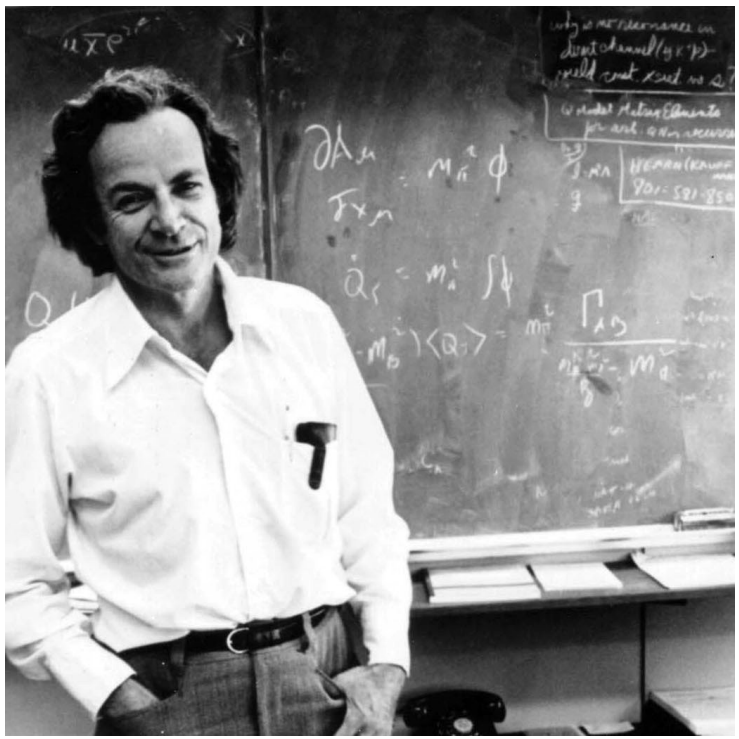
Quantum Computing (CST Part II)

Lecture 1: Bits and Qubits

Steven Herbert

What is quantum computing? (1)



https://it.wikipedia.org/wiki/File:Richard-feynman.jpg

. . . trying to find a computer simulation of physics seems to me to be an excellent program to follow out. . . . the real use of it would be with quantum mechanics. . . . Nature isn't classical . . . and if you want to make a simulation of Nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy.

Richard Feynman

What is quantum computing? (2)

https://quillette.com/2019/01/26/the-unconstrained-vision-of-david-deutsch

The theory of computation has traditionally been studied almost entirely in the abstract, as a topic in pure mathematics. This is to miss the point of it. Computers are physical objects, and computations are physical processes. What computers can or cannot compute is determined by the laws of physics alone, and not by pure mathematics.

David Deutsch

Why study quantum computing?

Article

Quantum supremacy using a programmable superconducting processor

https://doi.org/10.1038/s41586-019-1666-5

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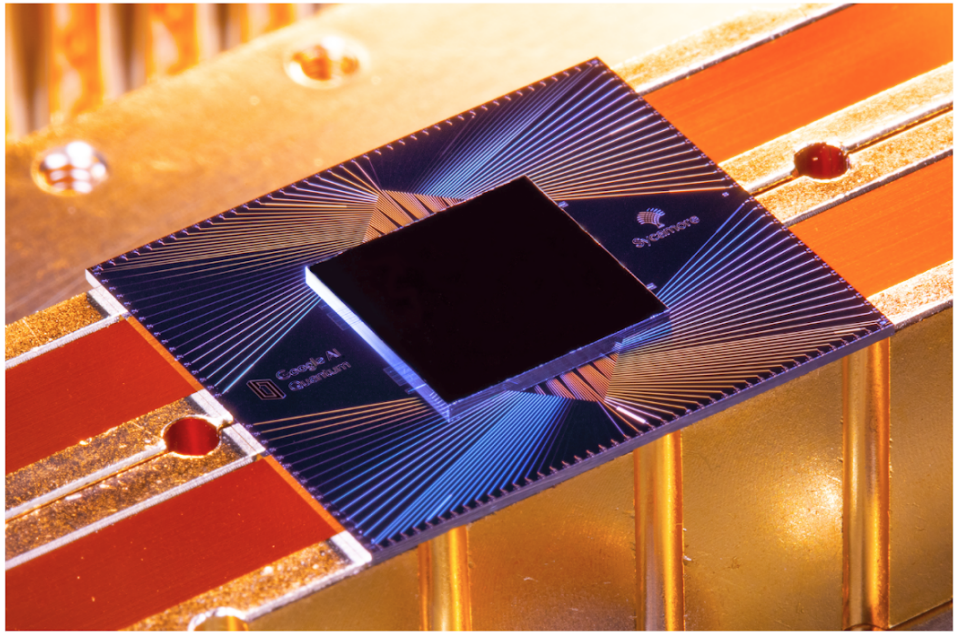


https://ai.googleblog.com/2019/10/quantum-supremacy-using-programmable.html

Why study quantum computing in computer science?

