Module L314 – Digital Signal Processing – take-home test

Answer all questions.

The completed answers should be submitted either via

https://www.ule.cam.ac.uk/course/view.php?id=174711

or handed in to the Graduate Education Office (room FS03), along with the completed cover sheet provided there, no later than 16:00 on Friday 15 November 2019.

Students will be required to sign an undertaking that work submitted will be entirely their own; no collaboration is permitted.
1 What type of discrete system (linear/non-linear, time-invariant/ non-time-invariant, causal/non-causal, memory-less) is:

(a) \( y_n = |x_{n-1} - x_{n+1}| \)
(b) \( y_n = \frac{1}{2}(x_{2n} + x_{2n+1}) \)
(c) \( y_n = \frac{3x_{n-1} + x_{n-2}}{x_{n-3}} \)
(d) \( y_n = \prod_{i=0}^{8} x_{n-i} \)

(e) \( y_n = \sum_{i=-\infty}^{\infty} x_i \cdot \delta_{i-n+2} \)
(f) \( y_n = x_n \cdot e^{n/14} \)
(g) \( y_n = x_n \cdot u_n \)

2 A finite-support sequence is non-zero only at a finite number of positions. If \( m \) and \( n \) are the first and last non-zero positions, respectively, then we call \( n - m + 1 \) the length of that sequence. What maximum length can the result of convolving two sequences of length \( k \) and \( l \) have?

3 (a) Find a pair of sequences \( \{a_n\} \) and \( \{b_n\} \), where each one contains at least three different values and where the convolution \( \{a_n\} \ast \{b_n\} \) results in an all-zero sequence.

(b) Does every LTI system \( T \) have an inverse LTI system \( T^{-1} \) such that \( \{x_n\} = T^{-1}T\{x_n\} \) for all sequences \( \{x_n\} \)? Why?
4  (a) Determine the Fourier transform of the triangular pulse

\[ \Lambda(t) = \begin{cases} 
1 - |t|, & \text{for } |t| < 1 \\
0, & \text{otherwise}
\end{cases} \]

using the convolution theorem and the Fourier transform of a rectangular function.

(b) You sample a 2 kHz sine wave at 12 kHz and later convert the resulting discrete series back into a continuous signal using linear interpolation.

(i) At what other frequencies besides 2 kHz is there signal energy in the resulting continuous waveform?

(ii) Consider among those other components the one with the lowest frequency. By what factor is its voltage lower compared to the 2 kHz component?

(iii) Your colleague records with a PC soundcard at 44.1 kHz sampling frequency 1024 samples of the continuous waveform, loads these into MATLAB as vector \( x \) and then attempts to plot an amplitude spectrum with the command

\[
\text{plot(real(fft(x)))};
\]

Name two problems that need to be fixed in this command before the resulting plot is likely to agree with the result of (b)(ii).

5 What is the \( z \)-transform \( H(z) \) of the impulse response of the following filter?

![Filter Diagram]
Consider the system $h : \{x_n\} \rightarrow \{y_n\}$ with $y_n + y_{n-1} = x_n - x_{n-4}$.

(a) What is the impulse response of $h$ (i.e., $h\{\delta_n\}$)?

(b) What is the step response of $h$ (i.e., $h\{u_n\}$)?

(c) Apply the $z$-transform to (the impulse response of) $h$ to express it as a rational function $H(z)$.

(d) Can you eliminate a common factor from numerator and denominator? What does this mean?

(e) For what values $z \in \mathbb{C}$ is $H(z) = 0$?

(f) How many poles does $H$ have in the complex plane?

(g) Write $H$ as a fraction using the position of its poles and zeros and draw their location in relation to the complex unit circle.

(h) If $h$ is applied to a sound file with a sampling frequency of 8000 Hz, sine waves of what frequency will be eliminated and sine waves of what frequency will be quadrupled in their amplitude?

END OF PAPER