Y (there seen as a primitive rather as the above arcane lambda-term) suitable for an interpreter is given in section 1.14.

1.13 Object-oriented languages

The view that lambda-calculus provides a fairly complete model for binding constructs in programming languages has generally been well-accepted. However, notions in inheritance in object-oriented languages seem to require a generalised notion of binding. Consider the following C++ program:

[1] The form

$$Y = \lambda f.\ (\lambda g.\ gg)(\lambda g.\ f(gg))$$

is usually quoted, but (for reasons involving the fact that our lambda-evaluator uses call-by-value and the above definition requires call-by-name) will not work on the lambda-evaluator presented here.

```
const int i = 1;
class A { const int i = 2; };
class B : A { int f(); };
int B::f() { return i; }
```

There are two `i` variables visible to `f()`: one being `i=1` by lexical scoping, the other `i=2` visible via inheritance. Which should win? C++ defines that the latter is visible (because the definition of `f()` essentially happens in the scope of `B` which is effectively nested within `A`). The `i=1` is only found if the inheritance hierarchy has no `i`. Note this argument still applies if the `const int i=1;` were moved two lines down the page. The following program amplifies that the definition of the order of visibility of variables is delicate:

```
const int i = 1;
class A { const int j = 2; };
void g()
{   const int i = 2;
    class B : A { int f() { return i; }; }
    // which i does f() see?
}
```

The lambda-calculus for years provided a neat understanding of scoping which language designers could follow simply; now such standards committees have to use their (not generally reliable!) powers of decision.

Note that here we have merely talked about (scope) *visibility* of identifiers; languages like C/Java also have declaration qualifier concerning *accessibility* (`public`, `private`, etc.). It is for standards bodies to determine whether, in the first example above, changing the declaration of `i` in `A` to be `private` should invalidate the program or merely cause the `private i` to become invisible so that the `i=1` declaration becomes visible within `B::f()`. (Actually draft ISO C++ checks accessibility after determining scoping.)

We will later return to implementation of objects and methods as data and procedures.

1.14 Mechanical evaluation of lambda expressions

We will now describe a simple way in which lambda expressions may be evaluated in a computer. We will represent the expression as an applicative structure and evaluate it in an environment that is initially empty. The applicative structure that we shall use has tree nodes representing variables, constants, addition, function abstraction and function application. In ML this can be written:

```
datatype Expr = Name of string |
                Numb of int |
                Plus of Expr * Expr |
                Fn of string * Expr |
                Apply of Expr * Expr;
```

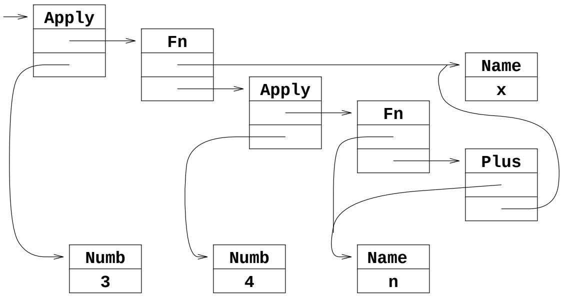
Later in the course we will see this also can be described by the context-free grammar

```
<Expr> -> <Name> | <Int> | <Expr>+<Expr>
                 | fn <Name>.<Expr> | <Expr>(<Expr>)
```

The expression: $\{\lambda$`x. (`λ`n. n+x)(4)`$\}$ `(3)` would be written in ML (or C, assuming appropriate (constructor) functions like `Apply`, `Fn` etc. were defined to allocated and initialise structures) as:

```
Apply(Fn("x", Apply(Fn("n", Plus(Name("n"), Name("x"))),
                    Numb(4))),
      Numb(3))
```

and be represented as follows:



When we evaluate such an `Expr` we expect to get a value which is either a integer or a function. For non-ML experts the details of this do not matter, but in ML we write this as

```
datatype Val = IntVal of int | FnVal of string * Expr * Env;
```

(the justification for why functions consist of more than simply their text will become apparent when we study the evaluator '`eval`' below).

We will represent the environment of defined names (names in scope) as a linked list with the following structure:

```
datatype Env = Empty | Defn of string * Val * Env;
```

(I.e. an `Env` value is either `Empty` or is a 3-tuple giving the most recent binding of a name to a value and the rest of the environment.) The function to look up a name in an environment[2] could be defined in ML as follows.

```
fun lookup(n, Defn(s, v, r)) =
            if s=n then v else lookup(n, r);
  | lookup(n, Empty) = raise oddity("unbound name");
```

We are now ready to define the evaluation function itself:

```
fun eval(Name(s), r) = lookup(s, r)
  | eval(Numb(n), r) = IntVal(n)
  | eval(Plus(e, e'), r) =
      let val v = eval(e,r);
          val v' = eval(e',r)
      in case (v,v') of (IntVal(i), IntVal(i')) => IntVal(i+i')
                      | (v, v') => raise oddity("plus of non-number")
      end
  | eval(Fn(s, e), r) = FnVal(s, e, r)
```

[2]There is a tradition of using letters like r or ρ for 'environment' to avoid clashing with the natural use of e for 'expression'.

```
| eval(Apply(e, e'), r) =
    case eval(e, r)
      of IntVal(i) => raise oddity("apply of non-function")
       | FnVal(bv, body, r_fromdef) =>
           let val arg = eval(e', r)
           in eval(body, Defn(bv, arg, r_fromdef))
           end;
```

The immediate action of `eval` depends on the leading operator of the expression it is evaluating. If it is `Name`, the bound variable is looked up in the current environment using the function `lookup`. If it is `Numb`, the value can be obtained directly from the node (and tagged as an `IntVal`). If it is `Plus`, the two operands are evaluated by (recursive) calls of `eval` using the current environment and their values summed (note the slightly tedious code to check both values correspond to numbers else to report an error). The value of a lambda expression (tagged as a `FnVal`) is called a *closure* and consists of three parts: the bound variable, the body and the current environment. These three components are all needed at the time the closure is eventually applied to an argument. To evaluate a function application we first evaluate both operands in the current environment to produce (hopefully) a closure (`FnVal(bv, body, r_fromdef)`) and a suitable argument value (`arg`). Finally, the body is evaluated in an environment composed of the environment held in the closure (`r_fromdef`) augmented by `(bv, arg)`, the bound variable and the argument of the call.

At this point it is appropriate to mention that recursion via the Y operator can be simply incorporated into the interpreter. Instead of using the gory definition in terms of λ, we can implement the recursion directly by

```
| eval(Y(Fn(f,e)), r) =
    let val fv = IntVal(999);
        val r' = Defn(f, fv, r);
        val v = eval(e, r')
    in
        fv := v;      (* updates value stored in r' *)
        v
    end;
```

This first creates an extended closure `r'` for evaluating `e` which is `r` extended by the (false) assumption that `f` is bound to 999. e (which should really be an expression of the form $\lambda x.\ e'$ to ensure that the false value of `f` is not used) is then evaluated to yield a closure, which serves as result, but only after the value for `f` stored in the closure environment has been updated to its proper, recursive, value `fv`. This construction is sometimes known as "tying the knot [in the environment]" since the closure for `f` is circular in that its environment contains the the closure itself (under name `f`).

A more detailed working evaluator including Y and *let*) can be found on the web page for this course (see front cover).

1.14.1 Static and dynamic scoping

This final point is worth a small section on its own; the normal state in modern programming languages is that free variables in are looked up in the environment of existing at the time the function was *defined* rather than when it is *called*. This is called *static scoping* or *static binding* or even *lexical scoping*; the alternative of using the calling environment is called *dynamic binding* and was used in many dialects of Lisp. The difference is most easily seen in the following example:

```
let a = 1;
```

```
let f() = a;
let g(a) = f();
print g(2);
```

Check your understanding of static and dynamic scoping by observing that this prints `1` under the former and `2` under the latter.

Exercises

1. Draw the tree structure representing the lambda expression form of the following program.

   ```
   { let x = 3
     in let f(n) = n+x
     in let x = 4
     in f(x)
   }
   ```

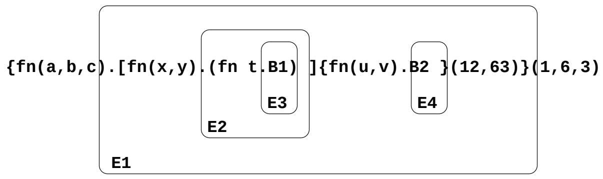
   Apply the `eval` function by hand to this tree and an empty environment and draw the structure of every environment that is used in the course of the evaluation.

2. Is it possible to write a finite program that would cause this evaluator to attempt to create an infinitely long environment?

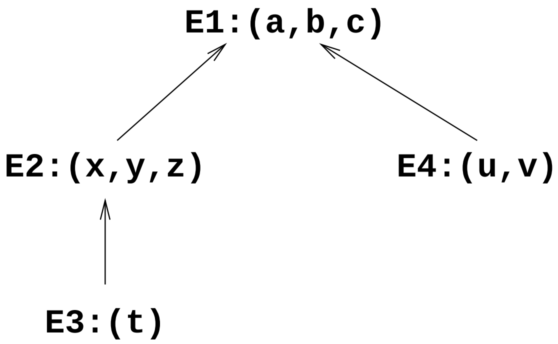
3. Modify the interpreter to use dynamic scoping; is it now possible to write a finite program that would cause this evaluator to attempt to create an infinitely long environment?

1.15 A more efficient implementation of the environment

The previous lambda evaluator (also known as an *interpreter*) is particularly inefficient in its treatment of names since it searches a potentially long environment chain every time a name is used. This search can be done much more efficiently if the environment were represented differently; moreover the technique we describe is much more appropriate for a *compiler* which generates machine code for a target machine. Consider the following:



The environment structure can be represented as a tree as follows (note that here the tree is logically backwards from usual in that each node has a single edge to its parent, rather than each node having an edge to its children):



The levels on the right give the depth of textual nesting of lambda bodies, thus the maximum number of levels can be determined by inspecting the given expression. When evaluating `B1`, we are in environment `E3` which looks like:

```
E1:(a,b,c)          level 1
E2:(x,y,z)          level 2
E3:(t)              level 3
```

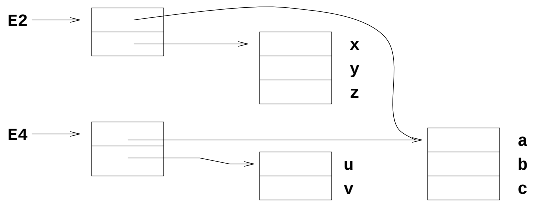
We can associate with any name used in B1 an 'address' consisting of a pair of numbers, namely, a level number and a position within that level. For example:

```
a:(1,1)   b:(1,2)   c:(1,3)
x:(2,1)   y:(2,2)   z:(2,3)
t:(3,1)
```

Similarly within B2:

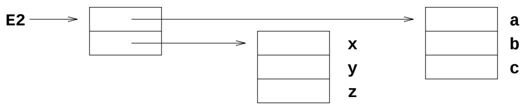
```
a:(1,1)   b:(1,2)   c:(1,3)
u:(2,1)   v:(2,2)
```

At execution time, the environment could be represented as a vector of vectors. For example,

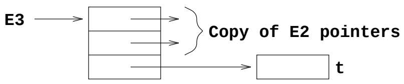


A vector such as the one pointed to by `E2` is called a *display* (first coined by Dijkstra). Notice that while evaluating `B2` in environment `E4` we may use the "address" (2,1) to access the variable u. It should be clear that the pointer to the current display provides sufficient information to access any currently declared variable and so may be used in place of the environment chain used in the `eval` function.

You will recall that a closure (the value representing a function) consists of three parts—the bound variable, the body and the environment information. If we are using the display technique then the environment part can be represented by a pointer to the appropriate display vector. For instance, the environment part of the closure for `(fn t.B1)` in the last example is the pointer to the display vector for `E2`.

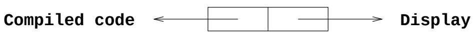


In order to apply this closure to an argument value `k`, we must first create a new display which consists of a copy of `E2` augmented with a new level. The new display will be as follows:



The body of `B1` is then evaluated in this new environment. When the application is complete the value is returned and the previous environment reinstated.

The compiled form of a function often consists of code which first constructs the new display and then evaluates the body; hence the closure is often represented as a pair of pointers:

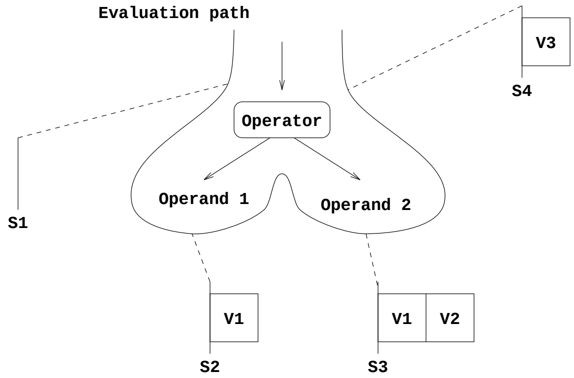


Exercise

Re-implement the `eval` function defined above using a display mechanism for the environment (instead of the linked list).

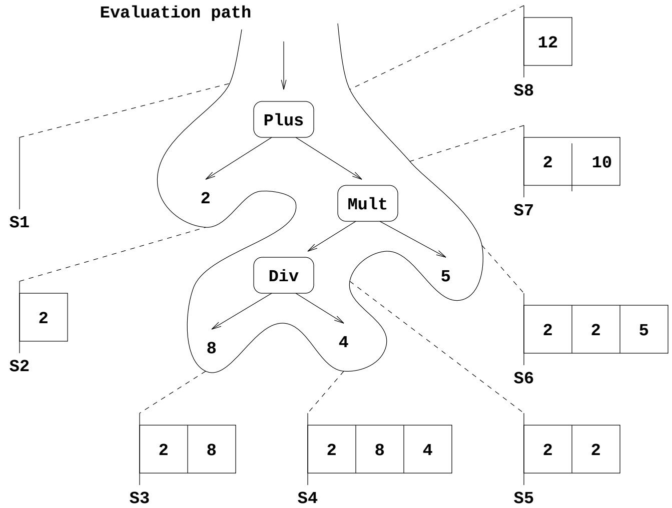
1.16 Evaluation using a stack

A stack has the property that items can be placed on the top or removed from the top. Such a storage organisation is sometimes called a LIFO (last in first out) store. Stacks are often used in the implementation of programming languages since they provide an efficient means of allocating work space used in the evaluation of expressions. To evaluate an expression consisting of an operator applied to two operands, the operands are first evaluated and then the operator applied to the results obtained. This can be shown diagrammatically as follows:



The evaluation proceeds along the evaluation path. The initial stack (S1) is empty. After evaluating the first operand the stack (S2) contains one value namely `V1`. The second operand causes a second value to be placed on the stack (S3) and, finally, the operator takes these two value from the stack replacing them by the single result `V3` as shown in stack (S4).

This mechanism works for expression of any complexity; for example, the evaluation of `2+(8/4)*5` is as follows:



Observe that we only use the applicative structure to help us find the evaluation path. To perform the evaluation it is sufficient to know the order in which the items appear in the evaluation path, namely:

```
2 8 4 / 5 * +
```

This order is called the *reverse polish form* of the expression `2+(8/4)*5` and it is important since

- it is easy to obtain
- and it is an order of evaluation that works.

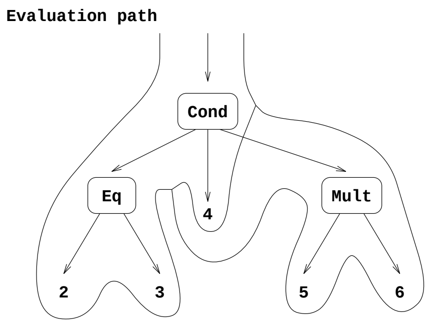
If we connect the operator to the path on the way down rather than on the way up we obtain the sequence:

```
+ 2 * / 8 4 5
```

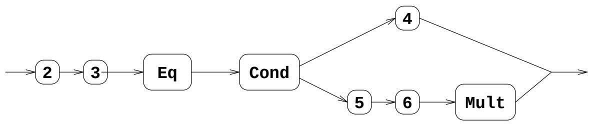
This is called the polish (or prefix) notation and is close to the functional form

```
plus(2, mult(div(8, 4), 5))
```

Notice that monadic operators can be transformed to reverse polish form. For example, the reverse polish form of `2 * (-3)` is `2 3 ~ *` where `~` denotes the monadic minus operator. Some expressions are not quite so susceptible. Consider the expression: `2=3 ? 4 : 5*6`.



