From Double-slits to Qubits
A note for Quantum Computing (Part II CST)

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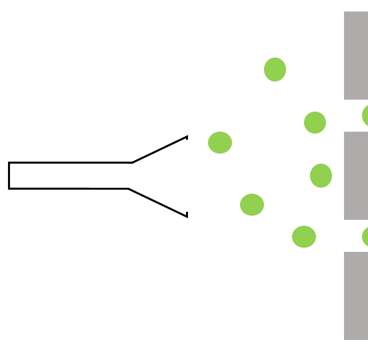
Abstract

This article uses the double-slit experiment to illustrate why qubits need to be represented as two-element complex unit vectors. It also provides a nice visualisation of why relative phase matters but global phase doesn't, and also gives some insight into exactly what measuring a quantum system entails.

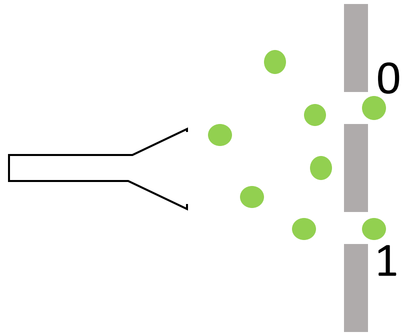
1 Particles and waves

1.1 Particles

Consider a machine firing out balls at a wall that has two holes in:

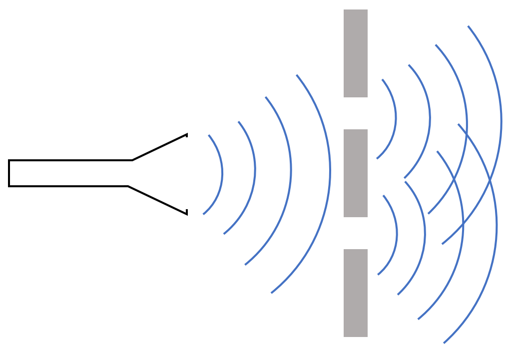


Any ball that passes through to the right-hand side (i.e., past the wall) must have passed through one of the holes. As there are two holes, there are two possibilities, so **we can record which hole a given ball has passed through with a single bit**. For example, we may use the value "0" to record that the ball has passed through the upper hole (as we view it on the page) and the value "1" to record that the ball has passed through the lower hole:



1.2 Waves

Now consider the situation in which a light source emits light (that is, an electromagnetic wave), which travels towards a screen with two slits:



The electric field can be used to describe the effect of a propagating electromagnetic wave at a given point in space, so we now consider the electric field at the transmitting source, which in general can be expressed (in a simplified manner) as:

$$E = ke^{i\omega t}, \tag{1}$$

where E is the electric field, k is a constant, ω is the angular frequency ($\omega = 2\pi f$ where f is the frequency) and t is time. This oscillating electric field then propagates at the speed of light, so if the distance to each of the slits is d, then the electric field at the slits is:

$$E_u = k_u e^{i(\omega(t-(d/c)))} = k_u e^{i\omega t} e^{-i\omega d/c} \tag{2}$$

at the upper slit, for some constant k_u, and where c is the speed of light (note the constant k_u is needed as, in general, there will be a reduction in magnitude as the wave propagates). Thus the d/c term simply represents the time lag incurred by the wave travelling to the slit. Likewise for the lower slit

$$E_l = k_l e^{i\omega t} e^{-i\omega d/c} \tag{3}$$

for some constant k_l.

So it is that the electric field at the screen with the double slits consists of two components (E_u and E_l), and subsequent electric fields arising from the electromagnetic wave propagation depends on these. That is, the electric field to the right of the screen will be identical to that should two sources (of appropriate magnitude and phase) have been placed at the two slits. Note that electric fields are linear, so to find the electric field at some point to the right of the double slit we can simply add the electric fields from the two components.

2 Wave particle duality

Quantum phenomena exhibit "wave-particle" duality – that is, unobserved a quantum system evolves as a wave (i.e., a wave passing through two slits and subsequently interfering with itself) – but when measured as if it has objective (classical) reality, the wave-function collapses and it does indeed have objective reality (i.e., as a photon or if you like a little "ball" passing through one of the two slits).

So it follows that our mathematical description of a quantum system should be sufficient to allow both possibilities – it should both enable the (probabilistic) measurement outcomes to be determined, and also fully capture the subsequent wave propagation (if a measurement isn't made). In particular, according to Postulate 1, *the system is completely described by its state vector*, thus the quantum state at the double slit must completely capture everything about the wave-particle duality. For a two-level quantum system (qubit), we can qualitatively appreciate that a complex superposition over computational basis vectors has the required ingredients. The computational basis vectors ($|0\rangle$ and $|1\rangle$) represent the binary states which can occur if measured (i.e., which slit the photon has passed through) – and the complex coefficients thereof allow the probabilities of each to be evaluated, but also are sufficient to allow the subsequent wave-propagation (i.e., to the right of the screen) to be determined if a measurement isn't made (and this is why they must be complex).

