Programming in C and C++

Lecture 8: The Memory Hierarchy and Cache Optimization

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(Materials by Neel Krishnaswami)
void increment_every(int *array) 
    for (int i = 0; i < BIG_NUMBER; i += 1) {
        array[i] = 0;
    }

void increment_8th(int *array) {
    for (int i = 0; i < BIG_NUMBER; i += 8)
        array[i] = 0;
}

void increment_16th(int *array) {
    for (int i = 0; i < BIG_NUMBER; i += 16)
        array[i] = 0;
}

- Which runs faster?
- ... and by how much?
The Memory Hierarchy

Main Memory (RAM)

Unified Level 3 Cache

Unified Level 2 Cache

Level 1 Instruction Cache

Level 1 Data Cache

CPU and Registers
### Latencies in the Memory Hierarchy

<table>
<thead>
<tr>
<th>Access Type</th>
<th>Cycles</th>
<th>Time</th>
<th>Human Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 cache reference</td>
<td>≈4</td>
<td>1.3 ns</td>
<td>1s</td>
</tr>
<tr>
<td>L2 cache reference</td>
<td>≈10</td>
<td>4 ns</td>
<td>3s</td>
</tr>
<tr>
<td>L3 cache reference, unshared</td>
<td>≈40</td>
<td>13 ns</td>
<td>10s</td>
</tr>
<tr>
<td>L3 cache reference, shared</td>
<td>≈65</td>
<td>20 ns</td>
<td>16s</td>
</tr>
<tr>
<td>Main memory reference</td>
<td>≈300</td>
<td>100 ns</td>
<td>80s</td>
</tr>
</tbody>
</table>

- Random accesses to main memory are *slow*
- This can dominate performance!
When a CPU looks up an address...:

1. It looks up the address in the cache
2. If present, this is a *cache hit* (cheap!)
3. If absent, this is a *cache miss*
   3.1 The address is then looked up in main memory (expensive!)
   3.2 The address/value pair is then stored in the cache
   3.3 ... along with the next 64 bytes (typically) of memory
   3.4 This is a *cache line* or *cache block*
Caching is most favorable:

- Each piece of data the program works on is near (in RAM) the address of the last piece of data the program worked on.
- This is the *principle of locality*
- Performance engineering involves redesigning data structures to take advantage of locality.
Consider the following Java linked list implementation

class List<T> {
    public T head;
    public List<T> tail;

    public List(T head, List<T> tail) {
        this.head = head;
        this.tail = tail;
    }
}
Pointers Are Expensive in C, too

typedef struct List* list_t;
struct List {
    void *head;
    list_t tail;
};
list_t list_cons(void *head, list_t tail) {
    list_t result = malloc(sizeof(struct list));
    r->head = head;
    r->tail = tail;
    return r;
}

- C uses `void *` for genericity, but this introduces pointer indirections.
- This can get expensive!
Suppose we use a list at a `Data *` type:

```c
struct data {
    int i;
    double d;
    char c;
};
typedef struct data Data;

struct List {
    Data *head;
    struct List *tail;
};
```
Technique #1: Intrusive Lists

We can try changing the list representation to:

```c
typedef struct intrusive_list ilist_t;
struct intrusive_list {
    Data head;
    ilist_t tail;
};

ilist_t ilist_cons(Data head, ilist_t tail) {
    list_t result = malloc(sizeof(struct intrusive_list));
    r->head = head;
    r->tail = tail;
    return r;
}
```

- The indirection in the head is removed
- But we had to use a specialized representation
- Can no longer use generic linked list routines
Linked lists are expensive:

1. Following a tail pointer can lead to cache miss
2. Cons cells requiring storing a tail pointer...
3. This reduces the number of data elements that fit in a cache line
4. This decreases data density, and increases cache miss rate
5. Replace ilist_t with Data[]!
Technique #2: Lists of Structs to Arrays of Structs

We can try changing the list representation to:

```c
Data *iota_array(int n) {
    Data *a = malloc(n * sizeof(Data));
    for (int i = 0; i < n; i++) {
        a[i].i = i;
        a[i].d = 1.0;
        a[i].c = 'x';
    }
    return a;
}
```

