Foundations of Computer Science:

Datatypes and Trees

Lecture 6

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23rd October 2019

SSCoF NOMINATIONS

Take the chance to become the voice of feedback to the department by standing to be on the SSCoF committee. We are looking for one representative from each of the following years to represent their year group :

Part IA NST, Part IA CST, Part IB, Part II, Part III/MPhil, PhD

Applications must be submitted by **Friday 25th October**

Meetings to be held twice termly on a Wednesday lunchtime, with lunch provided.

For more info or to collect a nomination form, please ask at the Student Administration hatch.

Custom Types

Exceptions

Recursive Types

Custom Types

Custom Types

- So far, our types have been basic: `int`, `float` or `bool` types that are built into OCaml.
- In this lecture we introduce one of the coolest features of ML-style languages in the form of **custom datatypes**!
- We continue to improve the **abstraction** of our data away from the details of its **representation**.

Let’s describe a vehicle

```
# let number_of_wheels = function
        “bike” -> 2
      | “motorbike” -> 2
      | “car” -> 4
      | “lorry” -> 18
```

Let’s describe a vehicle

```
# let number_of_wheels = function
      “bike” –> 2
    | “motorbike” –> 2
    | “car” –> 4
    | “lorry” –> 18
```

```
# number_of_wheels “bike”
– : int = 2

# number_of_wheels “motorbke”
???
```

Let’s describe a vehicle

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# let number_of_wheels = function
      “bike” -> 2
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```
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- : int = 2

# number_of_wheels “motorbke”
???
```
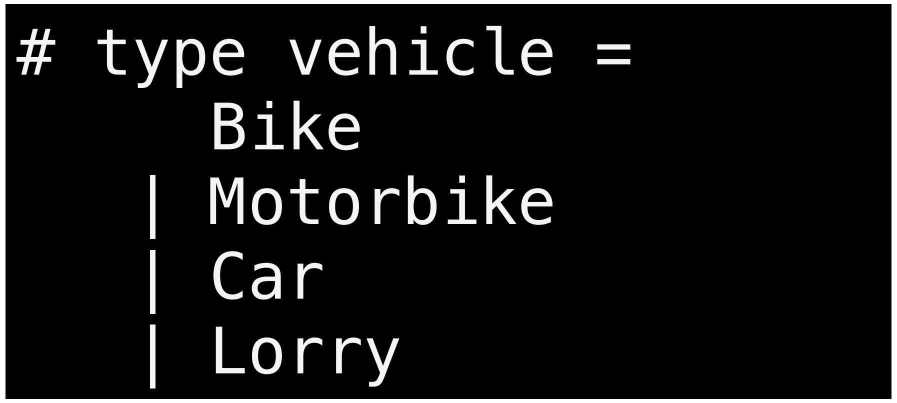

How can we make illegal states unrepresentable?

An Enumeration Type

```
# type vehicle =
     Bike
   | Motorbike
   | Car
   | Lorry
```

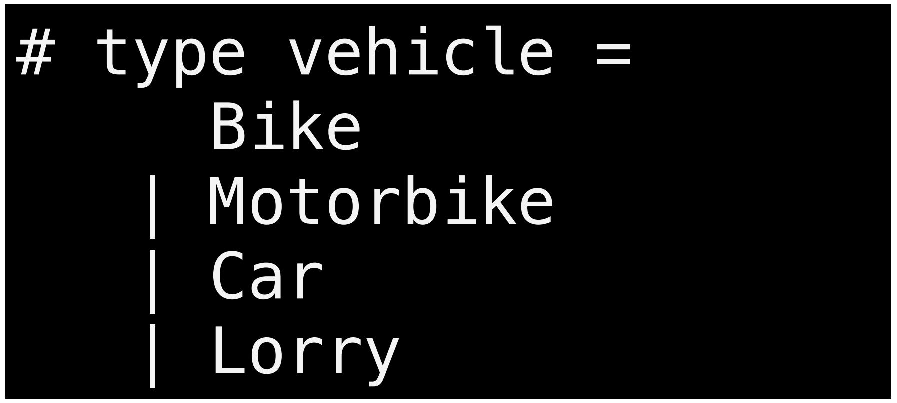
An Enumeration Type