The choice of evaluation path depends on the value of the boolean expression. The closest approach to reverse polish form might be



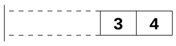
As far as compiled code is concerned, it does not matter that the reverse polish form cannot be written as a linear sequence of items.

1.17 The use of a stack in function calls

The observation that function calls are in many respects similar to applicative expression operators leads one to suspect that it possible to use a stack in the implementation of function applications. Consider the call `f(3,4)` of a function defined as follows:

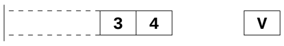
```
let f(x, y) = E
```

If we regard `f` as an ordinary operator we find that, at the time when `f` is to be applied, the top two items of the stack are `3` and `4`.

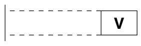


The evaluation of the call involves the evaluation of `E` and so one might provide a new stack on which to perform this evaluation.

Ultimately the evaluation of `E` will yield a value `V`, say, at which time the stacks will be as follows:



Then finally this result is carried back to the previous stack frame and the old control path resumed. At that moment the stack will look as follows:

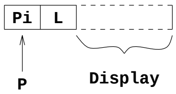


1.18 An implementation of function calls

What follows is a description of a simple mechanism for the implementation of function calls. Many variations and optimisations are possible and so the implementation given here should only be regarded as a guide.

When a function is called it is allocated a piece of stack (sometimes called a stack frame, or activation record) that is used to hold temporary values that are needed during the course of the evaluation of the body. A frame pointer (`P`) will be used to point to the base of the stack frame that is currently active.[3] When the function call is complete the previous stack frame must be re-instated and execution must resume at the point just after the call. For this to be possible the previous frame pointer (`Pi`) and the return address (`L`) are held in the current stack frame.

When a function is applied, a new display must be constructed so that the free variables of the function are accessible during the evaluation of the body. This new display could be placed in the stack frame allocated for the function call, and so combined with the previous frame pointer and return address the base of a stack frame is as follows:



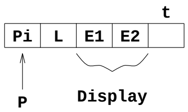
The size of the display depends on the textual depth of nesting of the function body and so is known when the function is being compiled. Local variables of the function can be allocated cells within the stack frame and accessed using constant offsets relative to the stack frame pointer.

The purpose of the display is to allow for access to the free variables of the function. Since every free variable is a local variable of some other function it would be sufficient for each element of the display to be a pointer to the appropriate stack frame. The display can, thus, be a vector of frame pointers. In the example in section 1.15, the environment `E3` was as follows:

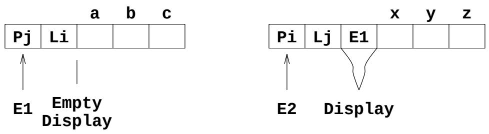
```
E1:(a,b,c)           level 1
E2:(x,y,z)           level 2
E3:(t)               level 3
```

When execution is in environment `E3`, the current stack will be as follows:

[3] It is convenient to assume the stack is essentially an array of 4-byte or 8-byte locations (capable of holding an integer or a pointer) and to describe accessing the 5th location in the current stack frame as `P[4]` etc.



where E1 and E2 are pointers to the stack frames containing (x,y,z) and (a,b,c).



Notice that all the variables a, b, c, x, y, z and t can be accessed via P.

```
a: P[2][2]   b: P[2][3]   c: P[2][4]
x: P[3][3]   y: P[3][4]   z: P[3][5]
t: P[4]
```

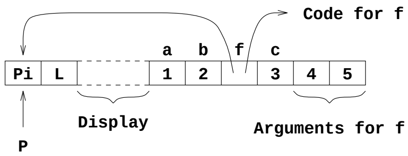
Thus P can be regarded as a representation of the current environment and so can be used, for example, as the environment part of a closure.

An example

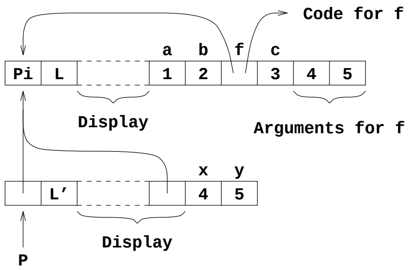
Consider the following fragment of program:

```
...
let r() =
{ let a, b = 1, 2
  let f(x, y) = a*x + b*y
  let c = 3
  c := f(4,5)
}
...
```

At the moment when f is just about to be entered the current stack frame is as follows:



At the moment just after f has been entered (when a*x+b*y is about to be evaluated) the state is as follows:



If E is the environment part of the closure for f, the display can be constructed by compiled code at the start of f consisting of a sequence of assignments (assuming the size of the new display is 3).

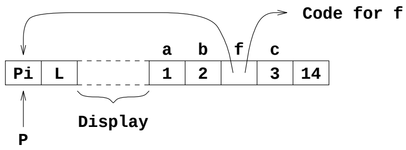
```
P[2] := E[2]       // make a copy
P[3] := E[3]       // of the display
P[4] := E          // add one more level
```

The arguments can now be popped from the calling stack frame and placed in their appropriate positions in the current frame.

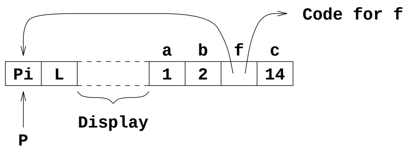
Notice that at this moment a, b, x and y can be addressed via P as follows:

```
a: P[4][4]      b: P[4][5]
x: P[5]         y: P[6]
```

Just after the return from the call f(4,5) the stack is as follows:



and after the assignment to c the stack is as follows:



The implementation of this mechanism can be reasonably efficient provided there are sufficient general registers available to hold the P pointer and all the display pointers. These registers could be set up on entry to the function. This would allow all the variables (both local and free) to be directly addressed. On some machines saving and restoring display registers can be done efficiently using the store-multiple and load-multiple instructions. However, given the relative rarity of variables which are neither top-level (global) nor local, it often suffices just to maintain a static link to which we now turn.

1.19 Static chain method

Instead of storing the entire display in each stack frame (of size 0 in a top-level function, of size n in a function textually nested n times etc.), the display can be held implicitly be merely keeping a chain of pointers (called the *static chain*) through the stack frames corresponding to the different levels of the display. The base of a stack frame (the *linkage information*) now consists of:

1. The previous frame pointer P.

2. The return address L.

3. The static chain pointer S.

Assuming the linkage information is of size 3 words, on entry to the above example call to f(), a, b, x and y can be addressed via P as follows:

```
a: P[2][3]      b: P[2][4]
x: P[3]         y: P[4]
```

The length of the static chain is equal to the textual nesting level of the current piece of code that is being executed and is, of course, equal to the size of the current display. The elements of the display are the pointers to all the stack frames that the static chain passes through.

A possible calling sequence for the call `f(args ...)` is as follows:

```
         code to calculate the arguments
         E    := <environment part of f>
         B    := <code address part of f>
         NEWP := <pointer to the first free stack position>
         L    := ret; JMP B       subroutine jump to f
    ret:
```

The code at the entry point of `f` could be:

```
NEWP[0] := P  save the previous P pointer
P := NEWP     change to the new P pointer
P[1] := L     save the return address
P[2] := E     save the static chain link
```

and the code to return from `f` could be:

```
L := P[1]     find the return address
P := P[0]     restore the previous P pointer
JMP L         branch to the return address
```

One way to examine the possible actual code sequences for (say) the Pentium architecture is to create a file `foo.c` containing C such as

```
extern int g(int);
int f(int x) { return g(x+1)+2; }
```

and then to compile it using (say) gcc with

```
gcc -S foo.c
```

This creates a file `foo.s` containing generated machine for for the host architecture (so chose a Pentium machine to see Pentium code!). Beware: 'gcc' constructs a downwards-growing stack as opposed to the upwards-growing stack used in these notes (see variations below) so everything will not correspond exactly.

1.20 Variations

The calling sequence outlined above should only be regarded as a guide, since many variations and optimisations are possible. A few of the possibilities are as follows.

1. Most modern machines have a fair number of registers. Some of these are very useful in holding pointers to the current stack frame, the limit of stack, arguments and local variables (the Part II course on optimising compilers looks at register allocation for user variables in more detail). It is then worth evaluating which of these should be preserved over procedure call by the called routine and which the caller has responsibility to save. Some registers may be used to cache outermore lexical levels obtained from the display (or from following the static chain). This course ignores such clever uses of register and uses `P[n]` to access local variables, `P[2][n]` to access variables via following the static chain once etc.

2. The outermost display level is the same for every display and so may be regarded as constant needing no code to repeatedly set it (some linkers arrange that a dedicated register points to (small) global variables which are held contiguously). The innermost display level is represented by the P pointer. In practice only about 3% of variable accesses belong to the intermediate levels and so many compilers accept longer access times for such variables instead of calculating them at entry to the procedure 'just in case they are needed'. This improves the efficiency of most procedures. This 3% fact also justifies the modern tendency to use static links in preference to displays.

3. An advantage of the static chain approach is that the position of the arguments in a stack frame depends only on their number and type and not on the textual depth of the procedure. This means that the code in the call to deal with the arguments can (possibly) place them directly in the correct positions in the new stack frame.

4. A decision must be made whether to allocate one (large enough) contiguous region of store for the stack, or to break the stack on each procedure call.

5. In most programming languages procedures are constant (not updatable by assignment) and so no storage cells need be allocated for them. When such a procedure is referenced the compiler can easily construct the environment pointer and entry address that would have been in the closure. Notice that the required environment pointer is the pointer to the stack frame that would have held the closure if a storage cell had been allocated. Proper closures are needed for procedure variables, and they are usually used when procedures are passed as parameters to other procedures.

6. Often extra information is stored in a stack frame to allow for improved runtime diagnostics. This requires a difficult compromise between space efficiency, execution efficiency and the effectiveness of the debugging aids.

1.21 Situations where a stack does not work

If the language allows the manipulation of pointers then erroneous situations are possible. Suppose we have the "address of" operator `&` which is defined so that `&x` yields the address of (or pointer to) the storage cell for `x`. Suppose we also have "contents of" operator `*` which takes a pointer as operand and yields the contents of the cell to which it refers. Naturally we expect `*(&x)=x`. Consider the program:

```
let f() = { let a = 0
            in &a
          }
let p = f()
...
```

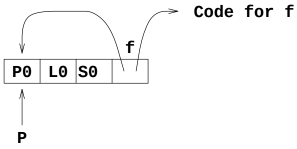
The result of `f` is a pointer to the local variable a but unfortunately when we return from the call this variable no longer exists and `p` is initialised to hold a pointer which is no longer valid and if used may cause an extremely obscure runtime error. Many languages (e.g. Pascal) avoid this problem by only allowing pointers into the heap.

Some other objects such as functions and arrays contain implicit pointers to the stack and so have to be restricted if a stack implementation is to work. Consider:

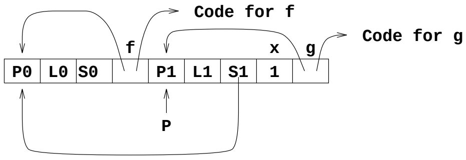
```
let f(x) = { let g(t) = x+t
             in g
           }
```

```
let add1 = f(1)
...
```

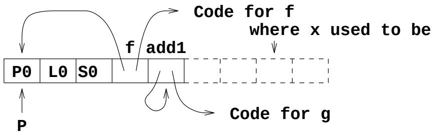
The result of `f(1)` should be a function which will add one to its argument. Thus one might hope that `add1(23)` would yield `24`. It would, however, fail if implemented using a simple stack. We can demonstrate this by giving the state of the stack at various stages of the evaluation. Just after the `f` has been declared the stack is as follows:



At the time when g has just been declared in the evaluation of `f(1)` the stack is as follows:



After the declaration of `add1` the stack would be as follows:



Thus if we now try to use `add1` it will fail since its implicit reference to `x` will not work. If `g` had free variables which were also free variables of `f` then failure would also result since the static chain for `g` is liable to be overwritten.

The simple safe rule that many high level languages adopt to make a stack implementation possible is that no object with implicit pointers into the stack (procedures, arrays or labels) may be assigned or returned as the result of a procedure call. Algol-60 first coined these restrictions as enabling a stack-based implementation to work.

ML clearly does allow objects to be returned from procedure calls. We can see that the problem in such languages is that the above implementation would forbid stack frames from being deallocated on return from a function, instead we have to wait until the last use of any of its bound variables.[4] This implementation is called a "Spaghetti stack" and stack-frame deallocation is handled by a garbage collector. However, the overhead of keeping a whole stack-frame for possibly a single variable is excessive and we now turn to an efficient implementation.

1.22 Implementing ML free variables

In ML programs like

```
val a = 1;
fun g(b) = (let fun f(x) = x + a + b in f end);
val p = g 2;
val q = g 3;
```

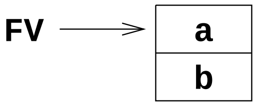
we have seen that an implementation which permanently allocates `b` to the stack location where it is passed will not work.

[4] More precisely, using static links, to the last use of any free variable of the called function.

A mechanism originally proposed by Strachey is as follows. To declare a function such as

```
let f(x) = x + a + b
```

a tuple is constructed (called the *free variable list*) which contains the values (Lvalues or Rvalues whichever is appropriate) of the free variables. A pointer to this list is sufficient environment information for the closure. For `f` defined above the list would be as follows:



During the evaluation of a function call, two pointers are needed: the `P` pointer, as before, to address the arguments and local variables, and a pointer `FV` to point to the free variable list (although note that the `FV` pointer could be treated as an additional hidden argument to functions—this would be appropriate for expressing the translation as C code rather than machine code).

This mechanism requires more work at function definition time but less work within the call since all free variables can be accessed via a simple indirection. It is used in the Edinburgh SML implementation. (An additional trick is to store a pointer to the function code in offset 0 of the free variable list as if it were the first free variable. A pointer to the free variable list can then represent the whole closure as a single word.)

Note that this works most effectively when free variables are Rvalues and hence can be copied freely. When free variables are Lvalues we need to enter a pointer to the actual aliased location in the free variable list of each function which references it. It is then necessary also to allocate the location itself on the heap. (For ML experts: note that ML's use of `ref` for updateable variables means that this is already the case in ML.)

1.23 Parameter passing mechanisms

When we apply a function to an argument we write:

```
f(arg)
```

where `f` is a function (or more strictly an expression yielding a function) and `arg` is any expression. In a referentially transparent language all we need to know about `arg` is its value; but, as we have seen, there are two sorts of value that might be considered, namely an Lvalue or an Rvalue. Many languages (e.g. Pascal, Ada) allow the user to specify which is to be used. For example:

```
let f(VALUE x) = ...
```

might declare a function whose argument is an Rvalue. The parameter is said to be *called by value*. Alternatively, the declaration:

```
let f(REF x) = ...
```

might declare a function whose argument is an Lvalue. The parameter is said to be *called by reference*. The difference in the effect of these two modes of calling is demonstrated by the following example.

```
let r(REF x) = { x := x+1 }      let r(VALUE x) = { x := x+1 }
let a = 10                       let a = 10
r(a)                             r(a)
// a now equals 11               // a now equals 10
```

1.24 Note on Algol call-by-name

Algol 60 is a language that attempted to be mathematically clean and was influenced by the simple calling mechanism of lambda calculus. In the standard report on Algol 60 the procedure calling mechanism is described in terms of textually replacing a call by a copy of the appropriate procedure body. Systematic renaming of identifiers was used to avoid problems with the scope of names. With this approach the natural treatment for an actual parameter of a procedure was to use it as a textual replacement for every occurrence of the corresponding formal parameter. This is precisely the effect of the lambda calculus evaluation rules and in the absence of the assignment command it is indistinguishable from call-by-value or call-by-reference.[5]

When an actual parameter in Algol is *called by name* it is not evaluated in either Lmode or Rmode but is passed to the procedure as an unevaluated expression. Whenever this parameter is used within the procedure, the expression is evaluated. Hence the expression may be evaluated many times (possibly yielding a different value each time). Consider the following Algol program.

```
INTEGER a,i,b;
PROCEDURE f(x) INTEGER;
  BEGIN  a := x;
         i := i+1;
         b := x
  END;
a:=i:=b:=10;
f(i+2);
COMMENT a=12, i=11 and b=13;
```

ML and C/C++ have no call-by-name mechanism, but the same effect can be achieved by passing a suitable function by value. The following convention works:

1. Declare the parameter as a parameterless function (a 'thunk').
2. Replace all occurrences of it in the body by parameterless calls.
3. Replace the actual parameter expression by a parameterless function whose body is that expression.

The above Algol example then transforms into the following C program:

```
int a = 10, i = 10, b = 10;
int pointlessname() { return i+2;}
void f(int x(void)) { a = x();
                      i = i+1;
                      b = x();
                    }
f(pointlessname);
```

[C experts might care to note that this trick only works for C when all variables free to the thunk are declared at top level; Java cannot even express passing a function as a parameter to another function.]