- No longer store tail pointers
- Every element comes after previous element in memory
- Can no longer incrementally build lists
- Have to know size up-front
struct data {
    int i;
    double d;
    char c;
};
typedef struct data Data;

void traverse(int n, Data *a) {
    for (int i = 0; i < n; i++)
        a[i].c += 'y';
}

• Note that we are only modifying character field c.
• We have “hop over” the integer and double fields.
• So characters are at least 12, and probably 16 bytes apart.
• This means only 4 characters in each cache line...
• Optimally, 64 characters fit in each cache line...
Technique #3: Arrays of Structs to Struct of Arrays

```c
typedef struct datavec *DataVec;
struct datavec {
    int *is;
    double *ds;
    char *cs;
};
```

- Instead of storing an array of structures...
- We store a struct of arrays
- Now traversing just the cs is easy
Technique #3: Traversing Struct of Arrays

```c
void traverse_datavec(int n, DataVec d) {
    char *a = d->cs;
    for (int i = 0; i < n; i++) {
        a[i] += 'y';
    }
}
```

- To update the characters...
- Just iterate over the character...
- Higher cache efficiency!
Technique #4: Loop Blocking

```c
#define SIZE 8192
#define dim(i, j) (((i) * SIZE) + (j))

double *add_transpose(double *A,
                      double *B) {
    double *dest =
        malloc(sizeof(double) * SIZE * SIZE);
    for (int i = 0; i < SIZE; i++) {
        for (int j = 0; j < SIZE; j++) {
            dest[dim(i,j)] =
                A[dim(i,j)] + B[dim(j,i)];
        }
    }
    return dest;
}
```

- The `add_transpose` function takes two square matrices `A` and `B`, and returns a new matrix equal to \( A + B^T \).
- C stores arrays in row-major order.
How Matrices are Laid out in Memory

\[ A \triangleq \begin{cases} 
0 & 1 & 4 \\
9 & 16 & 25 \\
36 & 49 & 64 \\
81 & 100 & 121 
\end{cases} \]

<table>
<thead>
<tr>
<th>Address</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
<td>36</td>
<td>49</td>
<td>64</td>
<td>81</td>
<td>100</td>
<td>121</td>
</tr>
</tbody>
</table>

- \( A \) is a 3 \times 4 array.
- \( A(i, j) \) is at address \( 3 \times i + j \) (0 based!)
- E.g., \( A(2, 1) = 49 \), at address 7
- E.g., \( A(3, 1) = 100 \), at address 10
# define SIZE 8192
# define dim(i, j) (((i) * SIZE) + (j))

double *add_transpose(double *A, double *B) {
    double *dest = malloc(sizeof(double)
        * SIZE * SIZE);
    for (int i = 0; i < SIZE; i++) {
        for (int j = 0; j < SIZE; j++) {
            dest[dim(i,j)] = A[dim(i,j)] + B[dim(j,i)];
        }
    }
    return dest;
}
How to Block a Loop, Concept

- We can see that A has a favorable traversal, and B is “jumpy”
- Let’s change the traversal order!
How to Block a Loop, Concept

- Since each nested iteration is acting on the same $n \times n$ submatrix, a cache miss on one lookup will bring memory into cache for the other lookup.
- This reduces the total number of cache misses.
double *add_transpose_blocked(double *m1,
                                 double *m2,
                                 int bsize) {

double *dest =
    malloc(sizeof(double) * SIZE * SIZE);
for (int i = 0; i < SIZE; i += bsize) {
    for (int j = 0; j < SIZE; j += bsize) {
        for (int ii = i; ii < i+bsize; ii++) {
            for (int jj = j; jj < j+bsize; jj++) {
                dest[dim(ii,jj)] =
                    m1[dim(ii,jj)] + m2[dim(jj, ii)];
            }
        }
    }
}
return dest;
}

• Doubly-nested loop goes to quadruply-nested loop
• Increment i and j by bsize at a time
• Do a little iteration over the submatrix with ii and jj
Conclusion

- Memory is hierarchical, with each level slower than predecessors
- Caching makes *locality assumption*
- Making this assumption true requires careful design
- Substantial code alterations can be needed
- But can lead to major performance gains