```
# type vehicle =
     Bike
   | Motorbike
   | Car
   | Lorry
```

- We have declared a new type `vehicle`
- Instead of representing any string, it can *only* contain the four constants defined.
- These four constants become the *constructors* of the `vehicle` type

An Enumeration Type



```
# type vehicle =
     Bike
   | Motorbike
   | Car
   | Lorry
```

- The *representation* in memory is more efficient than using strings.
- Adding new types of vehicles is straightforward by extending the definitions.
- Different custom types cannot be intermixed, unlike strings or integers.

Declaring functions on vehicles

```
# let wheels = function
  | Bike -> 2
  | Motorbike -> 2
  | Car -> 4
  | Lorry -> 18
val wheels : vehicle -> int = <fun>
```

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# let wheels = function
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val wheels : string -> int = <fun>
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Declaring functions on vehicles

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val wheels : vehicle -> int = <fun>
```

```
# let wheels = function
  | Bike -> 2
  | Motorbike -> 2
  | Car -> 4
Warning 8: this pattern-matching is not exhaustive.
Here is an example of a case that is not matched:
Orange
val wheels : vehicle -> int = <fun>
```

- Adding new types of vehicles is straightforward by extending the definitions and fixing warnings.

Declaring functions on vehicles

```
# type vehicle = Bike
               | Motorbike of int
               | Car       of bool
               | Lorry     of int
```

- OCaml generalises the notion of enumeration types to allow *data* to be stored alongside each variant.

```
# Bike
# Motorbike 250
# Car true
# Lorry 500
```

Declaring functions on vehicles

```
# type vehicle = Bike
               | Motorbike of int
               | Car       of bool
               | Lorry     of int
```

- OCaml generalises the notion of enumeration types to allow *data* to be stored alongside each variant.

```
# type vehicle = Bike
               | Motorbike of int  (* engine size in CCs *)
               | Car       of bool (* true if a Reliant Robin *)
               | Lorry     of int  (* number of wheels *)
```

Declaring functions on vehicles

```
# type vehicle = Bike
               | Motorbike of int
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```

- OCaml generalises the notion of enumeration types to allow *data* to be stored alongside each variant.
- Even though they have different data, they are all of type `vehicle` when wrapped by the constructor.

```
# [ Bike; Car true; Motorbike 450 ]
- : vehicle list
```

A finer wheel computation