Quantum software development

Quantum algorithm design

Quantum machine learning

Quantum computer architecture and compiler design

Quantum information theory research

Quantum communication system development

Quantum security system development

Scope of the course

- Theoretical basis for quantum computing.
- Practical basis for quantum computing.
- Near-term potential of quantum computing.
- Other applications of quantum information.

What the course isn't:

Philosophy

.. and physics is quite minimal.

Course outline

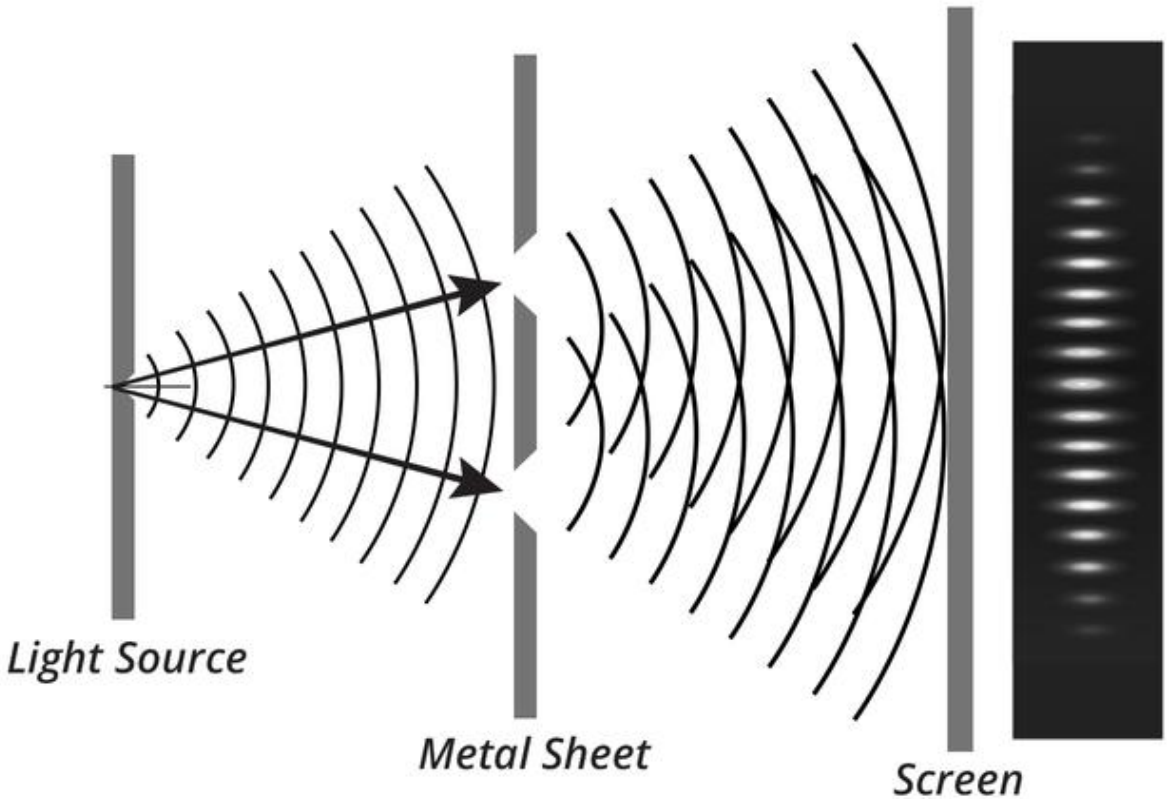
1	Bits and qubits	Introduction
2	Linear algebra	Fundamentals
3	Postulates of quantum mechanics	
4	Concepts in quantum mechanics	
5	The quantum circuit model	
6	Applications of quantum information	Quantum information
7	Deutsch-Jozsa algorithm	Theoretical QC
8	Quantum search	
9	QFT & QPE	
10	QFT & QPE: Factoring	
11	QFT & QPE: Quantum Chemistry	
12	Quantum complexity	
13	Quantum error correction	Practical QC
14	Fault tolerant quantum computing	
15	Adiabatic quantum computing	Near-term QC
16	Case studies in quantum computation	

Useful information

Full list of resources on the course website:
`https://www.cl.cam.ac.uk/teaching/1920/QuantComp`

Key reference book is Quantum computation and quantum information, *Nielsen and Chuang*.