[5] Well, there is a slight difference in that an unused call-by-name parameter will never be evaluated! This is exploited in so-called 'lazy' languages and the Part II course looks at optimisations which select most appropriate calling mechanism for each definition in such languages.

1.25 A source-to-source view of argument passing

Many modern languages only provide call-by-value. This invites us to explain, as we did above, other calling mechanisms in terms of call-by-value (indeed such translations, and languages capable of expressing them, have probably had much to do with the disappearance of such mechanisms!).

For example, values passed by reference (or by result—Ada's `out` parameter) typically have to be Lvalues. Therefore they can be address-taken in C. Hence we can represent:

```
void f1(REF int x) { ... x ... }
void f2(IN OUT int x) { ... x ... }     // Ada-style
void f3(OUT int x) { ... x ... }        // Ada-style
void f4(NAME int x) { ... x ... }
... f1(e) ...
... f2(e) ...
... f3(e) ...
... f4(e) ...
```

as

```
void f1'(int *xp) { ... *xp ... }
void f2'(int *xp) { int x = *xp; { ... x ... } *xp = x; }
void f3'(int *xp) { int x; { ... x ... } *xp = x; }
void f4'(int xf()) { ... xf() ... }
... f1'(&e) ...
... f2'(&e) ...
... f3'(&e) ...
... f4'(fn () => e) ...
```

It is a good exercise (and a frequent source of tripos questions) to write a program which prints different numbers based on which (unknown) parameter passing mechanism a sample language uses.

1.26 Modes of binding free variables

Consider a function definition such as

```
let f(x) = x+a
```

We have seen how `x` can be passed by value or by reference. It is also possible to distinguish the modes of association of free variables such as `a`. The language CPL provided syntactic means of specifying this process: a function whose free variables are called by value was defined using the = operator. If the free variables are called by reference then the function was defined using the == operator. For example,

```
let a = 3                          let a = 3
let f(x) = x+a                     let f(x) == x+a
// f(5) equals 8                   // f(5) equals 8
a := 10;                           a := 10;
// f(5) equals 8                   // f(5) equals 15
```

Nowadays languages provide the latter form only, leaving the former form to be simulated by the user by

```
let a = 3
    let private_a = a     // save ‘a’ at definition
    let f(x) = x + private_a
// f(5) equals 8
a := 10;
// f(5) equals 8
```

The variable `private_a` can be made truly private (visible in the body of `f` but not at `a:=10;`) by the ML ‘local’ construct or merely by taking a little care:

```
let a = 3
let f = (let private_a = a     // save ‘a’ at definition
         in fn x => x + private_a)
...
```

1.27 Labels and jumps

In Algol, if the destination label of a jump is in the same block as the jump, then the jump only involves a simple transfer of control. However, if the destination is in an outer block then the jump also involves the removal of some declared variables from the stack, and if the destination is global to the current procedure then the jump will cause an exit from one or more procedures. Consider:

```
 { let r(lab) = { ...
                  ... goto lab
                  ...
                }
   ...
   r(M)
   ...
M: ...
 }
```

The call `r(M)` is equivalent to obeying the body of `r` with appropriate parameter substitutions. Thus the call is equivalent to:

```
{ ...
  ... goto M
  ...
}
```

and so the jump clearly should cause control to be resumed at the point labelled `M`.

In terms of the stack implementation it is necessary to reset the `P` pointer to the value it had at the moment when execution entered the scope of `M`. Notice that, at the time when the jump is about to be made, the current `P` pointer may differ. One way to implement this kind of jump is to represent the value of a label as a pair of pointers—a pointer to compiled code and a P pointer (note the similarity to a function closure—we need to get to the correct code location and also to have the correct environment when we arrive). The action to take at the jump is then:

1. reset the P pointer,

2. transfer control.

We should notice that the value of a label (like the value of a function) contains an implicit frame pointer and so some restrictions must be imposed to avoid nonsensical situations. Typically labels (as in Algol) may not be assigned or returned as results of functions. This will ensure that all jumps are jumps to activations that dynamically enclose the jump statement.

1.28 Exceptions

ML and Java exceptions and their handlers are conveniently seen as a restricted form of *goto*, albeit with an argument.

This leads to the following implementation: a `try` (Java) or `handle` (ML) construct effectively places a label on the handler code. Entering the `try` block pushes the label value (recall a label/frame-pointer pair) onto a stack (`H`) of handlers and successful execution of the `try` block pops `H`. When an exception occurs its argument is stored in a reserved variable (just like a procedure argument) and the label at the top of `H` is popped and a `goto` executed to it. The handler code then checks its argument to see if matches the exceptions intended to be caught. If there is no match the exception is re-raised therefore invoking the next (dynamically) outermore handler. If the match succeeds the code continues in the handler and then with the statement following the `try-except` block.

For example given `exception foo;` we would implement

```
try C1 except foo => C2 end; C3
```

as

```
    push(H, L2);
    C1
    pop(H);
    goto L3:
L2: if (raised_exc != foo) doraise(raised_exc);
    C2;
L3: C3;
```

and the `doraise()` function looks like

```
void doraise(exc)
{   raised_exc = exc;
    goto pop(H);
}
```

Now, an alternative to an explicit stack is to implement `H` as a linked list of handlers `(label-value, next)` and keep a pointer to its top item. This has the advantage that each element can be stored in the stack frame which is active when the `try` block is entered; thus a single stack suffices for function calls and exception handlers.

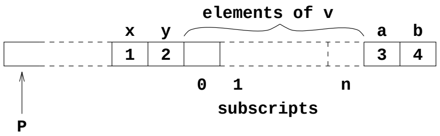
Finally, sadly ANSI C labels cannot be used as values as indicated above, and so code shown above would have to be implemented using the library function `setjmp()` instead.

1.29 Arrays

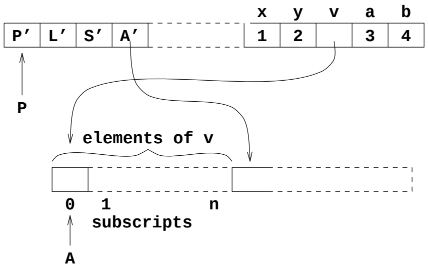
When an array is declared space must be allocated for its elements. In most languages the lifetime of an array is the same as that of a simple variable declared at the same point, and so it would be natural to allocate space for the array on the runtime stack. This is indeed what many implementations do. However, this is not always convenient for various reasons. Consider, for example, the following:

```
...
{ int x=1, y=2;
  int v[n];      // an array from 0 to n-1
  int a=3, b=4;
  ...
}
...
```

Within the body of the above block the current stack frame might look like the following:



(This assumes, as in C, that `v` itself is a constant and does not require a storage cell.) In this example, `n` may be large and so the variables `a` and `b` may be a great distance from `P`. On some machines access to such variables is less efficient. Moreover, if `n` is not a compile-time constant,[6] the position of `a` and `b` relative to `P` will not be known until runtime, again causing inefficiency. Many implementations allocate such array elements on a separate stack, often working from the other end of store. This necessitates a separate array-stack pointer (`A`, say) and space in the main stack to save and restore it. For the above example the two stacks may look as follows:



When execution leaves the current procedure `P`, `L`, `S` and `A` must all all restored. On many implementations `A` is restored at the end of the execution of any block in which an array is declared.

1.30 Object-oriented language storage layout

Declarations (in C++) like

```
class A { int a1,a2; } x;
```

allocate storage for two integers and record the fact that `a1` is at offset zero, and `a2` is at offset 4 (assuming `int`s are 4 bytes wide). Now after

[6] C requires `n` to be a compile-time constant.

```
class B : A { int b; };
```

objects of type B have 3 integer fields a1 and a2 (by inheritance) normally stored at offsets 0 and 4 so that (pointers to) objects of type B can be passed to functions expecting objects of type A with no run-time cost. The member b would then be at offset 8. The following definition is similar.

```
class C : A { int c; };
```

Now, suppose one has multiple inheritance (as in C++) so we can inherit the members and methods from two or more classes and writes:

```
class D : B,C { int d; };
```

Firstly there is the observation that passing an object of type D to a routine expecting C must involve a run-time cost of an addition so that element c can be accessed at offset 8 in the received C. (This assumes that B is stored at offset zero in D.)

There is also the more fundamental question as to what are the members of objects of type D. Does it have 7 (3 in both B and C and also d)? Or maybe 5 (a1, a2, b, c, d)? C++ by default has 7, i.e. the two copies of A are separate. In C++ we can cause the two copies of A to share by replacing the definitions for B and C by

```
class B : virtual A { int b; };
class C : virtual A { int c; };
class D : B,C { int d; };
```

But now the need to treat objects of type D as objects of type B or C means that the storage layout for D is likely to be implemented as

```
struct { A *__p, int b; A *__q, int c; A x; } s =
           { &s.x, 0, &s.x, 0, { 0, 0 }};
```

I.e. there is a single A object and both the __p field of the logical B object and the __q field of the logical C object share it. This is necessary so that a D object can be passed to routines which expect a B or a C object—but note that is causes declarations like B x to be of 16 bytes: 8 for the A, 4 for the indirect pointer (after all, routines need to be compiled which access the elements of a B not knowing whether is it a 'true' B or actually a D).

Such arguments are one reason why Java omits multiple inheritance. Its interface facility provides similar facilities.

The above details only dealt with ordinary members and inheritance. Suppose we now add member functions (methods). Firstly consider the implementation of a method like:

```
class C {
  int a;
  static int b;
  int f(int x) { return a+b+x;}
};
```

How is f() to access its variables? Recall that a static variable is per-class, and a non-static one per-instance. Hence the code could be re-written as:

```
int unique_name_for_b_of_C;
class C {
  int a;
  int f(int x) { return a + unique_name_for_b_of_C + x;}
};
```

Now consider a call to `f()` such as `c.f(x)` where `c` is of class `C`. This is typically implemented as an ordinary procedure call `unique_name_for_f_of_C(c,x)` and the definition of `f()` implemented as:

```
 int unique_name_for_f_of_C(C c, int x)
 {   return c.a                     // fixed offset from c
          + unique_name_for_b_of_C  // global variable
          + x;                      // argument
};
```

Let us now turn to how inheritance affects this model of functions, say in Java:

```
class A { void f() { printf("I am an A"); }};
class B:A { void f() { printf("I am a B"); }};
A x;
B y;
void g(A p) { p.f(); }
main() { x.f();        // gives: I am an A
         y.f();        // gives: I am a B
         g(x);         // gives I am an A
         g(y);         // gives what?
       }
```

There are two cases to be made; should the fact that in the call `p.f();` we have that `p` is of type `A` cause `A::f();` to be activated, or should the fact that the value of `p`, although now an `A` was originally a `B` cause `B::f();` to be activated and hence "I am a B" to be printed? In Java the latter happens; by default in C++ the former happens, to achieve the arguably more useful Java effect it is necessary to use the `virtual` keyword:

```
class A { virtual void f() { printf("I am an A"); }};
class B:A { virtual void f() { printf("I am a B"); }};
```

So how is this implemented? Although it appears that objects of type `A` have no data, they need to represent that fact that one or other `f` is to be called. This means that their underlying implementation is of a storage cell containing the address of the function to be called. (In practice, since there may be many virtual functions and so a *virtual function table* is used whereby a class which has one or more virtual functions has a single additional cell which points to a table of functions to be called by this object. This can be shared among all objects declared at that type, although each type inheriting the given type will in general need its own table).

For more details on this topic the interested reader is referred to Ellis and Stroustrup "The annotated C++ reference manual".

1.31 Data types

In the course so far we have essentially ignored the idea of data type. Indeed we have used '`int x = 1`' and '`let x = 1`' almost interchangeably. Now we come to look at the possibilities of typing.

One possibility (adopted in Lisp, Prolog and the like) is to decree that types are part of an Rvalue and that the type of a name (or storage cell) is the value last stored in it. This is a scheme of *dynamic types* and in general each operation in the language need to check whether the value stored in the cell is of the correct type. (This manifested itself in the lambda calculus evaluator in section 1.14 where errors occur if we apply an integer as a function or attempt to add a function to a value).

Most mainstream languages associate the concept of data type with that of an identifier. This is a scheme of *static types* and generally providing an explicit type for all identifiers leads to the data type of all expressions being known at compile time. The *type* of an expression can be thought of as a constraint on the possible values that the expression may have. The type is used to determine the way in which the value is represented and hence the amount of storage space required to hold it. The types of variables are often declared explicitly, as in:

```
float x;
double d;
int i;
```

Knowing the type of a variable has the following advantages:

1. It helps the compiler to allocate space efficiently, (`int`s take less space than `double`s).
2. It allows for *overloading*. That is the ability to use the same symbol (e.g. `+`) to mean different things depending on the types of the operands. For instance, `i+i` performs integer addition while `d+d` is a `double` operation.
3. Some type conversions can be inserted automatically. For instance, `x := i` is converted to `x := itof(i)` where `itof` is the conversion function from `int` to `float`. Similarly, `i+x` is converted to `itof(i)+x`.
4. Automatic type checking is possible. This improves error diagnostics and, in many cases, helps the compiler to generate programs that are incapable of losing control. For example, `goto L` will compile into a legal jump provided `L` is of type label. Nonsensical jumps such as `goto 42` cannot escape the check. Similar considerations apply to procedure calls.

Overloading, automatic type conversions and type checking are all available to a language with dynamic types but such operations must be handled at runtime and this is like to have a drastic effect on runtime efficiency. A second inherent inefficiency of such languages is caused by not knowing at compile time how much space is required to represent the value of an expression. This leads to an implementation where most values are represented by pointers to where the actual value is stored. This mechanism is costly both because of the extra indirection and the need for a garbage collecting space allocation package. In implementation of this kind of language the type of a value is often packed in with the pointer.

One advantage of dynamic typing over static typing is that it is easy to write functions which take a list of any type of values and applies a given function to it (usually called the `map` function). Many statically typed languages render this impossible (one can see problems might arise if lists of (say) characters were stored differently from lists of integers). Some languages (most notably ML) have `polymorphic types` which are static types but which retain some flexibility expressed as parameterisation. For example the above `map` function has ML type

$$(\alpha \texttt{ -> } \beta) \texttt{ * } (\alpha \texttt{ list}) \texttt{ -> } (\beta \texttt{ list})$$

If one wishes to emphasise that a statically typed system is not polymorphic one sometimes says it is a *monomorphic type system.*

Polymorphic type systems often allow for *type inference*, often called nowadays *type reconstruction* in which types can be omitted by the user and reconstructed by the system. Note that in a monomorphic type system, there is no problem in reconstructing the type of λ`x. x+1` nor λ`x. x ? false:true` but the simpler λ`x. x` causes problems, since a wrong 'guess' by the type reconstructor may cause later parts of code to fail to type-check.

We observe that overloading and polymorphism do not always fit well together: consider writing in ML λ`x. x+x`. The `+` function has both type

`(int * int -> int)` and `(real * real -> real)`

so it is not immediately obvious how to reconstruct the type for this expression (ML rejects it).

It may be worth talking about inheritance based polymorphism here.

Finally, sometimes languages are described as *typeless*. BCPL (a forerunner of C) is an example. The idea here is that we have a single data type, the word (e.g. 32-bit bit-pattern), within which all values are represented, be they integers, pointers or function entry points. Each value is treated as required by the operator which is applied to it. E.g. in `f(x+1,y[z])` we treat the values in `f`, `x`, `y`, `z` as function entry point, integer, pointer to array of words, and integer respectively. Although such languages are not common today, one can see them as being in the intersection of dynamically and statically type languages. Moreover, they are often effectively used as intermediate languages for typed languages whose type information has been removed by a previous pass (e.g. in intermediate code in a C compiler there is often no difference between a pointer and an integer, whereas there is a fundamental difference in C itself).

1.32 Source-to-source translation

It is often convenient (and you will have seen it done several times above in the notes) to explain a higher-level feature (e.g. exceptions or method invocation) in terms of lower-level features (e.g. gotos or procedure call with a hidden 'object' parameter).

This is often a convenient way to specify precisely how a feature behaves by expanding it into phrases in a 'core' subset language. Another example is the definition of

`while` e `do` e'

construct in Standard ML as being shorthand (syntactic sugar) for

`let fun f() = if` e `then (`e'`; f()) else () in f() end`

(provided that `f` is chosen to avoid clashes with free variables of e and e').

A related idea (becoming more and more popular) is that of compiling a higher-level language (e.g. Java) into a lower-level language (e.g. C) instead of directly to machine code. This has advantages of portability of the resultant system (i.e. it runs on any system which has a C compiler) and allows one to address issues (e.g. of how to implement Java `synchronized` methods) by translating them by inserting mutex function calls into the C translation instead of worrying about this and keeping the surrounding generated code in order.