```
# let wheels = function
  | Bike -> 2
  | Motorbike _ -> 2
  | Car robin -> if robin then 3 else 4
  | Lorry w -> w
```

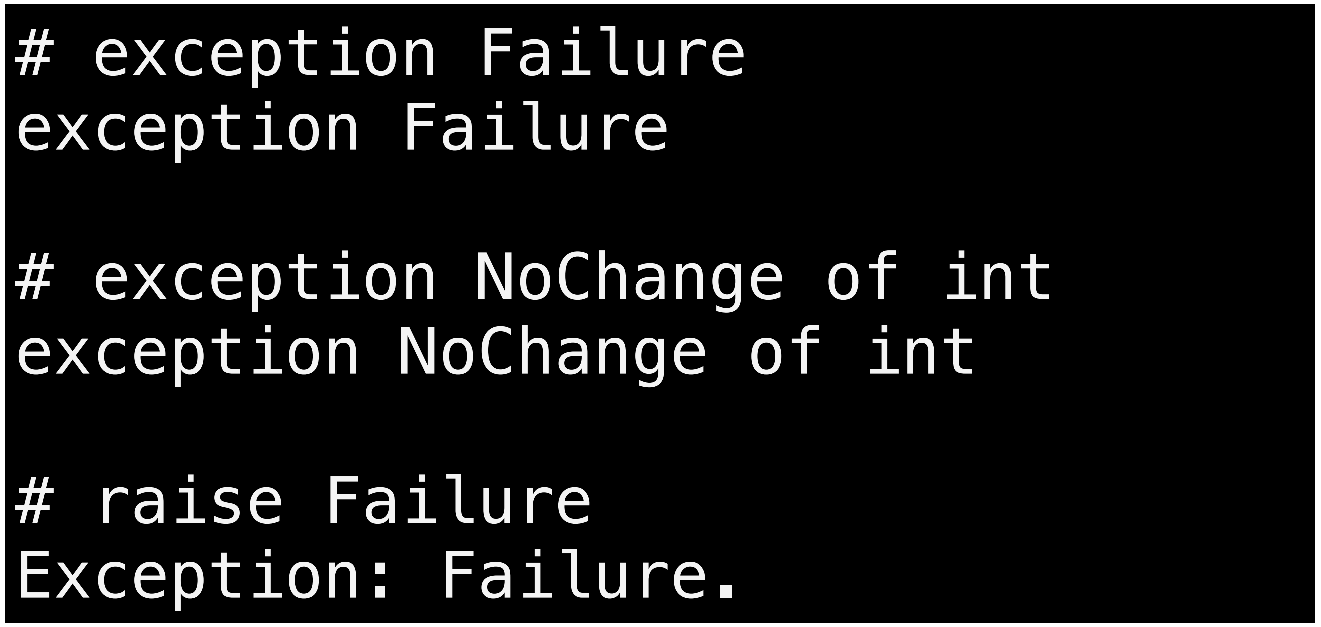
- A Bike has two wheels.
- A Motorbike has two wheels.
- A Reliant Robin has three wheels; all other cars have four.
- A Lorry has the number of wheels stored with its constructor.

Exceptions

Exceptions

- During a computation, what if **something goes wrong**?
