

3: Statistical Properties of Language

Machine Learning and Real-world Data (MLRD)

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Lent 2020

Last session: You implemented a naive Bayes classifier

- Smoothed vs Unsmoothed
- The accuracy of the un-smoothed classifier was seriously affected by unseen words.
- We implemented add-one (Laplace) smoothing:

$$\hat{P}(w_i|c) = \frac{\text{count}(w_i, c) + 1}{\sum_{w \in V} (\text{count}(w, c) + 1)} = \frac{\text{count}(w_i, c) + 1}{(\sum_{w \in V} \text{count}(w, c)) + |V|}$$

- Smoothing helped!

Today: frequency distributions in language

Questions:

- Why did smoothing help? (or in other words:)
- What is it about the distribution of words in a language that affected the performance of the un-smoothed classifier?

Word frequency distributions obey a power law

- there are a small number of very high-frequency words
- there are a large number of low-frequency words
- word frequency distributions obey a power law ([Zipf's law](#))
- Zipf's law: the n th most frequent word has a frequency proportional to $1/n$

“a word's frequency in a corpus is inversely proportional to its rank”

The parameters of Zipf's law are language-dependent

Zipf's law:

$$f_w \approx \frac{k}{r_w^\alpha}$$

where

f_w : frequency of word w

r_w : frequency rank of word w

α, k : constants (which vary with the language)

e.g. α is around 1 for English but 1.3 for German

The parameters of Zipf's law are language-dependent

Actually...

$$f_w \approx \frac{k}{(r_w + \beta)^\alpha}$$

where

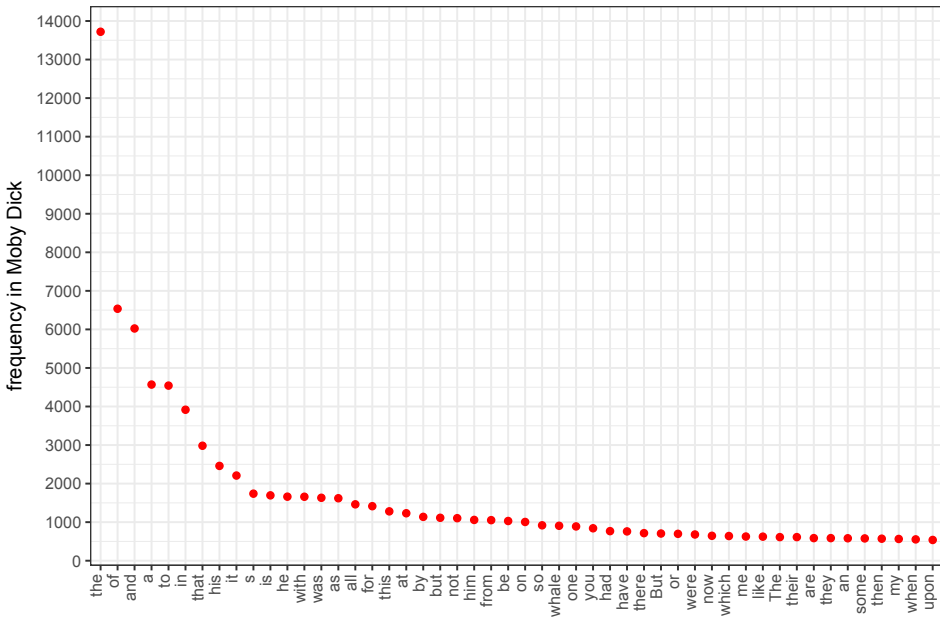
β : a shift in the rank

see summary paper by Piantadosi

<https://link.springer.com/article/10.3758/s13423-014-0585-6>

we won't worry about the rank-shift today

There are a small number of high-frequency words...



Similar sorts of high-frequency words across languages

Top 10 most frequent words in some large language samples:

Similar sorts of high-frequency words across languages

Top 10 most frequent words in some large language samples:

English

1	the	61,847
2	of	29,391
3	and	26,817
4	a	21,626
5	in	18,214
6	to	16,284
7	it	10,875
8	is	9,982
9	to	9,343
10	was	9,236

BNC,
100Mw

Similar sorts of high-frequency words across languages

Top 10 most frequent words in some large language samples:

English

German

1	the	61,847	1	der	7,377,879
2	of	29,391	2	die	7,036,092
3	and	26,817	3	und	4,813,169
4	a	21,626	4	in	3,768,565
5	in	18,214	5	den	2,717,150
6	to	16,284	6	von	2,250,642
7	it	10,875	7	zu	1,992,268
8	is	9,982	8	das	1,983,589
9	to	9,343	9	mit	1,878,243
10	was	9,236	10	sich	1,680,106