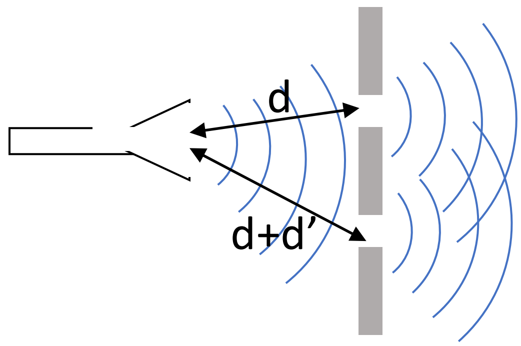
This also provides a nice way to think about computational bases, as in some sense representing "classical" events with objective reality, and measurement thereof as simply obtaining and ascertaining this classical reality by collapsing the wave-function. That is, quantum measurement is just normal measurement with a voltmeter, an ammeter, a signal analyser or whatever.

The fact that we introduce the general measurement postulate is for completeness, but in the part II CST quantum computing course, we almost invariably use computational basis measurement, which has this tangible physical interpretation.

2.1 What about the philosophy of the measurement problem?

Speaking of quantum *measurement* implicitly relies on an objective distinction between a quantum system being measured, and a classical system doing the measurement. But physically, all this comes down to is that "small" objects are quantum, and "large" (i.e., not microscopic) objects are classical. The inelegance of this distinction is known as the *measurement problem* in philosophy, and there have been a number of proposals for how to resolve it, most notably the *Copenhagen interpretation* and the *many worlds interpretation*. In this course we take an operational approach and (very reasonably) assume that this distinction is clear in practise, and thus quantum objects and classical measurement apparatus are well-defined, and we do not trouble ourselves with precisely how this distinction occurs.

3 Global and relative phase



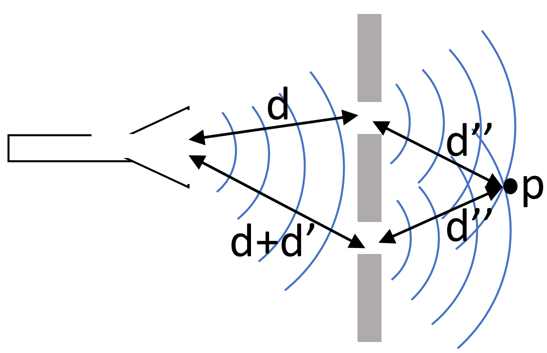
If we adjust the double slit so that the lower of the slits is now a distance $d + d'$ from the source, as shown above, we get electric fields at the upper slit

$$E_u = k_u e^{i\omega t} e^{-i\omega d/c}, \tag{4}$$

as before, but for the lower slit

$$E_l = k_l e^{i\omega t} e^{-i\omega d/c} e^{-i\omega d'/c} = k_l e^{i\omega t} e^{-i\omega d/c} e^{-i\phi}, \tag{5}$$

where we define $\phi = \omega d'/c$ (which we can do because the angular frequency is a constant). If we now want to know the electric field at some point "p" equidistant from the two slits (and to the right of by a distance d''),



we simply add the electric fields (i.e., because of linearity of electric field):

$$\begin{aligned} E_p =& E'_u + E'_l \\ =& k_{u'} e^{i\omega t} e^{-i\omega d/c} e^{-i\omega d''/c} + k_{l'} e^{i\omega t} e^{-i\omega d/c} e^{-i\omega\phi} e^{-i\omega d''/c} \\ =& e^{-i\omega d/c} e^{-i\omega d''/c} e^{i\omega t} (k_{u'} + k_{l'} e^{-i\omega\phi}), \end{aligned} \tag{6}$$

where the constants $k_{u'}$ and $k_{l'}$ have been defined to allow for further reduction in electric field magnitude owing to the further propagation. Therefore $e^{-i\omega d/c} e^{-i\omega d''/c}$ has been factored out, and the constant $-\omega(d + d'')/c$ is the global phase, which has only a classical effect, for example we have implicitly defined the phase at the transmitting source as zero, so we can measure the phase difference from this if we so desire. However, the quantum element of the wave's behaviour only concerns how the two superposed components interfere (and thus the probabilities of measurement outcomes at various points in the evolution), and this is determined only by the *relative magnitudes* $k_{u'}$ and $k_{l'}$ and the *relative phase* $-\phi$.

For example, if the difference in path length between the two components, d', is such that $\phi = \pi$, then the interference will be destructive, and if its such that $\phi = 2\pi$ it will be destructive. This is true regardless of the global phase.

4 Limitations of the analogy

The wave-particle duality exhibited in the double-slit experiment is useful for illustrating why qubits (and indeed general quantum systems) are represented as they are – as complex superpositions over the computational basis states, but in other ways the analogy is limited. In particular, the propagation of a photon is a complicated quantum event, and it is only by viewing it at the double-slit itself that we can (in a slightly contrived way) think of it as a two-level quantum system. Whilst the quantum state could be normalised such that at the double-slit it indeed appears as a 2-element complex unit vector, in reality the system is more complicated than this, and so it follows that there is no real interpretation within the double-slit / qubit analogy for the unitary evolution that an actual qubit can undergo. Furthermore, a crucial aspect of quantum information processing and computing is how qubits interact (including entangling interacts), but again the double-slit experiment cannot readily be used to explain this.

There exist in nature more genuine two-level quantum systems, that do indeed evolve unitarily and can interact with one another. For students who are interested, the *Stern Gerlach experiment* provides another example of a two-level quantum system, which allows better explanation of some of these further aspects of the behaviour and nature of qubits.