  - Division by zero
  - Pattern matching failure
- **Exception handling** allows us to recover from these:
  - **Raising** an exception abandons the current expression
  - **Handling** the exception attempts an alternative
- Raising and handling can be separated in the source code

Exceptions



```
# exception Failure
exception Failure

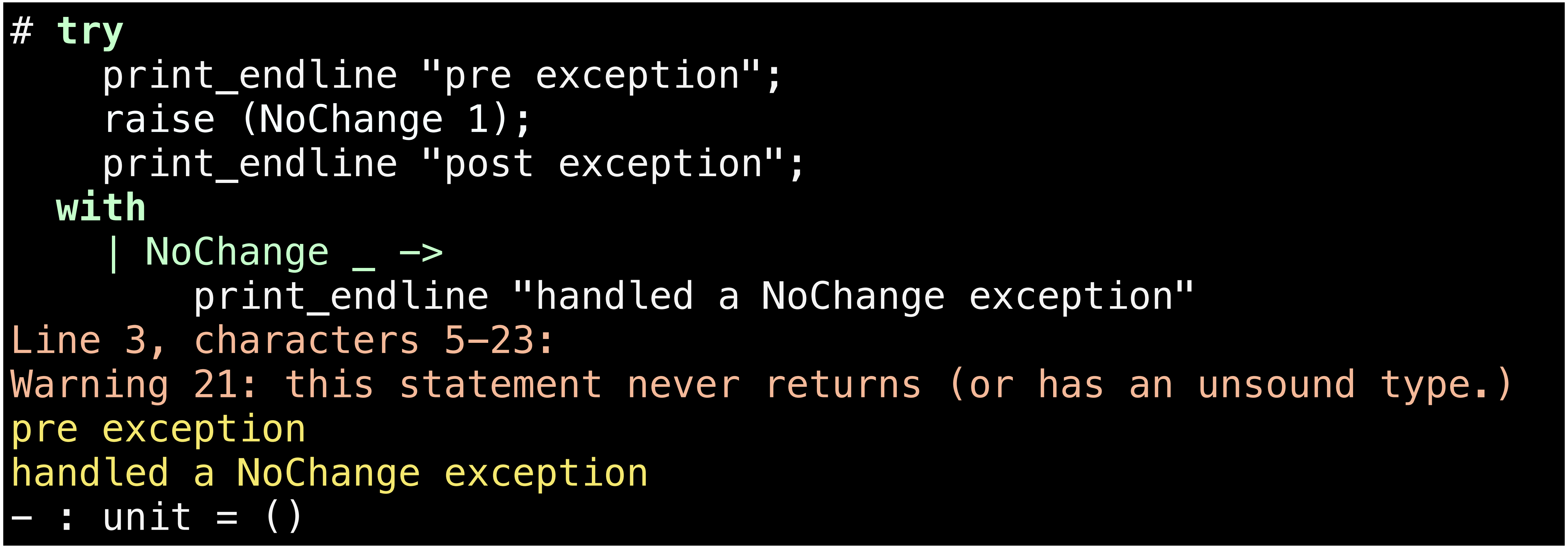
# exception NoChange of int
exception NoChange of int