BNC,
100Mw

“Deutscher
Wortschatz”,
500Mw

Similar sorts of high-frequency words across languages

Top 10 most frequent words in some large language samples:

English

German

Spanish

1	the	61,847	1	der	7,377,879	1	que	32,894
2	of	29,391	2	die	7,036,092	2	de	32,116
3	and	26,817	3	und	4,813,169	3	no	29,897
4	a	21,626	4	in	3,768,565	4	a	22,313
5	in	18,214	5	den	2,717,150	5	la	21,127
6	to	16,284	6	von	2,250,642	6	el	18,112
7	it	10,875	7	zu	1,992,268	7	es	16,620
8	is	9,982	8	das	1,983,589	8	y	15,743
9	to	9,343	9	mit	1,878,243	9	en	15,303
10	was	9,236	10	sich	1,680,106	10	lo	14,010

BNC,
100Mw

“Deutscher
Wortschatz”,
500Mw

subtitles,
27.4Mw

Similar sorts of high-frequency words across languages

Top 10 most frequent words in some large language samples:

English		German		Spanish		Italian	
1	the 61,847	1	der 7,377,879	1	que 32,894	1	non 25,757
2	of 29,391	2	die 7,036,092	2	de 32,116	2	di 22,868
3	and 26,817	3	und 4,813,169	3	no 29,897	3	che 22,738
4	a 21,626	4	in 3,768,565	4	a 22,313	4	è 18,624
5	in 18,214	5	den 2,717,150	5	la 21,127	5	e 17,600
6	to 16,284	6	von 2,250,642	6	el 18,112	6	la 16,404
7	it 10,875	7	zu 1,992,268	7	es 16,620	7	il 14,765
8	is 9,982	8	das 1,983,589	8	y 15,743	8	un 14,460
9	to 9,343	9	mit 1,878,243	9	en 15,303	9	a 13,915
10	was 9,236	10	sich 1,680,106	10	lo 14,010	10	per 10,501
BNC, 100Mw		"Deutscher Wortschatz", 500Mw		subtitles, 27.4Mw		subtitles, 5.6Mw	

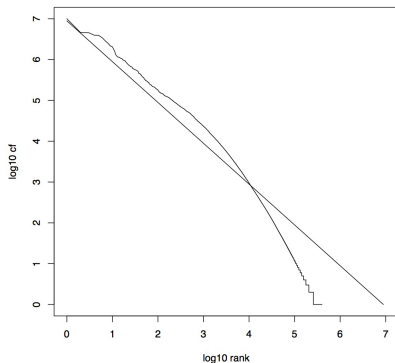
Similar sorts of high-frequency words across languages

Top 10 most frequent words in some large language samples:

English		German		Spanish		Italian		Dutch						
1	the	61,847	1	der	7,377,879	1	que	32,894	1	non	25,757	1	de	4,770
2	of	29,391	2	die	7,036,092	2	de	32,116	2	di	22,868	2	en	2,709
3	and	26,817	3	und	4,813,169	3	no	29,897	3	che	22,738	3	het/'t	2,469
4	a	21,626	4	in	3,768,565	4	a	22,313	4	è	18,624	4	van	2,259
5	in	18,214	5	den	2,717,150	5	la	21,127	5	e	17,600	5	ik	1,999
6	to	16,284	6	von	2,250,642	6	el	18,112	6	la	16,404	6	te	1,935
7	it	10,875	7	zu	1,992,268	7	es	16,620	7	il	14,765	7	dat	1,875
8	is	9,982	8	das	1,983,589	8	y	15,743	8	un	14,460	8	die	1,807
9	to	9,343	9	mit	1,878,243	9	en	15,303	9	a	13,915	9	in	1,639
10	was	9,236	10	sich	1,680,106	10	lo	14,010	10	per	10,501	10	een	1,637
BNC, 100Mw		"Deutscher Wortschatz", 500Mw		subtitles, 27.4Mw		subtitles, 5.6Mw		subtitles, 800Kw						

It is helpful to plot Zipf curves in log-space

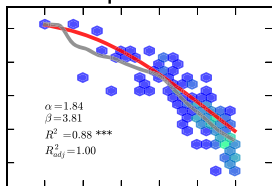
Reuters dataset: taken from <https://nlp.stanford.edu/IR-book/pdf/irbookonlinereading.pdf> – chapter 5



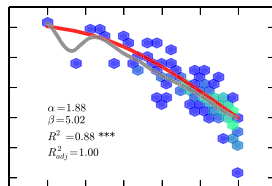
By fitting a simple line to the data in log-space we can estimate the language specific parameters α and k (we will do this today!)