1.33 Spectrum of Compilers and Interpreters

One might think that it is pretty clear whether a language is compiled (like C say) or interpreted (like BASIC say). Even leaving aside issues like microcoded machines (when the instruction set is actually executed by a lower-level program at the hardware level "Big fleas have little fleas upon their backs to bite them") this question is more subtle than first appears.

Consider Sun's Java system. A Java program is indeed compiled to instructions (for the Java Virtual Machine—JVM) which is then typically interpreted (one tends to use the word 'emulated' when the structure being interpreted resembles a machine) by a C program. One recent development is that of Just-In-Time—JIT compilers for Java in which the 'compiled' JVM code is translated to native code just before execution.

If you think that there is a world of difference between emulating JVM instructions and executing a native translation of them then consider a simple JIT compiler which replaces each JVM instruction with a procedure call, so instead of emulating

```
load_local_var 3
```

we execute

```
load_local_var(3);
```

where the procedure `load_local_var()` merely performs the code that the interpreter would have performed.

Similarly, does parsing our simple expression language into trees before interpreting them cause us to have a compiler, and should we reserve the word 'interpreter' for a system which interprets text (like some BASIC systems)?

So, we conclude there is no line-in-the-sand difference between a compiled system and and an interpreted system. Instead there is a spectrum whose essential variable is how much work is done statically (i.e. before execution starts) and how much is done during execution.

In our simple lambda evaluator earlier in the notes, we do assume that the program-reading phase has arranged the expression as a tree and faulted any mismatched brackets etcḢowever, we still arrange to search for names (see `lookup`) and check type information (see the code for $e_1 + e_2$) at run-time.

Designing a language (e.g. its type system) so that as much work as possible *can* be done before execution starts clearly helps one to build efficient implementations by allowing the compiler to generate good code.

Chapter 2

Compiling Techniques

A *compiler* is a program to translate the source form of a program into its equivalent machine code or relocatable binary form. The number of compilers that exist is very large and considerable human effort has been expended in constructing them. As a result most parts of the compilations process have become well understood and the job of writing a compiler is no longer the difficult task it once was. A compiler tends to be a large program (typically 10,000 to 50,000 machine instructions and sometimes as high as 300,000 to a million instructions) and it is wise to structure it in order to make its individual components small enough to think about and handle conveniently.

If a language is sufficiently simple it and is designed suitably, it can be compiled by a *one pass compiler*, that is, it can be compiled a small piece (often just a statement) at a time. During the compilation process the data types and allocated locations of variables are remembered together with any other information that may be needed later on during the compilation. The space required for this is substantially less than the space needed to hold the entire program and there is usually no limit on the size of the program than may be compiled in this way. In general such a compiler is simple in design and fast in execution.

Most languages have features that make compilation in a single pass either difficult or impossible. For example, in Algol 60 the declaration of names may occur many line or even pages after they are first used, as in:

```
BEGIN REAL m;
      PROC g;
      BEGIN REAL x;
            f(m);
            ...
         m: ...
            ...
      END
      ...
END
```

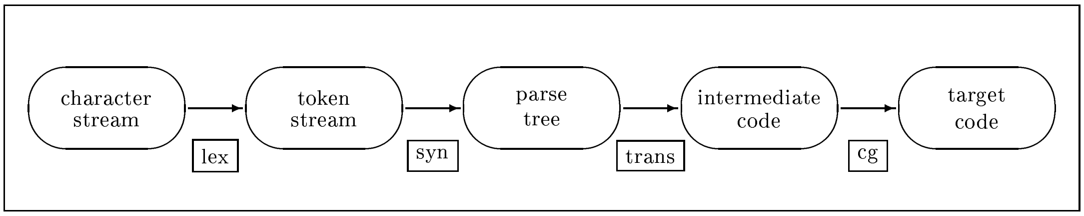
In this example it would be difficult to compile the call `f(m)` using a single pass compiler since although `m` seems to be declared as a REAL at the time of the call this is superseded by the label declaration some lines later. Similarly in ML, consider programs like:

```
val g = 3;
fun f(x) = ... g...
and g(x) = ...;
```

For most current programming languages, it is normal to compile in a number of stages (or passes) with the output of one pass being the input of the next. A compiler designed in this way is called a *multi-pass compiler*.

2.1 The structure of a typical multi-pass compiler

We will take as an example a compiler with four passes.



2.1.1 The lexical analyser

This reads the characters of the source program and recognises the basic syntactic components that they represent. It will recognise identifiers, reserved word, numbers, string constants and all other basic symbols (or tokens) and throw away all other ignorable text such as spaces, newlines and comments. For example, the result of lexical analysis of the following program:

```
{ let x = 1
  x := x + y
}
```

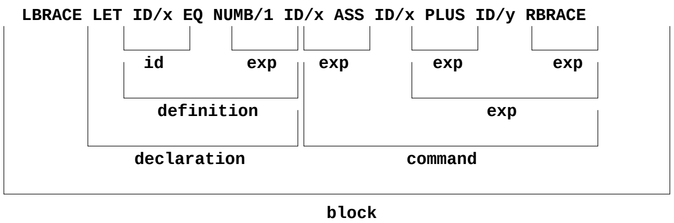
might be:

```
LBRACE LET ID/x EQ NUM/1 ID/x ASS ID/x PLUS ID/y RBRACE
```

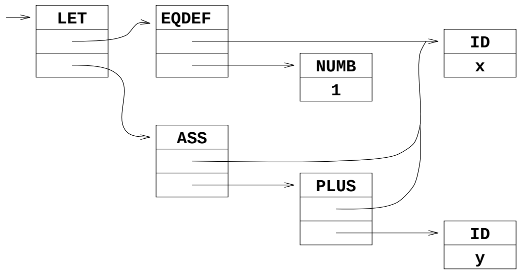
Lexical tokens are often represented in a compiler by small integers and for composite tokens such as identifiers, numbers, etc. additional information is passed by means of pointers into appropriate tables; for example calling a routine `lex()` might return the next token while setting a global variable `lex_aux_string` to the string form of an identifier when `ID` is returned, similarly `lex_aux_int` might be set to the binary representation of an integer when `NUM` is returned.

2.1.2 The syntax analyser

This will recognise the syntactic structure of the sequence of tokens delivered by the lexical analyser. The result of syntax analysis is often a tree representing the syntactic structure of the program. This tree is sometime called an *abstract syntax tree*. The syntax analyser would recognise that the above example parses as follows:



and it might be represented within the compiler by the following tree structure:



where the tree operators (e.g. LET and EQDEF) are represented as small integers.

In order that the tree produced is not unnecessarily large it is usually constructed in a condensed form as above with only essential syntactic features included. It is, for instance, unnecessary to represent the expression `x` as a `<sum>` which is a `<factor>` which is a `<primary>` which is an `<identifier>`. This would take more tree space space and would also make later processing less convenient. The phrase 'abstract syntax tree' refers to the fact the only semantically important items are incorporated into the tree; thus `a+b` and `((a)+(((b)))` might have the same representation, as might `while (e) C` and `for(;e;) C`.

2.1.3 The translation phase

This pass flattens the tree into a linear sequence of intermediate object code. At the same time it deals with

1. the scopes of identifiers,
2. declaration and allocation of storage,
3. selection of overloaded operators and the insertion of automatic type transfers.

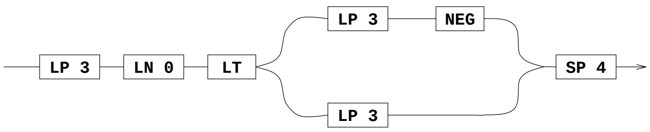
The intermediate object code for the command:

```
y := x<0 ? -x : x
```

might be as follows:

```
LP 3       load x
LN 0       load 0
LT         less than
JF L36     jump if false to L36
LP 3       load x
NEG        negate it
JUMP L37   jump to L37
LAB L36
LP 3       load x
LAB L37
SP 4       store y
```

Alternatively, the intermediate object code could be represented within the compiler as a directed graph as follows:



2.1.4 The code generator

This pass converts the intermediate object code into machine instructions and outputs them in either assembly language or relocatable binary form. The code generator is mainly concerned with local optimisation, the allocation of registers and the selection of machine instructions.

The four passes just described for a clear-cut logical divisions of the compiler, but are not necessarily applied in sequence. It is, for instance, common for the lexical analyser to be a subroutine of the syntax analyser and for it to be called whenever the syntax analyser requires another lexical token. It is also quite common for the translation phase and the code generator to be merged into one pass. Some compilers have additional passes, particularly if a high degree of optimisation is required.

2.1.5 Advantages of the multi-pass approach

1. It breaks a large and complicated task into smaller, more manageable pieces.
2. It is possible to reduce the store requirements of the compiler by overlaying the code for a later pass in the space occupied by the code of an earlier pass.
3. Modifications to the compiler (e.g. the addition of a synonym for a reserved word, or a minor improvement in compiler code) often require changes to one pass only and are thus simple to make.
4. A multi-pass compiler tends to be easier to describe and understand.
5. More of the design of the compiler is machine independent. It is sometimes possible to arrange that all machine dependent parts are in the code generator.
6. The job of writing the compiler can be shared between a number of programmers each working on separate passes. The interface between the passes is easy to specify precisely.

2.2 Lexical analysis

This is a critical part of a simple compiler since it can account for more than 50% of the compile time. This is because:

1. character handling tends to be expensive,
2. there are a large number of characters in a program compared with the number of lexical tokens, and
3. the lexical analyser usually constructs name tables and performs the binary conversion of constants.

For production compilers it was traditional to allocate one's most skillful programmers to the lexical analyser.

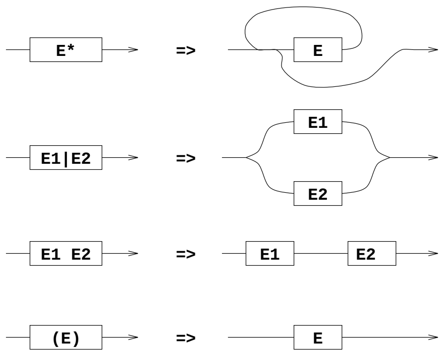
2.2.1 Regular expressions

The recognition of lexical tokens is straightforward and does not require a sophisticated analyser. This results from the simple syntax of lexical tokens. It is usually the case that all the lexical tokens of a language can be described by *regular expressions*, which then implies that the recognition can be performed by a *finite state algorithm.*

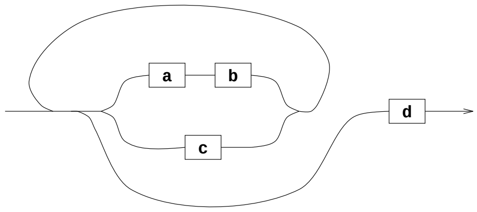
A *regular expression* is composed of characters, operators for concatenation (space), alternation (|) and repetition *, and parentheses are used for grouping. For example, `(a b | c)* d` is a regular expression. It can be regarded as a specification of a potentially infinite set of strings, in this case:

```
d
abd      cd
ababd    abcd    cabd    ccd
etc.
```

This is best derived by constructing the corresponding *transition diagram* by repeated application of the following rules.



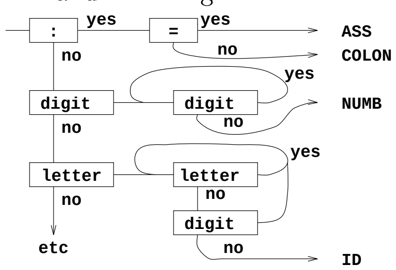
The transition diagram for the expression "`(a b | c)* d`" is:



This can be regarded as a generator of strings by applying the following algorithm:

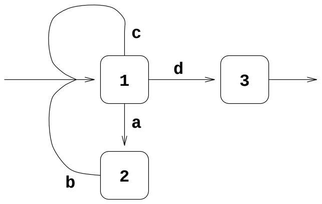
1. Follow any path from the starting point to any accessible box.
2. Output the character in the box.
3. Follow any path from that box to another box (possibly the same) and continue from step (2). The process stops when the exit point is reached.

We can also use the transition diagram as the basis of a recogniser algorithm. For example, an analyser to recognise `:=`, `:`, <numb> and <id> might have the following transition diagram:



Optimisation is possible (and needed) in the organisation of the tests. This method is only satisfactory if one can arrange that only one point in the diagram is active at any one time.

It is sometimes convenient to draw the transition diagram as a directed graph with labelled edges. For example, the graph for the expression "`(a b | c)* d`" can be represented as follows:

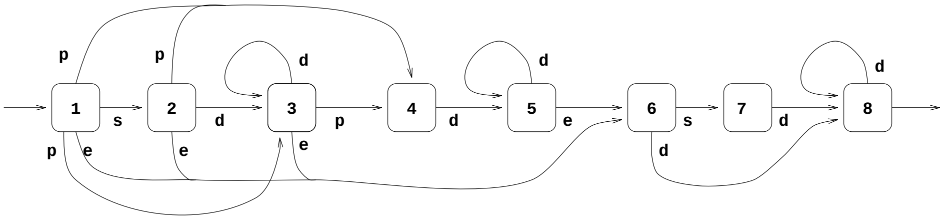


With state 3 designated an accepting state, this graph is a finite state acceptor for the given regular expression. The acceptor is easily implemented using a *transition matrix* to represent the graph.

We will demonstrate the method by considering the following syntax of floating point numbers (the '`->`' notation is introduced below in section 2.3):

```
    N  ->  U  |  s U             Number
    U  ->  D  |  E  |  D E       Unsigned number
    D  ->  J  |  F  |  J F       Unsigned decimal number
    E  ->  e I                   Exponent part
    F  ->  p J                   Decimal fraction
    I  ->  J  |  s J             Integer
    J  ->  d  |  d J             Unsigned integer

where  s  is a sign              + or -
       e  is the exponent symbol  E
       p  is the decimal point    .
       d  is a digit              0-9
```

The corresponding graph is:



The corresponding matrix is as follows:

	s	d	p	e	other
S1	S2	S3	S4	S6	.
S2	.	S3	S4	S6	.
S3	.	S3	S4	S6	acc
S4	.	S5	.	.	.
S5	.	S5	.	S6	acc
S6	S7	S8	.	.	.
S7	.	S8	.	.	.
S8	.	S8	.	.	acc

In a program that uses this technique each matrix entry would specify the address of some code to deal with the transition[1] and note the next matrix row to be used. The entry `acc` would point to

[1]E.g. multiply the current total by 10 and add on the current digit

the code that processes a complete floating point number. Blank entries correspond to syntactic error conditions.

In general, this technique is fast and efficient, but if used on a large scale it requires skill and cunning to reduce the size of the matrix and to reduce the number of separate transition routines.

2.3 Phrase structured grammars

A *grammar* consists of an *alphabet* of symbols (think of these as characters to lexing or tokens resulting from lexing) and set of rules for generating a *language* (set of strings) of such symbols. For example, if the alphabet were the set of all letters `{a ... z}` and the rule were "generate all strings of length three" we would have a language whose strings are:

```
aaa, aab, ... zzy, zzz
```

A more useful form of grammar is the *phrase structured grammar* where the generation rule is given as a set of *productions*. It is first necessary to break the alphabet into two sets of symbols: *terminal symbols* like `a`, `b`, `c` above which may occur in the input text and *non-terminals* like `Term` or `Declaration` which do not occur in input text but summarise the structure of a sequence of symbols. The most general form of a production is:

```
A B ... C  ->  P Q ... R
```

where `A...Z` are symbols and `A B ... C` contains at least one non-terminal. This rule specifies that if `A B ... C` occurs in a string belonging to the grammar then the string formed by replacing `A B ... C` by `P Q ... R` also belongs to the grammar (note that the symbol '`::=`' is sometimes used as an alternative to '`->`'). There must be a unique non-terminal `S`, say, called the *sentence symbol* that occurs by itself on the left hand side of just one production. Any string that can be formed by the application of productions is called a *sentential form*. A sentential form containing no non-terminals is called a *sentence*. The problem of syntax analysis is to discover which series of applications of productions that will convert the sentence symbol into the given sentence.

It is useful to impose certain restrictions on `A B ... C` and `P Q ... R` and this has been done by Chomsky to form four different types of grammar. The most important of these in the Chomsky Type 2 grammar.