# raise Failure
Exception: Failure.
```

- Each `exception` declaration introduces a distinct type of exception that can be handled separately.

- Exceptions are like enumerations and can have data attached to them.

Exceptions

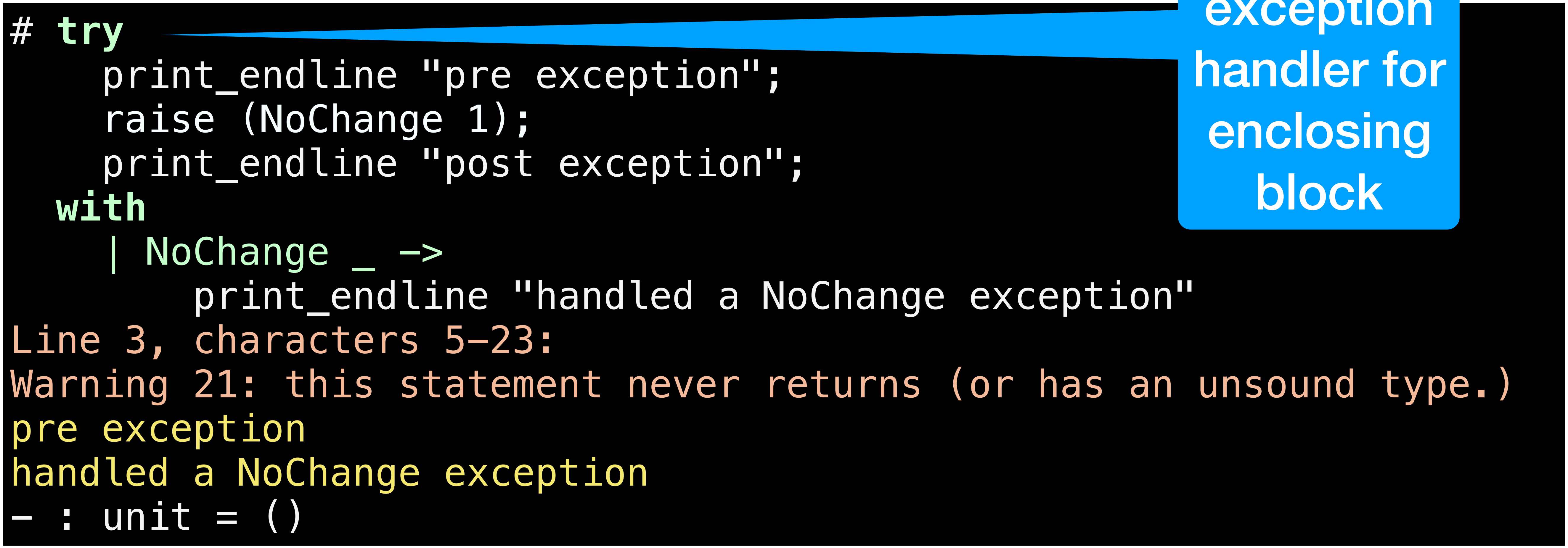