In log-space we can more easily estimate the language specific parameters

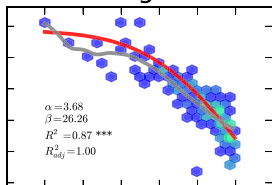
Spanish



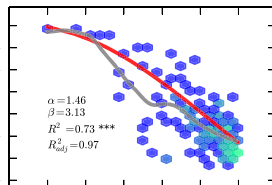
Russian



Portuguese



Chinese

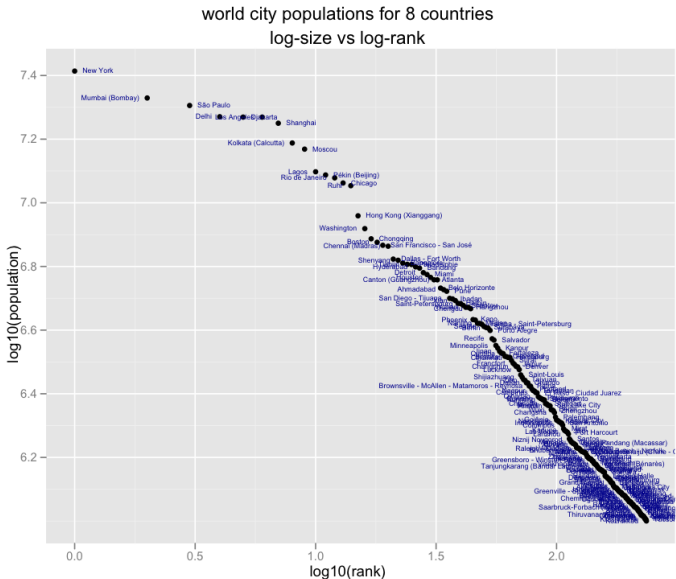


From Piantadosi <https://link.springer.com/article/10.3758/s13423-014-0585-6>

Zipfian (or near-Zipfian) distributions occur in many collections

- Sizes of settlements
- Frequency of access to web pages
- Size of earthquakes
- Word senses per word
- Notes in musical performances
- machine instructions
- ...

Zipfian (or near-Zipfian) distributions occur in many collections



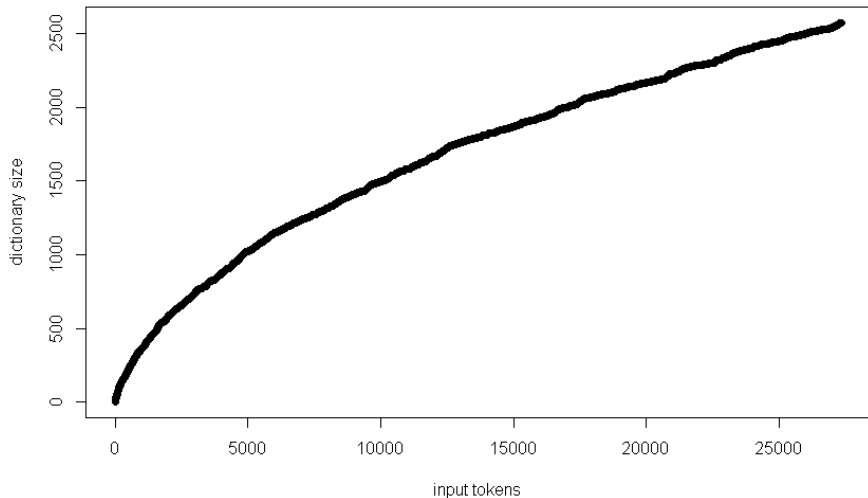
There is a relationship between vocabulary size and text length

So far we have been thinking about frequencies of particular words:

- we call any unique word a **type**: *the* is a word type
- we call an instance of a type a **token**: there are 13721 *the* tokens in Moby Dick
- the number of types in a text is the vocabulary (or dictionary size) for the text

Today we will explore the relationship between vocabulary size and the length of a text.