2.3.1 Type 2 grammar

In the Chomsky type 2 grammar the left hand side of every production is restricted to just a single non-terminal symbol. Such symbols are often called *syntactic categories*. Type 2 grammars are known as *context free grammars* and have been used frequently in the specification of the syntax of programming languages, most notably Algol 60 where it was first used. The notation is sometime called *Backus Naur Form* or BNF after two of the designers of Algol 60. A simple example of a type 2 grammar is as follows:

```
S  ->  A B
A  ->  a
A  ->  A B b
B  ->  b c
B  ->  B a
```

A slightly more convenient way of writing the above grammar is:

```
S  ->  A B
A  ->  a    |  A B b
B  ->  b c  |  B a
```

The alphabet for this grammar is `{S, A, B, a, b, c, d}`. The non-terminals are `S, A, B` being the symbols occurring on the left-hand-side of productions, with `S` being identified as the start symbol. The terminal symbols are `a, b, c, d`, these being the characters that only appear on the right hand side. Sentences that this grammar generates include, for instance:

```
abc
abcbbc
abcbca
abcbbcaabca
```

Where the last sentence, for instance, is generated from the sentence symbol by means of the following productions:

```
S
|
A---------------B
|               |
A-------B-----b B---a
|       |     | |   |
A-B---b B---a | b-c |
| |   | |   | | | | |
a b-c | b-c | | | | |
| | | | | | | | | | |
a b c b b c a b b c a
```

A grammar is ambiguous if there are two or more ways of generating the same sentence. Convince yourself that the follow three grammars are ambiguous:

```
a)   S  ->  A B
     A  ->  a  |  a c
     B  ->  b  |  c b

b)   S  ->  a T b  |  T T
     T  ->  a b  |  b a

c)   C  ->  if E then C else C  |  if E then C
```

Clearly every type 2 grammar is either ambiguous or it is not. However, it turns out that it is not possible to write a program which, when given an arbitrary type 2 grammar, will terminate with a result stating whether the grammar is ambiguous or not. It is surprisingly difficult for humans to tell whether a grammar in ambiguous. One example of this is that the productions in (c) above appeared in the original Algol 60 published specification. As an exercise, determine whether the example grammar given above is ambiguous.

For completeness, the other grammars in the Chomsky classification are as follows.

2.3.2 Type 0 grammars

Here there are no restrictions on the sequences on either side of productions. Consider the following example:

```
S     ->  a S B C  |  a B C
C B   ->  B C
a B   ->  a b
b B   ->  b b
b C   ->  b c
c C   ->  c c
```

This generates all strings of the form $a^n b^n c^n$ for all $n \geq 1$.

To derive `aaaaabbbbbccccc`, first apply `S -> aSBC` four times giving:

```
aaaaSBCBCBCBC
```

Then apply `S -> aBC` giving:

```
aaaaaBCBCBCBCBC
```

Then apply `CB -> BC` many times until all the `C`s are at the right hand end.

```
aaaaaBBBBBCCCCC
```

Finally, use the last four productions to convert all the `B`s and `C`s to lower case giving the required result. The resulting parse tree is as follows:

```
S
a-S-----------------------B-C
| a-S-----------------B-C | |
| | a-S-----------B-C | | | |
| | | a-S-----B-C | | | | | |
| | | | a-B-C | | | | | | | |
| | | | a-b B-C B-C B-C B-C |
| | | | | b-b B-C B-C B-C | |
| | | | | | b-b B-C B-C | | |
| | | | | | | b-b B-C | | | |
| | | | | | | | b-b | | | | |
| | | | | | | | | b-c | | | |
| | | | | | | | | | c-c | | |
| | | | | | | | | | | c-c | |
| | | | | | | | | | | | c-c |
| | | | | | | | | | | | | c-c
| | | | | | | | | | | | | | |
a a a a a b b b b b c c c c c
```

As a final remark on type 0 grammars, it should be clear that one can write a grammar which essentially specifies the behaviour of a Turing machine, and syntax analysis in this case is equivalent to deciding whether a given string is the answer to some program. This is undecidable and syntax analysis of type 0 grammars is thus, in general, undecidable.

2.3.3 Type 1 grammars

A production in a type 1 grammar takes the following form:

```
A ... B P C ... D  -> A ... B U ... V C ... D
```

where `P` is a single non-terminal symbol, and the sequences `A ... B`, `C ... D` and `U ... V` are sequences of terminal and non-terminal symbols. The sequence `U ... V` may not be empty. These grammars are called *context sensitive* since `P` can only be replaced by `U ... V` if it occurs in a suitable context.

2.3.4 Type 3 grammars

This is the most restrictive of the phrase structured grammars. In it all productions are limited to being in one of the following forms:

```
       A  ->  a
   or  A  ->  a B
```

That is, the right hand side must consist of a single terminal symbol possibly followed by a single non-terminal. It is sometimes possible to convert a type 2 grammar into an equivalent type 3 grammar. Try this for the grammar for floating point constants given earlier.

Type 3 grammars can clearly be parsed using a finite state recogniser, and for this reason they are often called *regular grammars*. [To get precise correspondence to regular languages it is necessary also to allow the empty production `S ->` ϵ otherwise the regular language consisting of the empty string (accepted by an automaton whose initial state is accepting, but any non-empty input sequence causes it to move to a non-accepting state) cannot be represented as a type 3 grammar.]

Finally, note that clearly every Type 3 grammar is a Type 2 grammar and every Type 2 grammar is a Type 1 grammar etc. Moreover these inclusions are strict in that there are languages which can be generated by (e.g.) a Type 2 grammar and which cannot be generated by any Type 3 grammar. However, just because a particular language can be described by (say) a Type 2 grammar does not automatically mean that there is no Type 3 grammar which describes the language. An example would be the grammar G given by

```
S  ->  a
S  ->  S a
```

which is of Type 2 (and not Type 3) but the grammar G' given by

```
S  ->  a
S  ->  a S
```

clearly generates the same set of strings (is *equivalent* to G) and is Type 3.

2.4 Syntax analysis

The type 2 (or context free) grammar is the most useful for the description of programming languages since it is powerful enough to describe the constructions one typically needs and yet is sufficiently simple to be analysed by a small and generally efficient algorithm. Some compiler writing systems use BNF (often with slight extensions) as the notation in which the syntax of the language is defined. The parser is then automatically constructed from this description.

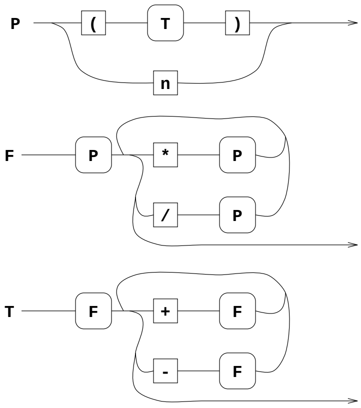
We will now look at three main parsing techniques, namely: *recursive descent*, *precedence* and *SLR(1)*.

2.5 Recursive descent

In this method the syntax is converted into transition diagrams for some or all of the syntactic categories of the grammar and these are then implemented by means of recursive functions. Consider, for example, the following syntax:

```
P  ->  ( T )  |  n
F  ->  F * P  |  F / P  |  P
T  ->  T + F  |  T - F  |  F
```

The corresponding transition diagrams are:



Notice that the original syntax has been modified to avoid left recursion[2] to avoid the possibility of a recursive loop in the parser. The recursive descent parsing functions are outlined below (implemented in C):

```
void RdP()
{ switch (token)
  { case '(': lex(); RdT();
              if (token != ')') error("expected ')');
              lex(); return;
    case 'n': lex(); return;
    default:  error("unexpected token");
  }
}

void RdF()
{ RdP();
  for (;;) switch (token)
  { case '*': lex(); RdP(); continue;
    case '/': lex(); RdP(); continue;
    default:  return;
  }
```

[2]By replacing the production

```
F -> F * P | F / P | P
```

with

```
F -> P * F | P / F | P
```

which has no effect on the strings accepted, although it does affect their parse tree—see later.

```
}

void RdT()
{ RdF();
  for (;;) switch (token)
  { case '+': lex(); RdF(); continue;
    case '-': lex(); RdF(); continue;
    default:  return;
  }
}
```

2.6 Data structures for parse trees

It is usually best to use a data structure for a parse tree which corresponds closely to the *abstract syntax* for the language in question rather than the *concrete syntax*. The abstract syntax for the above language is

```
E  ->  E + E  |  E - E  |  E * E  |  E / E  |  ( E ) | n
```

This is clearly ambiguous seen as a grammar on strings, but it specifies parse trees precisely and corresponds directly to ML's

```
datatype E = Plus of E * E  |  Minus of E * E  |
             Mult of E * E  |  Div of E * E  |
             Paren of E  |  Num of int;
```

Indeed one can go further and ignore the `( E )` construct in the common case parentheses often have no semantic import beyond specifying grouping. In C the construct tends to look like:

```
struct E {
  enum { T_Plus, T_Minus, T_Mult, T_Div, T_Paren, T_Numb } flavour;
  union { struct { struct E *left, *right; } diad;
            // selected by T_Plus, T_Minus, T_Mult, T_Div.
          struct { struct E *child; } monad;
            // selected by T_Paren.
          int num;
            // selected by T_Numb.
         } u;
};
```

It is not generally helpful to reliability and maintainability to make a single datatype which can represent all sub-structures of a parse tree. For parsing C, for example, one might well expect to have separate abstract parse trees for `Expr`, `Cmd` and `Decl`.

It is easy to augment a recursive descent parser so that it builds a parse tree while doing syntax analysis. The ML datatype definition defines constructor functions, e.g. `Mult` which maps two expression trees into one tree which represents multiplying their operands. In C one needs to work a little by defining such functions by hand:

```
struct E *mkE_Mult(E *a, E *b)
{   struct E *result = malloc(sizeof (struct E));
    result->flavour = T_Mult;
```

```
    struct E *RdP()
    {   struct E *a;
        switch (token)
        {   case '(': lex(); a = RdT();
                      if (token != ')') error("expected ')'");
                      lex(); return;
            case 'n': a = mkE_Numb(lex_aux_int); lex(); return a;
/* do names by
**          case 'i': a = mkE_Name(lex_aux_string): lex(); return a;
*/
            default:  error("unexpected token");
        }
    }

    /* We should have also had a right-associative '**' operator here! */

    struct E *RdF()
    {   struct E *a = RdP();
        for (;;) switch (token)
        {   case '*': lex(); a = mkE_Mult(a, RdP()); continue;
            case '/': lex(); a = mkE_Div(a, RdP()); continue;
            default:  return a;
        }
    }

    struct E *RdT()
    {   struct E *a = RdF();
        for (;;) switch (token)
        {   case '+': lex(); a = mkE_Plus(a, RdF()); continue;
            case '-': lex(); a = mkE_Minus(a, RdF()); continue;
            default:  return a;
        }
    }
```

Figure 2.1: Recursive descent parser yielding a parse tree

```
        result->u.diad.left = a;
        result->u.diad.right = b;
        return result;
    }
```

A recursive descent parser which builds a parse tree for the parsed expression is given in Figure 2.1.

When there are many such operators like `+`, `-`, `*`, `/` with similar syntax it can often simplify the code to associate a binding power with each operator and to define a single routine `RdE(int n)` which will read an expression which binds at least as tightly as `n`. In this case `RdT()` might correspond to `RdE(0)`, `RdT()` to `RdE(1)` and `RdP()` to `RdE(2)`.

This idea is can be pushed further to produce a table-driven parser to which topic we now turn.

2.7 Simple precedence

For simple arithmetic grammars a parser based on the precedence of the operators is possible. Consider the token stream:

```
[ x * y + a / t - c ** d ]
```

Let us define two relations LT and GT which hold as follows:

	+	-	*	/	**	]
[	LT	LT	LT	LT	LT	
+	GT	GT	LT	LT	LT	GT
-	GT	GT	LT	LT	LT	GT
*	GT	GT	GT	GT	LT	GT
/	GT	GT	GT	GT	LT	GT
**	GT	GT	GT	GT	LT	GT

then we can parse the above expression by means of the following steps:

```
We start with:    [ x * y + a / t - c ** d ]
[ LT *  GT +   => [ (x*y) + a / t - c ** d ]
+ LT /  GT -   => [ (x*y) + (a/t) - c ** d ]
[ LT +  GT -   => [ ((x*y)+(a/t)) - c ** d ]
- LT ** GT ]   => [ ((x*y)+(a/t)) - (c**d) ]
[ LT -  GT ]   => [ (((x*y)+(a/t))-(c**d)) ]
```

It is worth noting that this method allows both the precedence and associativity of operators to be specified. For example `a-b-c` parses as `(a-b)-c`, but `a**b**c` as `a**(b**c)`.

2.8 General precedence

For some grammars it is possible to define relations `EQ`, `GT` and `LT` between alphabet characters (terminal or non-terminal) of the grammar in such a way that, if

```
... U A B C D V ...
```

is a sentential form, and

```
U LT A EQ B EQ C EQ D GT V
```

then there must be a production `X -> A B C D` and so

```
... U X V ...
```

is a simpler sentential form. A matrix defining these relations then forms the basis of a very simple parser.

The definitions of `EQ`, `LT` and `GT` are as follows:

```
1)   A EQ B  <=>  P -> ... A B ...   is a production
```

```
2)   A LT B  <=>  P -> ... A U ...   is a production
                  and  U => B ...    (using one or more
                                      productions)

3a)  A GT B  <=>  P -> ... U B ...   is a production
                  and  U => ... A

3b)  A GT B  <=>  P -> ... U V ...   is a production
                  and  U => ... A
                  and  V => B ...
```

The grammar is a *precedence grammar* if for all pairs of alphabet characters at most one of the relations holds. The following grammar in not a precedence grammar.

```
S  ->  < E >
E  ->  T  |  E + T
T  ->  P  |  T * P
P  ->  ( E )  |  I
```

since, for instance `< EQ E` and `< LT E`. It can, however, be modified into the following equivalent grammar which is a precedence grammar.

```
S  ->  < E' >
E' ->  E
E  ->  T'  |  E + T'
T' ->  T
T  ->  P  |  T * P
P  ->  ( E' )  |  I
```

2.8.1 Construction of the precedence matrix

Before we construct the matrix it is convenient to form, for each non-terminal `U` in the grammar, two sets `Left(U)` and `Right(U)` of symbols that can start and end strings derived from `U`. If `U => B ...` then `B` is in `Left(U)`. Similarly, if `U => ... B` then `B` is in `Right(U)`.

`Left(U)` can be derived for all non-terminals in the grammar by the following algorithm:

```
1)  Initialise all sets Left(U) to empty.

2)  For each production  U -> A ...
    enter A into Left(U)

3)  For each production  U -> V ...
    enter all the elements of Left(V) into Left(U)

4)  Repeat (3) until no further change.
```

`Right(U)` can be derived similarly. For the example grammar the sets are as follows:

U	Left(U)	Right(U)
E'	E T' T P (I	E T' T P) I
E	E T' T P (I	T' T P) I
T'	T P (I	T P) I
T	T P (I	P) I
P	(I	) I

The following algorithm constructs the precedence matrix. For each pair `A B` occurring consecutively in a production (i.e. `P -> ... A B ...` is a production) do the following:

```
1)   Enter  A EQ B  into the matrix

2)   Enter  A LT X  into the matrix
                    for all X in Left(B)

3a)  Enter  X GT B  into the matrix
                    for all X in Right(A)

3b)  Enter  X GT Y  into the matrix
                    for all X in Right(A)
                    and all Y in Left(B)
```

For the example grammar, there are 8 pairs to consider:

< E'	E' >	E' +	+ T'	T *	* P	(E'	E')

and the resulting matrix is:

	E'	E	T'	T	P	(	I	*	+	)	>
E'	-	-	-	-	-	-	-	-	-	EQ	EQ
E	-	-	-	-	-	-	-	-	EQ	GT	GT
T'	-	-	-	-	-	-	-	-	GT	GT	GT
T	-	-	-	-	-	-	-	EQ	GT	GT	GT
P	-	-	-	-	-	-	-	GT	GT	GT	GT
)	-	-	-	-	-	-	-	GT	GT	GT	GT
I	-	-	-	-	-	-	-	GT	GT	GT	GT
*	-	-	-	-	EQ	LT	LT	-	-	-	-
+	-	-	EQ	LT	LT	LT	LT	-	-	-	-
(	EQ	LT	LT	LT	LT	LT	LT	-	-	-	-
<	EQ	LT	LT	LT	LT	LT	LT	-	-	-	-

For this grammar it is possible to find two functions `f` and `g` with the property that:

```
if  A EQ B   then   f(A) = g(B)
if  A LT B   then   f(A) < g(B)
if  A GT B   then   f(A) > g(B)
```

Define such functions for this grammar and show that it is not possible in general. Why are such functions useful?