```
# try
    print_endline "pre exception";
    raise (NoChange 1);
    print_endline "post exception";
  with
    | NoChange _ ->
        print_endline "handled a NoChange exception"
Line 3, characters 5-23:
Warning 21: this statement never returns (or has an unsound type.)
pre exception
handled a NoChange exception
- : unit = ()
```

- `raise` dynamically jumps to the nearest `try/with` handler that matches that exception

- Unlike some languages, OCaml does not mark a function to indicate that an exception might be raised.

Exceptions



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Install exception handler for enclosing block

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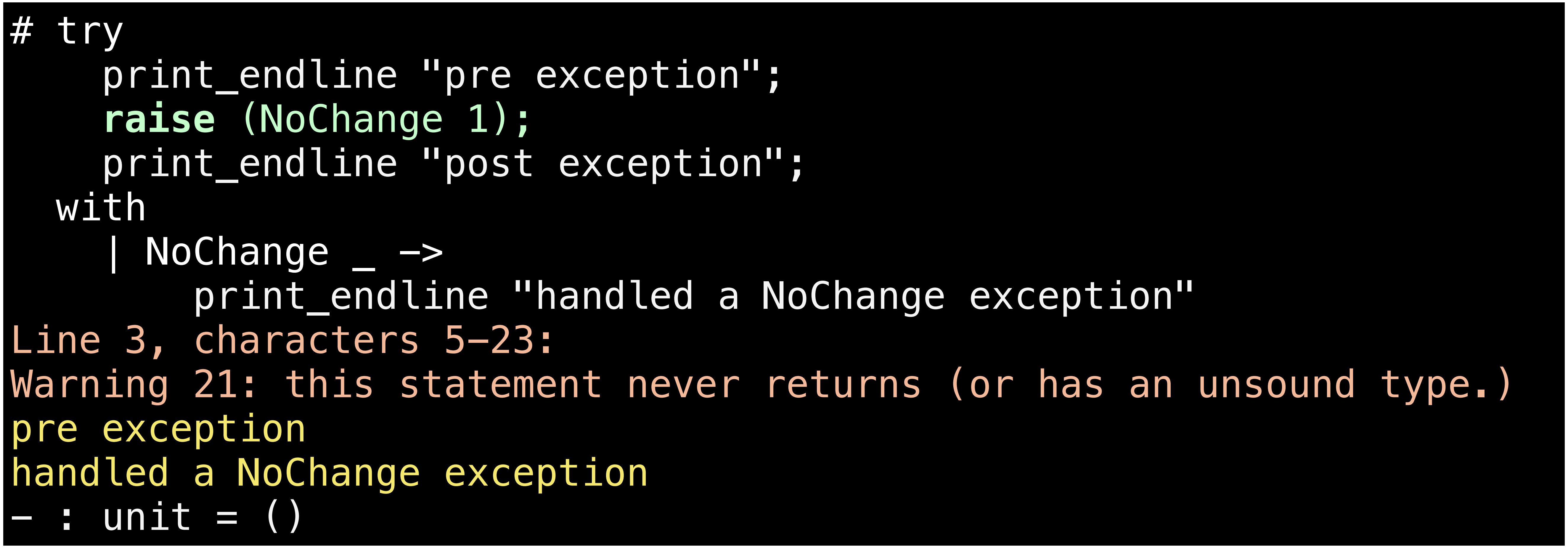
Exceptions



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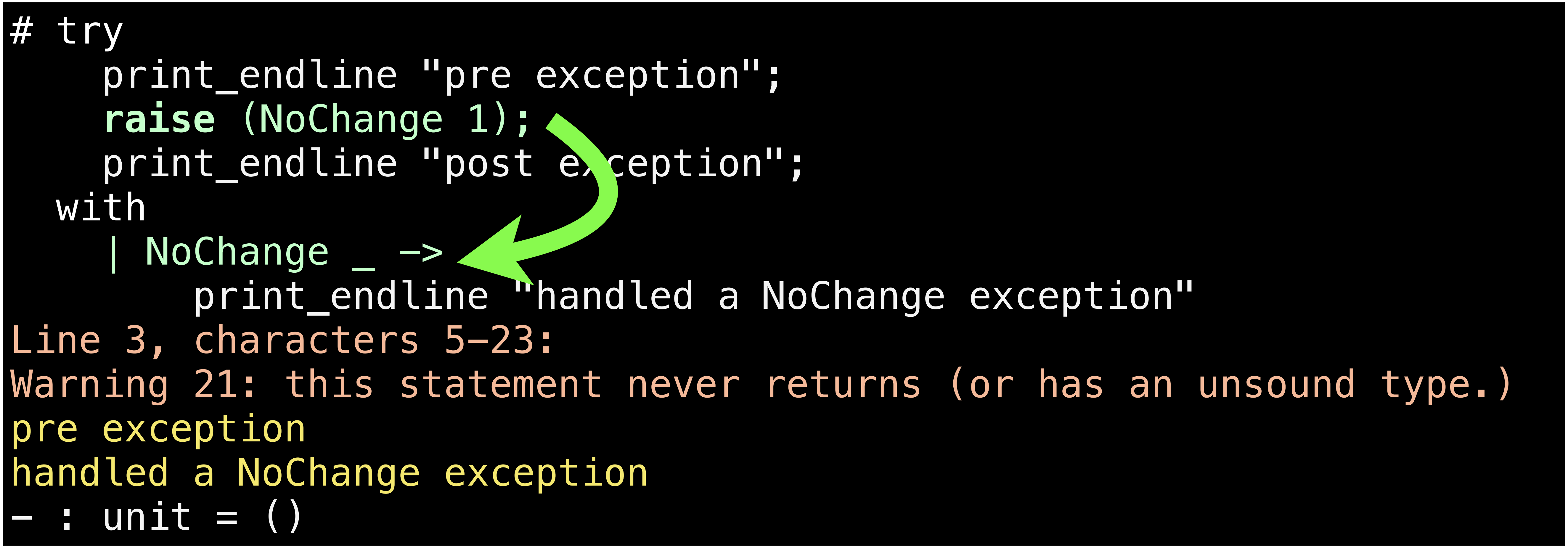

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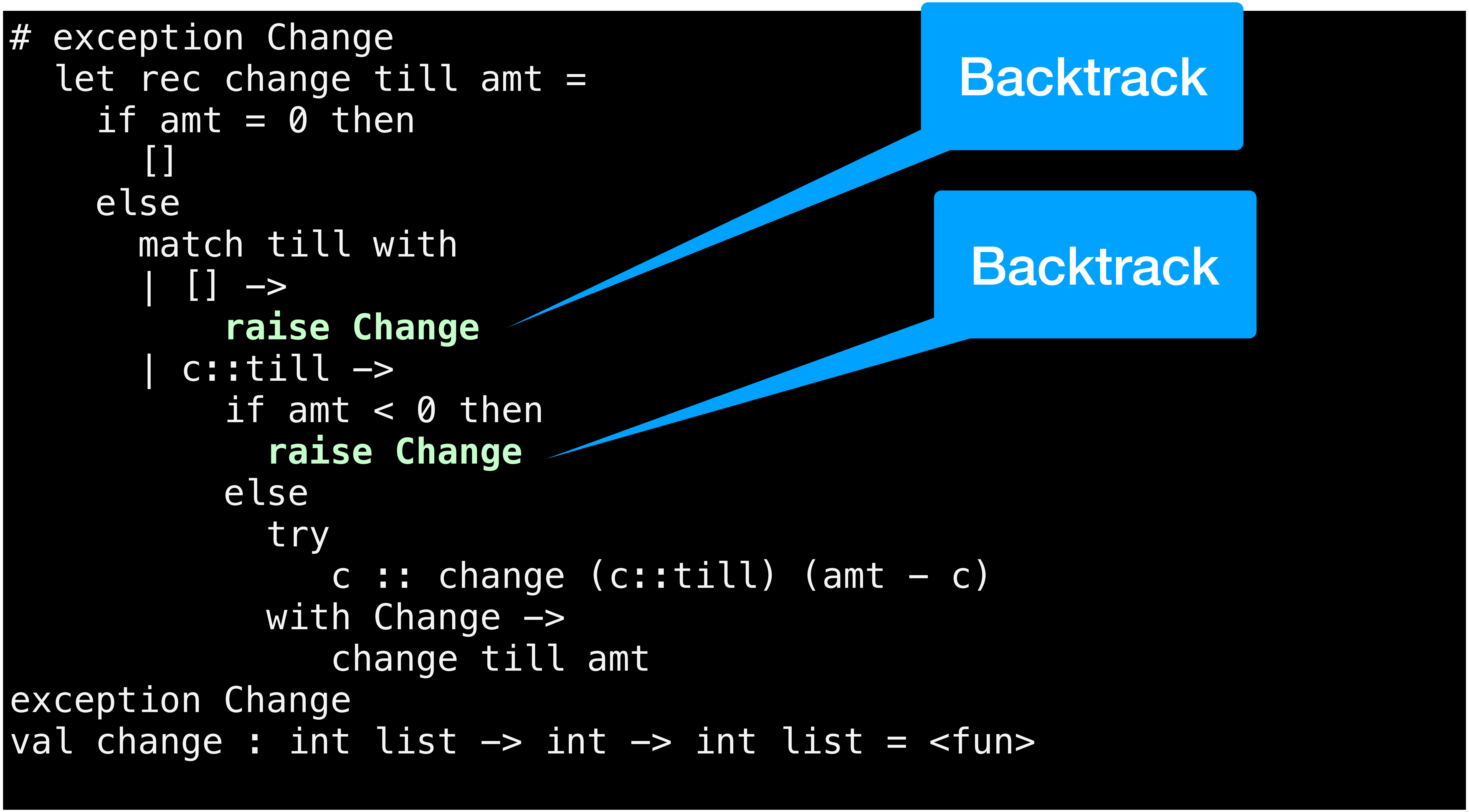
Change with *backtracking*