A little bit of physics: the double-slit experiment



https://curiosity.com/topics/the-double-slit-experiment-cracked-reality-wide-open-curiosity

The qubit: an information theoretic way to represent superposition

A classical bit is an intuitive concept, it is either equal to:

$$0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle \text{ or } 1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

Even if we are uncertain about whether a classical bit, B, is in state 0 or 1, we can characterise it by a probability mass function, or a *mixture*

$$p(B = 0) = p_0 \;\; ; \;\; p(B = 1) = p_1$$

where $p_0 + p_1 = 1$. A qubit, $|\psi\rangle$, is quite different, it can be in a *superposition* of the 0 and 1 states:

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

where α and β are complex numbers such that $|\alpha|^2 + |\beta|^2 = 1$.

Measuring a qubit

Any attempt to measure the state

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

results in $|0\rangle$ with probability $|\alpha|^2$, and $|1\rangle$ with probability $|\beta|^2$.

After the measurement, the system is in the measured state! That is, the post-measurement state, $|\psi'\rangle$, will be:

$$|\psi'\rangle = |0\rangle \text{ or } |\psi'\rangle = |1\rangle$$

Further measurements will always yield the same value. We can only extract one bit of information from the state of a qubit.

The state of a qubit describes more than just its measurement probabilities

The superposition of $|0\rangle$ and $|1\rangle$ states describes a physical structure, and not merely a probability mass function over possible measurement outcomes. For example:

$$\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$
$$\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$
$$\frac{1}{\sqrt{2}}\left(|0\rangle + i\,|1\rangle\right)$$
$$\frac{1}{\sqrt{2}}\left(|0\rangle - i\,|1\rangle\right)$$

all have a 50% chance of being in either in the $|0\rangle$ or $|1\rangle$ state if measured, but all correspond to different superpositions, which will evolve differently.

It is crucial to appreciate this point to grasp the essence of quantum computing.

The Hadamard gate: an example of interference

The Hadamard gate, H, has the following function on the state $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$:

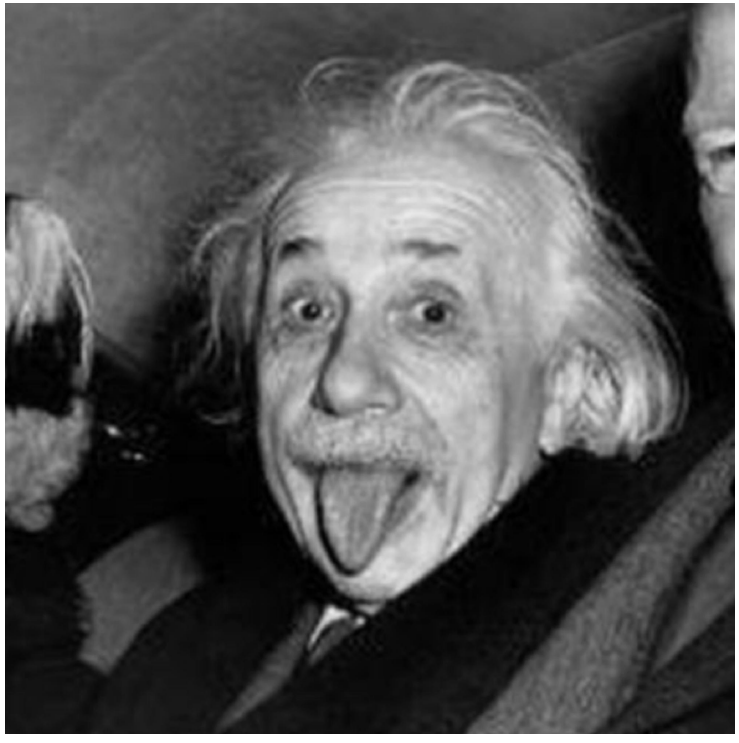
$$H|+\rangle \rightarrow |0\rangle$$

and also the function on the state $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$:

$$H|-\rangle \rightarrow |1\rangle$$

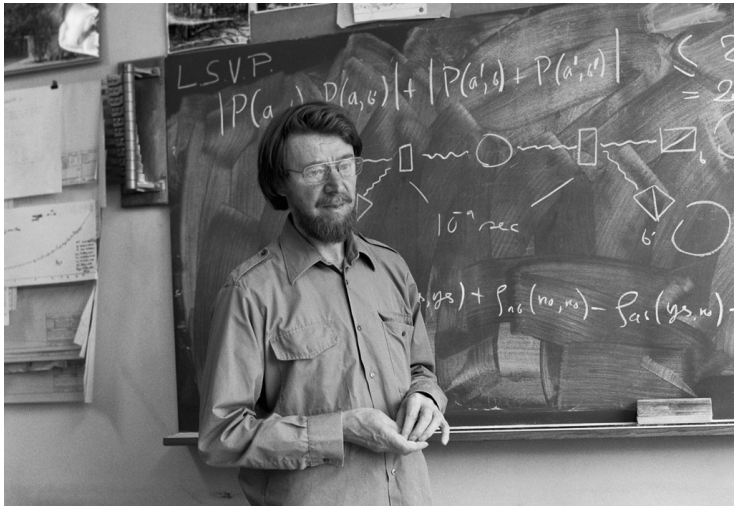
- This is an example of 'interfering' two states in superposition, to yield a deterministic outcome.
- It is also an example of a fundamental difference between two states with the same measurement outcome probabilities ($|+\rangle$ and $|-\rangle$).

A bit more physics: entanglement and Bell inequalities



Getty images

Spooky action at a distance.



https://www.belfasttelegraph.co.uk

No theory of reality compatible with quantum theory can require spatially separate events to be independent.

The Bell state: an information theoretic way to represent entanglement

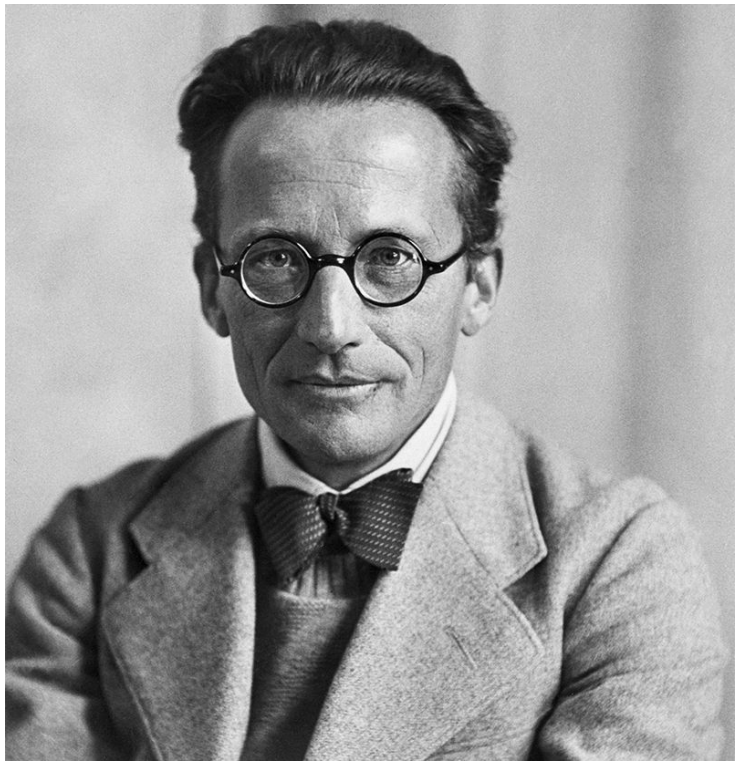
The (two-qubit) Bell state $|\Phi^+\rangle$ is defined:

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

What this says is:

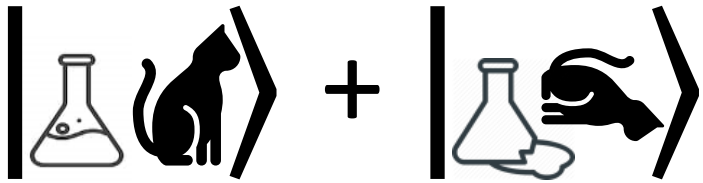
- Each of the two qubits are in an equal superposition of the $|0\rangle$ and $|1\rangle$ states.
- However, they are *entangled*, as soon as one qubit is measured (say the outcome is 1) then the second qubit *collapses* into the state $|1\rangle$.
- There is no requirement that the two qubits are *local*, in the spatial sense, in order for this to occur.

Schrödinger's cat



Historia/REX/Shutterstock.com

Erwin Schrödinger's famous thought experiment tells of a cat whose life has been entangled with a vial of deadly poison.



Three things to remember

Quantum systems are distinguished from their classical counterparts by three phenomena:

- Superposition
- Interference
- Entanglement

These can be represented in information theoretical terms by a qubit, and the operations thereon, and throughout this course we will see how these three phenomena give rise to powerful quantum communication, security and computing systems.