The following program will perform the parse using the precedence matrix. `P[k]` is the `k`th symbol of source text, and `S[i]` is the `i`th element of a stack.

```
S[0] = P[0]; i = 0, k = 1;
while (P[k] != '>')
{ S[++i] = P[k++];
  while (S[i] GT P[k])
  { int j = i;
    while (S[j-1] EQ S[j]) j--;
    S[j] = Leftpart(S[j] ,..., S[i]);
    i = j;
  }
}
```

The function `Leftpart` finds the subject of the production whose right part is given as its argument(s). Syntactic errors are detected by either encountering a blank entry in the matrix or by a failure in the function `Leftpart`.

NB. Note that the parsing program above is independent of the grammar to be used—all the grammatical details are stored in the tables. One can say that here the grammar is coded as data whereas in the recursive descent parser it was coded as program.

2.9 SLR parsing

Various parsing algorithms based on the so called LR(k) approach have become become popular. These are specifically LR(0), SLR(1), LALR(1) and LR(1). These four methods can parse a source text using a very simple program controlled by a table derived from the grammar. The methods only differ in the size and content of the controlling table.

To exemplify this style of syntax analysis, consider the following grammar (here `E`, `T`, `P` abbreviate 'expression', 'term' and 'primary'—an alternative notation would use names like `<expr>`, `<term>` and `<primary>` instead):

```
#0     S  ->  E  eof
#1     E  ->  E  +  T
#2     E  ->  T
#3     T  ->  P  **  T
#4     T  ->  P
#5     P  ->  i
#6     P  ->  (  E  )
```

The form of production `#0` is important. It defines the sentence symbol `S` and its RHS consists of a single non-terminal followed by the special terminal symbol `eof` which must not occur anywhere else in the grammar.

We first construct what is called the *characteristic finite state machine* or CFSM for the grammar. Each state in the CFSM corresponds to a different set of *items* where an *item* consists of a production together with a position marker (represented by `.`) marking some position on the right hand side. There are, for instance, four possible items involving production `#1`, as follows:

```
E  -> .E  +  T
E  ->  E .+  T
E  ->  E  + .T
E  ->  E  +  T .
```

If the marker in an item is at the beginning of the right hand side then the item is called an *initial* item. If it is at the right hand end the the item is called a *completed* item. In forming item sets a *closure* operation must be performed to ensure that whenever the marker in an item of a set precedes a non-terminal, `E` say, then initial items must be included in the set for all productions with `E` on the left hand side.

The first item set is formed by taking the initial item for the production defining the sentence symbol (`S  -> .E  eof`) and then performing the closure operation, giving the item set:

```
1: {  S  -> .E  eof
      E  -> .E  +  T
      E  -> .T
      T  -> .P  **  T
      T  -> .P
      P  -> .i
      P  -> .(  E  )
   }
```

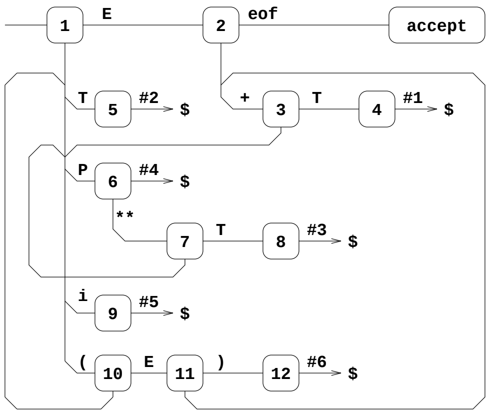
States have *successor* states formed by advancing the marker over the symbol it precedes. For state 1 there are successor states reached by advancing the marker over the symbols `E`, `T`, `P`, `i` or `(`. Consider, first, the `E` successor (state 2), it contains two items derived from state 1 and the closure operation adds no more (since neither marker precedes a non terminal). State 2 is thus:

```
2: {  S  ->  E .eof
      E  ->  E .+  T
   }
```

The other successor states are defined similarly, except that the successor of `eof` is always the special state `accept`. If a new item set is identical to an already existing set then the existing set is used. The successor of a completed item is a special state represented by `$` and the transition is labeled by the production number (`#i`) of the production involved. The process of forming the complete collection of item sets continues until all successors of all item sets have been formed. This necessarily terminates because there are only a finite number of different item sets.

For the example grammar the complete collection of item sets given in Figure 2.2. Note that for completed items the successor state is reached via the application of a production (whose number is given in the diagram).

The CFSM can be represented diagrammatically as follows:



Before we can construct an SLR(1) parser we must define and compute the sets `FOLLOW(A)` for all non-terminal symbols `A`. `FOLLOW(A)` is defined to be the set of all symbols (terminal and non-terminal) that can immediately follow the non-terminal symbol `A` in a sentential form. They can be formed iteratively by repeated application of the following rules.

```
1: {  S  -> .E  eof          \
      E  -> .E  +  T         /    E  => 2
      E  -> .T                    T  => 5
      T  -> .P  **  T        \
      T  -> .P               /    P  => 6
      P  -> .i                    i  => 9
      P  -> .(  E  )              (  => 10
   }
2: {  S  ->  E .eof               eof => accept
      E  ->  E .+  T              +  => 3
   }
3: {  E  ->  E  + .T              T  => 4
      T  -> .P  **  T        \
      T  -> .P               /    P  => 6
      P  -> .i                    i  => 9
      P  -> .(  E  )              (  => 10
   }
4: {  E  ->  E  +  T .            #1 => $
   }
5: {  E  ->  T .                  #2 => $
   }
6: {  T  =>  P .**  T             ** => 7
      T  ->  P .                  #4 => $
   }
7: {  T  ->  P  ** .T             T  => 8
      T  -> .P  **  T        \
      T  -> .P               /    P  => 6
      P  -> .i                    i  => 9
      P  -> .(  E  )              (  => 10
   }
8: {  P  ->  P  **  T .           #3 => $
   }
9: {  P  ->  i .                  #5 => $
   }
10:{  P  ->  ( .E  )         \
      E  -> .E  +  T         /    E  => 11
      E  -> .T                    T  => 5
      T  -> .P  **  T        \
      T  -> .P               /    P  => 6
      P  -> .i                    i  => 9
      P  -> .(  E  )              (  => 10
   }
11:{  P  ->  (  E .)              )  => 12
      E  ->  E .+  T              +  => 3
   }
12:{  P  ->  (  E  ) .            #6 => $
   }
```

Figure 2.2: CFSM item sets

1. If there is a production of the form `X -> ... Y Z ...` put `Z` and all symbols that can start `Z` into `FOLLOW(Y)`.
2. If there is a production of the form `X -> ... Y` put all symbols in `FOLLOW(X)` into `FOLLOW(Y)`.

We are assuming here that no production in the grammar has an empty right hand side. For our example grammar, the `FOLLOW` sets are as follows:

```
FOLLOW(E)  =  { eof  +  ) }
FOLLOW(T)  =  { eof  +  ) }
FOLLOW(P)  =  { eof  +  )  ** }
```

From the CFSM we can construct the two matrices `action` and `goto`:

1. If there is a transition from state `i` to state `j` under the terminal symbol `k`, then set `action[i,k]` to `Sj`.
2. If there is a transition under a non-terminal symbol `C`, say, from state `i` to state `j`, set `goto[i, C]` to `j`.
3. If state `i` contains a transition under `eof` set `action[i, eof]` to `acc`.
4. If there is a reduce transition `#p` from state `i`, set `action[i, k]` to `#p` for all terminals `k` belonging to `FOLLOW(A)` where `A` is the subject of production `#p`.

If any entry is multiply defined then the grammar is not SLR(1). Blank entries are represented by dash (`-`).

	action						goto		
state	eof	(	i	)	+	**	P	T	E
S1	-	S10	S9	-	-	-	S6	S5	S2
S2	acc	-	-	-	S3	-	-	-	-
S3	-	S10	S9	-	-	-	S6	S4	-
S4	#1	-	-	#1	#1	-	-	-	-
S5	#2	-	-	#2	#2	-	-	-	-
S6	#4	-	-	#4	#4	S7	-	-	-
S7	-	S10	S9	-	-	-	S6	S8	-
S8	#3	-	-	#3	#3	-	-	-	-
S9	#5	-	-	#5	#5	#5	-	-	-
S10	-	S10	S9	-	-	-	S6	S5	S11
S11	-	-	-	S12	S3	-	-	-	-
S12	#6	-	-	#6	#6	#6	-	-	-

The parsing algorithm used for all LR methods uses a stack that contains alternately state numbers and symbols from the grammar, and a list of input terminal symbols terminated by `eof`. A typical situation is represented below:

```
a A b B c C d D e E f  |  u v w x y z eof
```

Here `a ... f` are state numbers, `A ... E` are grammar symbols (either terminal or non-terminal) and `u ... z` are the terminal symbols of the text still to be parsed. If the original text was syntactically correct, then

```
A B C D E u v w x y z
```

will be a sentential form.

The parsing algorithm starts in state `S1` with the whole program, i.e. configuration

```
1  |  the whole program upto eof
```

and then repeatedly applies the following rules until either a syntactic error is found or the parse is complete.

1. If `action[f, u] = Si`, then transform

   ```
   a A b B c C d D e E f  |  u v w x y z eof
   ```

   to

   ```
   a A b B c C d D e E f u i  |  v w x y z eof
   ```

2. If `action[f, u] = #p`, and production `#p` is of length 3, say, then it will be of the form `P -> C D E` where `C D E` exactly match the top three symbols on the stack, and `P` is some non-terminal, then assuming `goto[c, P] = g`

   ```
   a A b B c C d D e E f  |  u v w x y z eof
   ```

   will transform to

   ```
   a A b B c P g  |  u v w x y z eof
   ```

   Notice that the symbols in the stack corresponding to the right hand side of the production have been replaced by the subject of the production and a new state chosen using the `goto` table.

3. If `action[f, u] = acc` then the situation will be as follows:

   ```
   a Q b  |  eof
   ```

   and the parse will be complete. (Here `Q` will necessarily be the single non-terminal in the start symbol production (`#0`).

4. If `action[f, u] = -` then the text being parsed is syntactically incorrect.

Note again that there is a single program for all grammars; the grammar is coded in the `action`and `goto` matrices.

As an example, the following steps are used in the parsing of `i + i`:

Stack	text	production used
1	i + i eof	
1 i 9	+ i eof	
1 P 6	+ i eof	P -> i
1 T 5	+ i eof	T -> P
1 E 2	+ i eof	E -> T
1 E 2 + 3	i eof	
1 E 2 + 3 i 9	eof	
1 E 2 + 3 P 6	eof	P -> i
1 E 2 + 3 T 4	eof	T -> P
1 E 2	eof	E -> E + T

```
%%
[ \t]  /* ignore blanks and tabs  */ ;

[0-9]+ { yylval = atoi(yytext); return NUMBER; }

"mod" return MOD;
"div" return DIV;
"sqr" return SQR;
\n|.  return yytext[0]; /* return everything else */
```

Figure 2.3: `calc.l`

2.9.1 Errors

A syntactic error is detected by encountering a blank entry in the `action` or `goto` tables. If this happens the parser can recover by systematically inserting, deleting or replacing symbols near the current point in the source text, and choosing the modification that yields the most satisfactory recovery. A suitable error message can then be generated.

2.9.2 Table compaction

In a typical language we can expect there to be over 200 symbols in the grammar and perhaps rather more states in the CFSM. The table `action` and `goto` are thus like to require over 40000 entries between them. There are good ways of compacting these by about a factor of ten.

2.10 Lex

Lex and Yacc are programs that run on Unix and provide a convenient system for constructing lexical and syntax analysers.

Lex takes as input a file (e.g. `calc.l`) specifying the syntax of the lexical tokens to be recognised and it outputs a C program (normally `lex.yy.c`) to perform the recognition. The syntax of each token is specified by means of a regular expression and the corresponding action when that token is found is supplied as a fragment of C program that is incorporated into the resulting lexical analyser. Consider the lex program `calc.l` in Figure 2.3. The regular expressions obey the usual unix conventions allowing, for instance, `[0-9]` to match any digit, the character `+` to denote repetition of one or more times, and dot (`.`) to match any character other than newline. Next to each regular expression is the fragment of C program for the specified token. This may use some predefined variables and constants such as `yylval, yytext` and `NUMBER`. `yytext` is a character vector that holds the characters of the current token (its length is held in `yyleng`). The fragment of code is placed in the body of an external function called `lex`, and thus a `return` statement will cause a return from this function with a specified value. Compound tokens such as `NUMBER` return auxiliary information in suitably declared variables. For example, the converted value of a `NUMBER` is passed in the variable `lexlval`. If a code fragment does not explicitly return from `lex` then after processing the current token the lexical analyser will start searching for the next token.

In more detail, a Lex program consists of three parts separated by %%s.

```
declarations
%%
translation rules
%%
auxiliary C code
```

The declarations allows a fragment of C program to be placed near the start of the resulting lexical analyser. This is a convenient place to declare constants and variables used be the lexical analyser. One may also make regular expression definitions in this section, for instance:

```
ws        [ \t\n]+
letter    [A-Za-z]
digit     [0-9]
id        {letter}({letter}|{digit})*
```

These named regular expressions may be used by enclosing them in braces (`{ or }`) in later definitions or in the translations rules.

The translation rules are as above and the auxiliary C code is just treated as a text to be copied into the resulting lexical analyser.

2.11 Yacc

Yacc (yet another compiler compiler) is like Lex in that it takes an input file (e.g. `calc.y`) specifying the syntax and translation rule of a language and it output a C program (usually `y.tab.c`) to perform the syntax analysis.

A Yacc program has three part separated by `%%`s.

```
declarations
%%
translation rules
%%
auxiliary C code
```

Within the declaration one can specify fragments of C code (enclosed within special brackets `%{` and `%}`) that will be incorporated near the beginning of the resulting syntax analyser. One may also declare token names and the precedence and associativity of operators in the declaration section by means of statements such as:

```
%token NUMBER
%left '*' DIV MOD
```

The translation rules consist of BNF-like productions that include fragments of C code for execution when the production is invoked during syntax analysis. This C code is enclosed in braces (`{ and }`) and may contain special symbols such as `$$`, `$1` and `$2` that provide a convenient means of accessing the result of translating the terms on the right hand side of the corresponding production.

The auxiliary C code section of a Yacc program is just treated as text to be included at the end of the resulting syntax analyser. It could for instance be used to define the main program.

An example of a Yacc program (that makes use of the result of Lex applied to `calc.l`) is `calc.y` listed in Figure 2.4.

```
%{
#include <stdio.h>
%}

%token NUMBER

%left '+' '-'
%left '*' DIV MOD
  /* gives higher precedence to '*', DIV and MOD  */
%left SQR

%%
comm: comm '\n'
    | /* empty  */
    | comm expr '\n'  { printf("%d\n", $2); }
    | comm error '\n' { yyerrok; printf("Try again\n"); }
    ;

expr: '(' expr ')'    { $$ = $2; }
    | expr '+' expr   { $$ = $1 + $3; }
    | expr '-' expr   { $$ = $1 - $3; }
    | expr '*' expr   { $$ = $1 * $3; }
    | expr DIV expr   { $$ = $1 / $3; }
    | expr MOD expr   { $$ = $1 % $3; }
    | SQR expr        { $$ = $2 * $2; }
    | NUMBER
    ;

%%

#include "lex.yy.c"

yyerror(s)
char *s;
{printf("%s\n", s);
}

main()
{ return yyparse();
}
```

Figure 2.4: `calc.y`

Yacc parses using the LALR(1) technique. It has the interesting and convenient feature that the grammar is allowed to be ambiguous resulting in numerous shift-reduce and reduce-reduce conflicts that are resolved by means of the precedence and associativity declarations provided by the user. This allows the grammar to be given using fewer syntactic categories with the result that it is in general more readable.

The above example uses Lex and Yacc to construct a simple interactive calculator; the translation of each expression construct is just the integer result of evaluating the expression. Note that in one sense it is not typical in that it does not construct a parse tree—instead the value of the input expression is evaluated as the expression is parsed. The first two productions for '`expr`' would more typically look like:

```
expr: '(' expr ')'    { $$ = $2; }
    | expr '+' expr   { $$ = mkbinop('+', $1, $3); }
```

where `mkbinop()` is a C function which takes two parse trees for operands and makes a new one representing the addition of those operands.

Chapter 3

Translation

The translation phase of a compiler normally converts the abstract syntax tree representation of a program into intermediate object code which is usually either a linear sequence of statements or an internal representation of a flowchart. We will assume that the translation phase deals with (1) the scope and allocation of variables, (2) determining the type of all expressions, (3) the selection of overloaded operators, and (4) generating of the intermediate code.