As we progress through a text we see fewer new types



Heaps' law describes the vocabulary / text-length relationship

Heaps' Law:

Describes the relationship between the size of a vocabulary and the size of text that gave rise to it:

$$u_n = kn^\beta$$

where

u_n : number of types (unique items)—i.e. vocabulary size

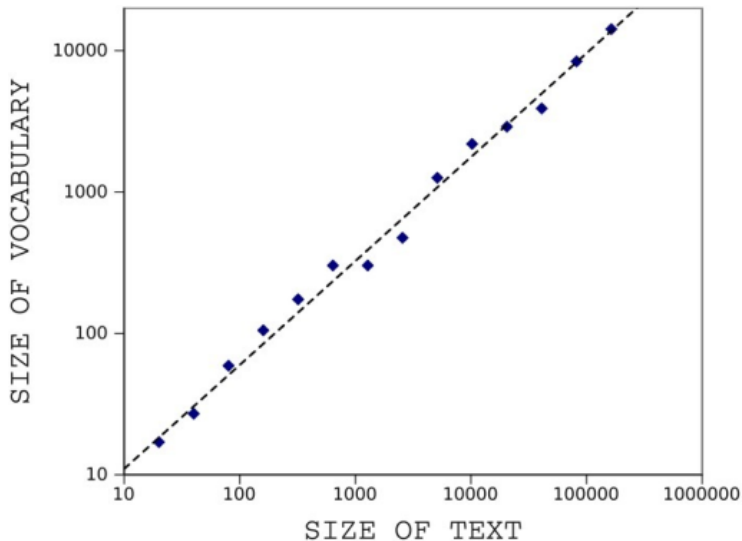
n : total number of tokens—i.e. text size

β, k : constants (language-dependent)

β is around $\frac{1}{2}$

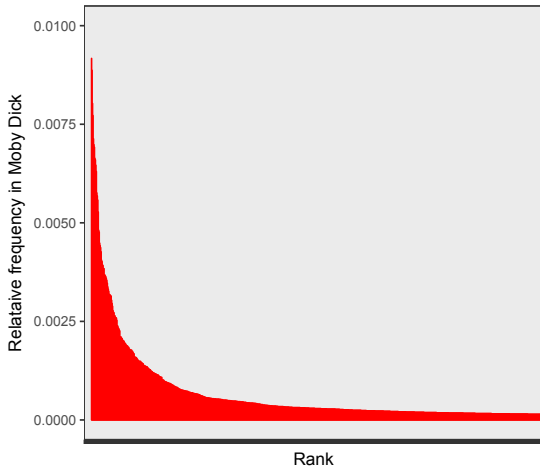
$30 \leq k \leq 100$

It is helpful to plot Heaps' law in log-space

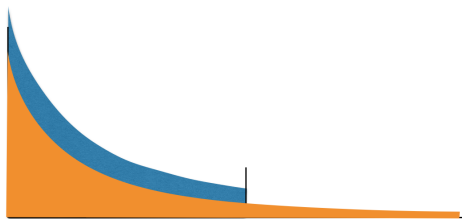


Zipf's law and Heaps' law affected our classifier

- Zipf curve has a lot of probability mass in the **long tail**.
- By Heaps' law, we need increasing amounts of text to see new word types in the tail



Zipf's law and Heaps' law affected our classifier



- With MLE, only seen types receive a probability estimate:
e.g. we used:

$$P_{MLE}^{\hat{}}(w_i|c) = \frac{\text{count}(w_i, c)}{\sum_{w \in V_{training}} \text{count}(w, c)}$$

- True probability: orange; MLE: blue
- Total probabilities must sum to 1; in MLE all that probability mass is given to seen types
- MLE overestimates the probability of seen types (as opposed to unseen)

Smoothing redistributes the probability mass

- Add-one smoothing redistributes the probability mass.

e.g. we used:

$$\hat{P}(w_i|c) = \frac{\text{count}(w_i, c) + 1}{\sum_{w \in V} (\text{count}(w, c) + 1)} = \frac{\text{count}(w_i, c) + 1}{(\sum_{w \in V} \text{count}(w, c)) + |V|}$$

- It takes some portion away from the MLE overestimate.
- It redistributes this portion to the unseen types.
- Better estimate; still not perfect.

Today we will investigate Zipf's and Heaps' law in movie reviews

Follow task instructions on moodle to:

- Plot a frequency vs rank graph for larger set of movie reviews (you are given chart plotting code)
- Plot a log frequency vs log rank graph
- Use least-squares algorithm to fit a line to the log-log plot (you are given best fit code)
- Estimate the parameters of the Zipf equation
- Plot type vs token graph for the movie reviews

Ticking for Task 3

There is no automatic ticker for Task 3

- Write everything in your notebook
- Save all your graphs (as screenshots or otherwise)
- Where do your 10 words end up in the plot?