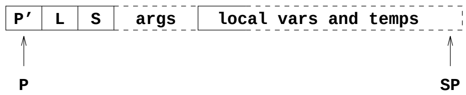
In this section we will use a simple but real-feeling example language (it is actually based on BCPL) with the following abstract syntax tree structure:

```
type N = string;   (* shorthand for 'name' *)
datatype E = Var of N | Num of int | Ap of N * (E list) |
             True | False |
             Neg of E  |  Pos of E  |  Not of E  |
             Subscript of E * E |
             Plus of E * E  |  Minus of E * E  |
             Mult of E * E  |  Div of E * E  |
             Eq of E * E  |  Ne of E * E  |
             Lt of E * E  |  Gt of E * E  |
             Le of E * E  |  Ge of E * E  |
             And of E * E  |  Or of E * E  |
             Cond of E * E * E  | Valof of C

     and C = Seq of C * C  |  Unless of E * C  |
             If1 of E * C  |  If2 of E * C * C  |
             While of E * C  |  Until of E * C  |
             Assign of N * E  | AssignSubscript of E * E * E  |
             For of N * E * E * C  | Result of E  |
             Call of N * E list  |  Let of D * C

     and D = And of D * D  |  Valdef of N * E  | Arraydef of N * int  |
             Fndef of N * (N list) * E  |  Rtdef of (N * N list) * C;
```

An example program fragment that can be represented by the above abstract syntax tree is given later.

We will assume that this language is essentially typeless and so all expression values are of the same size. It is to be implemented using a simple stack and the static chain method will be used for accessing free variables. We will also assume that functions may not be passed as arguments or returned as results. Most of these restrictions could easily be removed by decorating the abstract syntax tree with more type information.

The intermediate object code for this language which we adopt is a linear sequence of simple statements that describe instructions acting on the runtime stack. This is based on OCODE from the BCPL compiler but note the Java Virtual Machine code is similar. A stack frame will has the following form:



Here S is the static link, P' is the dynamic link (previous frame pointer) and L is the return address. Variables are accessed by the following intermediate code statements:

```
Statement      Meaning

Lv n 0         SP++; SP[0] := P[n];
Sv n 0         P[n] := SP[0]; SP--;
Lv n 1         SP++; SP[0] := P[2][n];
Sv n 1         P[2][n] := SP[0]; SP--;
Lv n 2         SP++; SP[0] := P[2][2][n];
Sv n 2         P[2][2][n] := SP[0]; SP--;
etc.
```

The address of a variable (or zeroth element of an array) can be computed using `Laddr`, for example

```
Laddr n 2      SP++; SP[0] := &P[2][2][n]
```

Some of the other intermediate code statements are as follows:

```
Ln k           SP++; SP[0] := k;
Plus           SP[-1] := SP[-1] + SP[0]; SP--;
Neg            SP[0] := - SP[0];
Eq             SP[-1] := SP[-1] = SP[0]; SP--;
Jt Ln          B := SP[0]; SP--; if  B goto Ln;
Jf Ln          B := SP[0]; SP--; if !B goto Ln;
Jmp Ln         goto Ln;
Lab Ln         Ln:
Setsp n        SP = P+n;
Call Ln d k    // Call the function or routine whose
               // entry point is labelled Ln and which
               // was declared at a textual depth d levels
               // less than the current depth,
               // incrementing the P pointer by k positions.
               NP := P+k
               NP[0] := P   // the old P
               NP[1] := Li  // the return address
               if d=0 do NP[2] := P        // current level
               if d=1 do NP[2] := P[2]     // one level out
               if d=2 do NP[2] := P[2]P[2] // two levels out
               etc.
               P := NP
               goto Ln            // Jump to the entry point
            Li:SP := P+k-1        // return here
```

```
Entry Ln    Ln:
Rtn            L := P[1]    // get return address
               P := P[0]    // get old P pointer
               goto L
SetRes         // RES for function results
               RES := SP[0]; SP--;
LdRes          SP++; SP[0] := RES;
```

The use of this intermediate code can be shown by considering the following C program fragment:

```
int i = 36;                         // i in P[3]

int g(a,b) { i = a*i+b; }           // Entry L1
int f(x,y) { return x==0 ? 0 :      // Entry L2
             x*y + f(x-1,y+1); }

int p(int n)                        // Entry L3 n in P[3]
{ if (n==0) return 1;
  else
  {   int v[11];                    // v in P[4]..P[14]
      int h(x) { return f(n,x); }   // Entry L4
      int j;                        // j in P[15]
      for (j = 0; j<11; j++)
      { g(n, j); v[j] = h(j) }
      return v[10];
  }
}
```

The code generated for the function g might be:

```
Entry L1  // entry to g(a,b)
Setsp 4   // Leave space for a in P[3] and b in P[4]
Lv 3 0    //     a
Lv 3 1    //          i
Mult      //       *
Lv 4 0    //               b
Add       //            +
Sv 3 1    // i :=
Rtn
```

and the code for the definition of h might be:

```
Entry L4    // entry to h(x)
Setsp 3     // Leave space for x in P[3]
Setsp 7     // Leave linkage space for call f(n, x)
Lv 3 1      // n  (one level out, pos 3)
Lv 3 0      // x  (current level, pos 3)
Call L2 1 4 // call f (entry L2, declared one level out,
            //        new P at pos P+4)
Rtn         // the result will be in RES
```

These statements will be generated within the translation phase by means of calls such as the following:

```
Gen3(Lv, 1, 3);
Gen3(Lv, 0, 3);
gen4(Call, 2, 1, 4);
gen1(Rtn);
```

The different intermediate code operators can be represented by distinct integers and so the intermediate code can be written to a file as just a sequence of integers.

3.1 Scope and allocation of variables

In a block structured language, variables can be declared and their scope is limited to the region of program for which the declaration is valid. When an identifier is encountered during translation it must be looked up in a stack of currently declared identifiers to discover its datatype and how it is to be accessed (i.e. its address). This stack of name cells is maintained by the translation phase. New cells are added when a declaration is encountered and they are removed when the translator moves out of the scope of the declaration. When an identifier is encountered during translation it is looked up by searching down the name cell stack. The first matching name cell will be the one required.

For our example language, name cells would contain: a pointer to the name node, the type (integer, function or routine), the textual level, and the allocated position within its stack frame if it were an integer or the entry label number if it were a function or routine.

3.2 Translation of expressions

Some of the functions used during translation are as follows:

```
trexp(x)          translate an expression in Rmode
trexplist(x)      translate an expression list in Rmode
trlexp(x)         tranlate an expression in Lmode
trname(op,x)      translate a name, op is one of
                  Lv, Sv, Laddr or Call
jumpcond(x,b,n) translate a conditional jump
trcom(x)          translate a command
declnames(x)      declare the names in a declaration,
                  allocate stack space where necessary, and put
                  suitable items in the declaration vector
trdecl(x)         perform a second pass over a
                  declaration to compile code to
                  initialise variables
```

During translation certain global variables hold information about the current state. For instance, `ssp` is the *simulated stack pointer* holding the distance between `P` and `SP` at the current point in the compiled code. Another variable is `depth` that hold the textual depth of the current piece of code being compiled. It is incremented when starting to translate the body of a function, and decremented when the translation of the body is complete. The argument to `trexp` is the tree for the expression being translated. An outline of its definition is as follows:[1]

[1] We have adopted ML to describe this code since we can exploit pattern matching to make the code more concise than C or Java would be. For ML experts there are still things left undone, like defining the `++` and `--` operators of type `int ref -> int`.

```
fun trexp(Num(k))      = (gen2(Ln, k); ++ssp)
  | trexp(Id(s))       = (trname(Lv,s); ++ssp)
  | trexp(Plus(x,y))   = (trexp(x); trexp(y); gen1(Plus); --ssp)
  | trexp(Minus(x,y))  = (trexp(x); trexp(y); gen1(Minus); --ssp)
  | trexp(Mult(x,y))   = (trexp(x); trexp(y); gen1(Mult); --ssp)
  | trexp(Div(x,y))    = (trexp(x); trexp(y); gen1(Div); --ssp)
  | trexp(Neg(x))      = (trexp(x); gen1(Neg))
  | trexp(Not(x))      = (trexp(x); gen1(Not))
  | trexp(Ap(f, el))   =
                 let val s = ssp in
                   gen2(Setsp, s+3);  // leave space for linkage
                   ssp := s+3;        // ditto for compiler state
                   trexplist(el);     // translate args
                   trname(Call, f);   // Compile Call Lf d
                   gen1(s+1);         // Compile           k
                   ssp := s;          // Restore saved ssp
                   gen1(Ldres);
                   ++ssp
                 end
  | trexp(Cond(b,x,y)) =
                 let val p = ++label;  // Allocate two labels
                     val q = ++label in
                   jumpcond(b,false,p);
                   trexp(x);           // code to put x on stack
                   gen2(Jmp,q);        // jump to common point
                   --ssp;              // at Lab stack is one less
                   gen2(Lab,p);
                   trexp(y);           // code to put y on stack
                   gen2(Lab,q)         // common point; result on stack
                 end;
etc...

fun trexplist[] = ()
  | trexplist(e::es) = (trexp(e); trexplist(es));
```

3.3 Translation of boolean expressions

If a boolean expression occurs in a context where a conditional jump is to be made on the outcome of the evaluation then some optimisation is possible. For example, consider code like

```
IF x>0 AND A[i]=0 THEN ...
```

If `x>0` is false then there is no need to evaluate `A[i]=0` (assuming it is side-effect free). Some languages (like C) even provide a special operator '`&&`' which prescribes such *short-cut* evaluation. We will use the function `jumpcond` to compile such expressions. Its first argument is the tree structure of the expression, the second is a truth value stating whether a jump is to be made on true or on false, and the third argument is the number of the label to jump to. The definition of `jumpcond` is outlined below:

```
fun jumpcond(True,       true, n) =  gen2(Jmp, n)
  | jumpcond(True,          _, _) =  ()
  | jumpcond(False,     false, n) =  gen2(Jmp, n)
```

```
| jumpcond(False,         _, _) =  ()
| jumpcond(Not(x),        b, n) =  jumpcond(x, not b, n)
| jumpcond(And(x, y),  true, n) =
                       let val m = ++label in
                         jumpcond(x, false, m);
                         jumpcond(y,  true, n);
                         gen2(Lab, m)
                       end
| jumpcond(And(x, y), false, n) =  (jumpcond(x, false, n);
                                    jumpcond(y, false, n))
| jumpcond( Or(x, y),  true, n) =  (jumpcond(x,  true, n);
                                    jumpcond(y,  true, n))
| jumpcond( Or(x, y), false, n) =
                       let val m = ++label in
                         jumpcond(x,  true, m);
                         jumpcond(y, false, n);
                         gen2(Lab, m);
                       end
| jumpcond(x, b, n) =  ( trexp(x);
                         gen2( (b ? Jt:Jf), n);
                         --ssp)
```

3.4 Translation to machine code from intermediate code

The part II course on 'Optimising Compilation' will cover this topic in an alternative manner, but let us for now merely observe that each intermediate instruction listed above can be mapped into a small number of Pentium instructions. For example, if the current offset of SP above P is 4 words (i.e. the stack has 5 words used) then[2,3]

```
LAB L3                  ssp = 4 here (3 linkage,x,y)
LP 3                    ssp = 5 here (3 linkage,x,y,temp)
LN 7                    ssp = 6
MINUS                   ssp = 5
SP 3                    ssp = 4
LP 3
LN 0
LT
JF L4
```

can be translated in a instruction-by-instruction manner (using a Pentium register here called fp as the frame pointer P since the Pentium stack pointer does not fit well into our upward growing stack model):

```
L3: movl 12(fp),%eax           ; LP 3 (get value)
    movl %eax,20(fp)           ; LP 3 (write to stack at ssp=5)
    movl #7,24(fp)             ; LN 7 (and write to stack at ssp=6)
    movl 20(fp),%eax           ; MINUS (operand 1)
```

[2]Note that this code could result from compiling the body of

```
int f(int x, int y) { x = x-7; if (x < 0) ...}
```

[3]Here we abbreviate "Lv k 0" as "LP k" and similarly "Sv k 0" as "SP k".

```
subl 24(fp),%eax            ; MINUS (operand 2)
movl %eax,20(fp)            ; MINUS (write result to ssp=5)
movl 20(fp),%eax            ; SP 3  ([pointlessly!] get value)
movl %eax,12(fp)            ; SP 3  (do the store)
...
```

With a little more care in remembering values in registers between translations of individual intermediate instructions quite reasonable, though far from optimal, code can be produced.

3.5 Translation using tree matching and rewriting

This section gives an alternative method of generating code for CISC-like target architectures directly from parse trees.

Ref: Code Generation Using tree matching and Dynamic programming by Aho, A.V., Ganapathi, M. and Tjiang, S.W.K. ACM Transactions on Programming Languages and Systems, Vol 11, No 4, October 1989.

A slightly simplified version of the algorithm is presented here. The algorithm uses a collection of tree rewrite rules to define the resulting translation. Each rule has four components as follows:

```
replacement  <-  template        cost      code
```

where `replacement` is a single node, `template` is a tree, `cost` is the cost of using this rule, `code` is a fragment of compiled code. For example:

```
      Rule                           Cost      Code

#1  Ri  <-  Kc                         2      MOV #c,Ri
#2  Ri  <-  Ma                         2      MOV a,Ri
#3  C   <-  Ass(Ma, Ri)                2      MOV Ri,a
#4  C   <-  Ass(Ind(Ri),Ma)            2      MOV a,*Ri
#5  Ri  <-  Ind(Add(Kc,Rj))            2      MOV c(Rj),Ri
#6  Ri  <-  Add(Ri, Ind(Add(Kc,Rj)))   2      ADD c(Rj),Ri
#7  Ri  <-  Add(Ri, Rj)                1      ADD Rj,Ri
#8  Ri  <-  Add(Ri, K1)                1      INC Ri
```

The tree pattern could be drawn as in Figure 3.1. The tree for the command: `v[i] := x` might be as in Figure 3.2. A third representation of the same tree is

```
Ass(Ind(Add(Add(Kv,Rp),Ind(Add(Ki,Rp))), Mx)
```

This tree can be 'covered' by the templates in several ways, but one that gives the least cost is given in Figure 3.3. The total cost in this case is 7. Using this covering, the code and resulting trees produced by a depth first left to right scan are as follows:

```
               Ass(Ind(Add(Add(Kv,Rp), Ind(Add(Ki,Rp))), Mx)
MOV #v,R0      Ass(Ind(Add(Add(R0,Rp), Ind(Add(Ki,Rp))), Mx)
ADD Rp,R0      Ass(Ind(Add(R0,         Ind(Add(Ki,Rp))), Mx)
ADD i(Rp),R0   Ass(Ind(R0),                              Mx)
MOV x,*R0      C
```

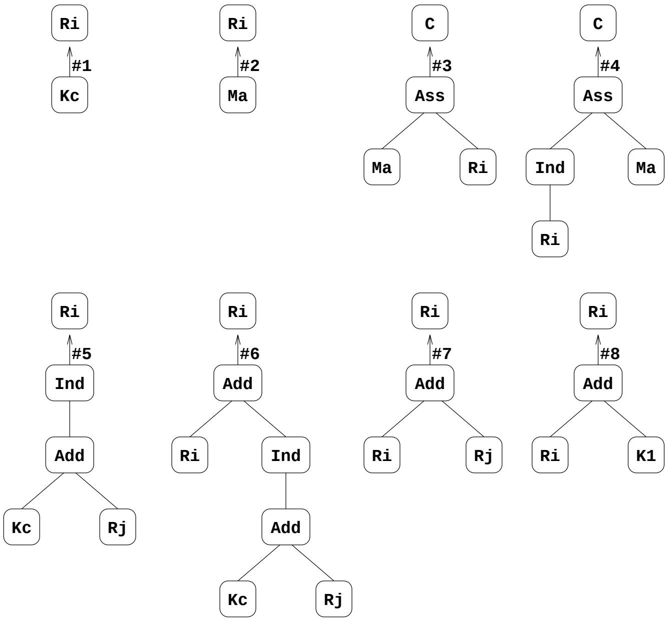


Figure 3.1: Tree version of rules

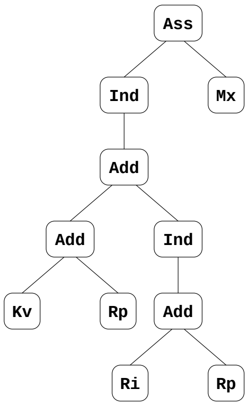


Figure 3.2: Tree for `v[i] := x`

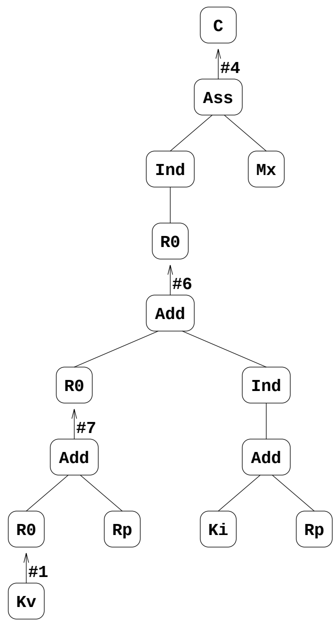


Figure 3.3: Minimum cost covering

Other coverings are possible but these would give different costs and different code sequences. The algorithm given here is designed to find a least cost covering efficiently.

The template `#6 Add(R?, Ind(Add(K?,R?)))` contains paths from the root to its three leaf nodes, namely:

```
Add.1--R?
Add.2--Ind.1--Add.1--K?
Add.2--Ind.1--Add.2--R?
```

The integers specify which edge is being taken from an operator node. The length of a path is the number of edges it contains. The path `Add.1--R?` is of length 1 and occurs in template `#6` (this is denoted by `#6/1`). We can combine all the paths from all the templates to form a tree as follows:

```
                                          Rule/Length

n00-Add.1-n01-R?--a00                     #8/1,#7/1,#6/1
  |    .2-n02-Ind.1-n03-Add.1-n04-K1-a01  #6/3,#5/2,#1/0
  |         |              |    *-K?-a02  #6/3,#5/2,#1/0
  |         |              .2-n05-R?-a03  #7/1,#6/3,#5/2
  |         *-K1--a04                     #8/1,#1/0
  |         *-R?--a05                     #7/1
  *-Ass.1-n06-Ind.1-n07-R?--a06           #4/2
  |    |    *-M?--a07                     #3/1,#2/0
  |    .2-n08-M?--a08                     #4/1,#2/0
  |         *-R?--a09                     #3/1
  *-Ind.1-n09-Add.1-n10-K1--a10           #5/2,#1/0
  |              |    *-K?--a11           #5/2,#1/0
  |              .2-n11-R?--a12           #7/1,#5/2
  *-K1--a13                               #1/0
  *-K?--a14                               #1/0
  *-M?--a15                               #2/0
```

The internal nodes of the tree are labelled `n00` to `n11`, and the leaf nodes are labelled `a00` to `a15`. Notice that the path `Add.1--R?` is of length 1 and occurs in three different templates, namely `#8`, `#7` and `#6` which accounts for why `#8/1,#7/1,#6/1` is attached to node `a00`. The node `a01` is at the end of the path `Add.2--Ind.1--Add.1--K1` which is a path of length three belonging to template `#6`. However, `a01` is also at the end of `Ind.1--Add.1--K1` which is a path of length 2 belonging to template `#5`, and it also ends a path of length zero belonging to template `#1`. This accounts for `#6/3,#5/2,#1/0` being attached to `a01`.

This tree forms the basis of a finite state acceptor with the leaf nodes being the accepting states. Extra transitions must be added, but the number of nodes remains unchanged. Consider the path string: `Ind.1--Add.2--Ind.1--Add.1--K1`. Starting from `n00`, `Ind.1` gets us to `n09`, then `add.2` gets us to `n11`, but from here the only transition in the tree is on `R?` to `a12`. This tells us that there are no path strings starting with `Ind.1--Add.2--Ind.1` belonging to any of our set of templates. What we should do is delete the first item of our path string, giving us `Add.2--Ind.1--Add.1--K1` and see if this leads to an accepting state. Thus from state `n11`, a transition on `Ind.1` should lead to `n03`. One way of viewing this is if state `n11` does not encounter `R?` then it should behave like state `n02`. Looking carefully at the tree we see that:

State	Not followed by	Behaves like
n01	R?	n00
n02	Ind.1, K1 or R?	n00
n03	Add.1 or Add.2	n09
n04	K1 or K?	n10
n05	R?	n11
n06	Ind.1 or M?	n00
n07	R?	n09
n08	M? or R?	n00
n09	Add.1 or Add.2	n00
n10	K1 or K?	n01
n11	R?	n02

This easily leads to the resulting finite state acceptor described by the following table:

	Add.1	Add.2	Ass.1	Ass.2	Ind.1	K1	K?	R?	M?	like
n00	n01*	n02*	n06*	n08*	n09*	a13*	a14*	-	a15*	-
n01	n01	n02	n06	n08	n09	a13	a14	a00*	a15	n00
n02	n01	n02	n06	n08	n03*	a04*	a14	a05*	a15	n00
n03	n04*	n05*	n06	n08	n09	a13	a14	-	a15	n09
n04	n01	n02	n06	n08	n09	a01*	a02*	a00	a15	n10
n05	n01	n02	n06	n08	n03	a04	a14	a03*	a15	n11
n06	n01	n02	n06	n08	n07*	a13	a14	-	a07*	n00
n07	n10	n11	n06	n08	n09	a13	a14	a06*	a15	n09
n08	n01	n02	n06	n08	n09	a13	a14	a09*	a08*	n00
n09	n10*	n11*	n06	n08	n09	a13	a14	-	a15	n00
n10	n01	n02	n06	n08	n09	a10*	a11*	a00	a15	n01
n11	n01	n02	n06	n08	n03	a04	a14	a12*	a15	n02

The asterisks (*) indicate transitions that are in the original tree. The non-asterisked entries of a row are copied from the row it (otherwise) behaves like. Compare, for instance, row `n11` with row `n02`.

The rule/length information associated with accepting states are encoded as bit patterns, as shown in the following table.

```
     #8 #7   #6  #5   #4 #3 #2 #1  Path                    Rule/Length

a00  10 10 0010 000 000 00  0  0  Add.1-R?                #8/1,#7/1,#6/1
a01  00 00 1000 100 000 00  0  1  Add.2-Ind.1-Add.1-K1  #6/3,#5/2,#1/0
a02  00 00 1000 100 000 00  0  1  Add.2-Ind.1-Add.1-K?  #6/3,#5/2,#1/0
a03  00 10 1000 100 000 00  0  0  Add.2-Ind.1-Add.2-R?  #7/1,#6/3,#5/2
a04  10 00 0000 000 000 00  0  1  Add.2-K1                #8/1,#1/0
a05  00 10 0000 000 000 00  0  0  Add.2-R?                #7/1
a06  00 00 0000 000 100 00  0  0  Ass.1-Ind.1-R?          #4/2
a07  00 00 0000 000 000 10  1  0  Ass.1-M?                #3/1,#2/0
a08  00 00 0000 000 010 00  1  0  Ass.2-M?                #4/1,#2/0
a09  00 00 0000 000 000 10  0  0  Ass.2-R?                #3/1
a10  00 00 0000 100 000 00  0  1  Ind.1-Add.1-K1          #5/2,#1/0
a11  00 00 0000 100 000 00  0  1  Ind.1-Add.1-K?          #5/2,#1/0
a12  00 10 0000 100 000 00  0  0  Ind.1-Add.2-R?          #7/1,#5/2
a13  00 00 0000 000 000 00  0  1  K1                      #1/0
a14  00 00 0000 000 000 00  0  1  K?                      #1/0
a15  00 00 0000 000 000 00  1  0  M?                      #2/0
```

The number of bits allocated for a template is one greater than the length of the longest path in the template. The position of a one indicates the length of an accepted path string. Bit strings allow overlapping matches to the same tree template to be recorded.

3.5.1 The Algorithm

Perform a left to right depth first scan over the subject tree attaching (context) states of the acceptor to each node. For the given example this gives:

```
Replacement               Ri Ri   Ri  Ri   C  C Ri Ri
Rule number               #8 #7   #6  #5  #4 #3 #2 #1

Ass                  n00  00 00 0000 000 001 00  0  0   #4/0
(#4->C)                   00 00 0000 000 000 00  0  0
*-Ind                n06  00 00 0000 000 010 00  0  0   #4/1
| *-Add              n01  00 01 0001 000 000 00  0  0   #6/0, #7/0
|   (#6->Ri)              00 00 0000 000 100 00  0  0   #4/2
|   *-Add            n01  00 01 0000 000 000 00  0  0   #7/0
|   | (#7->Ri)            10 10 0010 000 000 00  0  0   #8/1,#7/1,#6/1
|   | *-Kv           n01  00 00 0000 000 000 00  0  1   #1/0
|   | | (#1->Ri)          10 10 0010 000 000 00  0  0   #8/1,#7/1,#6/1
|   | *-Rp           n02  00 10 0000 000 000 00  0  0   #7/1
|   *-Ind            n02  00 00 0010 001 000 00  0  0   #6/1,5/0
|     (#5->Ri)            00 10 0000 100 000 00  0  0   #7/1,#5/2
|     *-Add          n03  00 01 0100 010 000 00  0  0   #7/0,#6/2,#5/1
|       (#7->Ri)          00 00 0000 000 000 00  0  0
|       *-Ki         n04  00 00 1000 100 000 00  0  1   #6/3,#5/2,#1/0
|       | (#1->Ri)        10 10 0010 000 000 00  0  0   #8/1,#7/1,#6/1
|       *-Rp         n05  00 10 1000 100 000 00  0  0   #7/1,#6/3,#5/2
*-Mx                 n08  00 00 0000 000 010 00  1  0   #4/1,#2/0
  (#2->Ri)                00 00 0000 000 000 10  0  0   #3/1
```

An item of the form `(#i->Op)` indicates that the current branch of the parse tree can be matched by rule `#i`, and if it is, the branch would be replaced by a leaf node with operator `Op`.

The bit patterns indicate for each position in the parse tree which rules are matched and to what depths. When the least significant bit is a one then that point in the tree, it can be matched by the corresponding rule. For instance, the bit pattern indicates that the root node can be matched by rule #4. The value obtained for a node is computed by ANDing together the bit patterns for its children and shifting the result right by one position. When a template matches that position in the tree can be replaced by a leaf node. This may lead to another accepting state whose bit pattern should be regarded as being ORed with the bit pattern for the node itself. Careful study of the above table should make the mechanism clear.

As each possible replacement is found its cost is computed and, if found to be lower than the cost of a previously discovered replacement (yielding the same leaf node), the new cost and the number of the rule that made it possible is recorded in the tree.

When the depth first scan is complete the root node will contain a list of possible leaf nodes that it can be replaced by, together with the minimum cost for each replacement and the corresponding rule that was used. A second pass over the tree can generate the code corresponding to this minimum cost covering.

Chapter 4

Object Modules and Linkers

We have shown how to generate assembly-style code for a typical programming language using relatively simple techniques. What we have still omitted is how this code might be got into a state suitable for execution. Usually a compiler (or an assembler, which after all is only the word used to describe the direct translation of each assembler instruction into machine code) takes a source language and produces an *object file* or *object module* (`.o` on Unix and `.OBJ` on MS-DOS). These object files are linked (together with other object files from program libraries) to produce an *executable file* (`.EXE` on MS-DOS) which can then be loaded directly into memory for execution. Here we sketch briefly how this process works.

Consider the C source file:

```
int m = 37;
extern int h(void);
int f(int x) { return x+1; }
int g(int x) { return x+m+h(); }
```

Such a file will produce a *code segment* (often called a *text segment* on Unix) here containing code for the functions `f` and `g` and a *data segment* containing static data (here `m` only).

The data segment will contain 4 bytes probably `[0x25 00 00 00]`.

The code for `f` will be fairly straightforward containing a few bytes containing bit-patterns for the instruction to add one to the argument (maybe passed in a register like `%eax)`) and return the value as result (maybe also passed in `%eax`). The code for `g` is more problematic. Firstly it invokes the procedure `h()` whose final location in memory is not known to `g` so how can we compile the call? The answer is that we compile a 'branch subroutine' instruction with a dummy 32-bit address as its target; we also output a *relocation entry* in a *relocation table* noting that before the module can be executed, it must be linked with another module which gives a definition to `h()`.

Of course this means that the compilation of `f()` (and `g()`) cannot simply output the code corresponding to `f`; it must also register that `f` has been defined by placing an entry to the effect that `f` was defined at (say) offset 0 in the code segment for this module.

It turns out that even though the reference to `m` within `g()` is defined locally we will still need the linker to assist by filling in its final location. Hence a relocation entry will be made for the 'add `m`' instruction within `g()` like that for 'call `h`' but for 'offset 0 of the current data segment' instead of 'undefined symbol `h`'.

A typical format of an object module is shown in Figure 4.1 for the format ELF often used on Linux (we only summarise the essential features of ELF).

Header information; positions and sizes of sections
`.text` segment (code segment): binary data
`.data` segment: binary data
`.rela.text` code segment relocation table: list of (offset,symbol) pairs showing which offset within `.text` is to be relocated by which symbol (described as an offset in `.symtab`)
`.rela.data` data segment relocation table: list of (offset,symbol) pairs showing which offset within `.data` is to be relocated by which symbol (described as an offset in `.symtab`)
`.symtab` symbol table: List of external symbols used by the module: each is listed together with attribute 1. undef: externally defined; 2. defined in code segment (with offset of definition); 3. defined in data segment (with offset of definition). Symbol names are given as offsets within `.strtab` to keep table entries of the same size.
`.strtab` string table: the string form of all external names used in the module

Figure 4.1: Summary of ELF

4.1 The linker

Having got a sensible object module format as above, the job of the linker is relatively straightforward. All code segments from all input modules are concatenated as are all data segments. These form the code and data segments of the executable file.

Now the relocation entries for the input files are scanned and any symbols required, but not yet defined, are searched for in (the symbol tables of) the library modules. (If they still cannot be found an error is reported and linking fails.) Object files for such modules are concatenated as above and the process repeated until all unresolved names have been found a definition.

Now we have simply to update all the dummy locations inserted in the code and data segments to reflect their position of their definitions in the concatenated code or data segment. This is achieved by scanning all the relocation entries and using their definitions of 'offset-within-segment' together with the (now know) absolute positioning of the segment in the resultant image to replace the dummy value references with the address specified by the relocation entry.

(On some systems exact locations for code and data are selected now by simply concatenating code and data, possibly aligning to page boundaries to fit in with virtual memory; we want code to be read-only but data can be read-write.)

The result is a file which can be immediately executed by *program fetch*; this is the process by which the code and data segments are read into virtual memory at their predetermined locations and branching to the *entry point* which will also have been marked in the executable module.

4.2 Dynamic linking

Consider a situation in which a user has many small programs (maybe 50k bytes each in terms of object files) each of which uses a graphics library which is several megabytes big. The classical idea of linking (*static linking*) presented above would lead to each executable file being megabytes big too. In the end the user's disc space would fill up essentially because multiple copies of library code rather than because of his/her programs. Another disadvantage of static linking is the following. Suppose a bug is found in a graphics library. Fixing it in the library (`.OBJ`) file will only fix it in my program when I re-link it, so the bug will linger in the system in all programs which have not been re-linked—possibly for years.

An alternative to static linking is *dynamic linking*. We create a library which defines *stub* procedures for every name in the full library. The procedures have forms like the following for (say) `sin()`:

```
static double (*realsin)(double) = 0;  /* pointer to fn */
double sin(double x)
{   if (realsin == 0)
    {   FILE *f = fopen("SIN.DLL");    /* find object file */
        int n = readword(f);           /* size of code to load */
        char *p = malloc(n);           /* get new program space */
        fread(p, n, 1, f);             /* read code */
        realsin = (double (*)(double))p;  /* remember code address */
    }
    return (*realsin)(x);
}
```

Essentially, the first time the `sin` stub is called, it allocates space and loads the current version of the object file (`SIN.DLL` here) into memory. The loaded code is then called. Subsequent calls essentially are only delayed by two or three instructions.

In this scheme we need to distinguish the stub file (`SIN.OBJ`) which is small and statically linked to the user's code and the dynamically loaded file (`SIN.DLL`) which is loaded in and referenced at run-time. (Some systems try to hide these issues by using different parts of the same file or generating stubs automatically, but it is important to understand the principle that (a) the linker does some work resolving external symbols and (b) the actual code for the library is loaded (or possibly shared with another application on a sensible virtual memory system!) at run-time.)

Dynamic libraries have extension `.DLL` (dynamic link library) on Microsoft Windows and `.so` (shared object file) on Linux. Note that they should incorporate a version number so that an out-of-date DLL file cannot be picked up accidentally by a program which relies on the features of a later version.

The principal disadvantage of dynamic libraries is the management problem of ensuring that a program has accept to acceptable versions of all DLL's which it uses. It is sadly not rare to try to run a Windows `.EXE` file only to be told that given DLL's are missing or out-of-date because the distributor forgot to provide them or assumed that you kept your system up to date by loading newer versions of DLL's from web sites! Probably static linking is more reliable for executables which you wish still to work in 10 years' time.

[The end]