```
# exception Change
  let rec change till amt =
    if amt = 0 then
      []
    else
      match till with
      | [] ->
          raise Change
      | c::till ->
          if amt < 0 then
            raise Change
          else
            try
               c :: change (c::till) (amt - c)
            with Change ->
               change till amt
exception Change
val change : int list -> int -> int list = <fun>
```

Change with *backtracking*



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Backtrack

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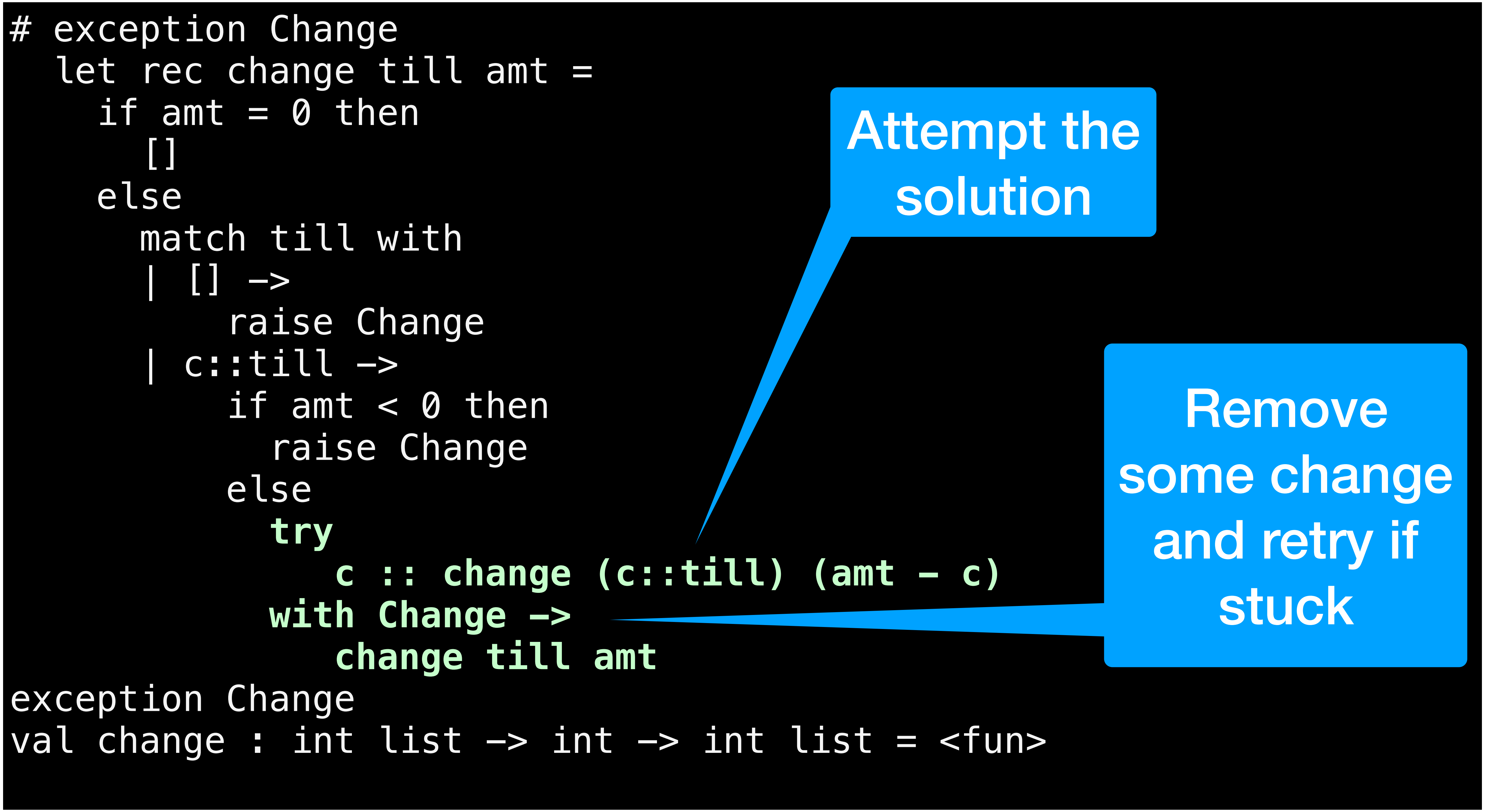
Change with *backtracking*



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            try
               c :: change (c::till) (amt - c)
            with Change ->
               change till amt
exception Change
val change : int list -> int -> int list = <fun>
```

Attempt the solution

Remove some change and retry if stuck

Change wit

```
# exception Change
  let rec change till amt =
    if amt = 0 then
      []
    else
      match till with
      | [] ->
          raise Change
      | c::till ->
          if amt < 0 then
            raise Change
          else
            try
              c :: change (c::till) (amt - c)
            with Change ->
              change till amt
exception Change
val change : int list -> int -> int list = <fun>
```

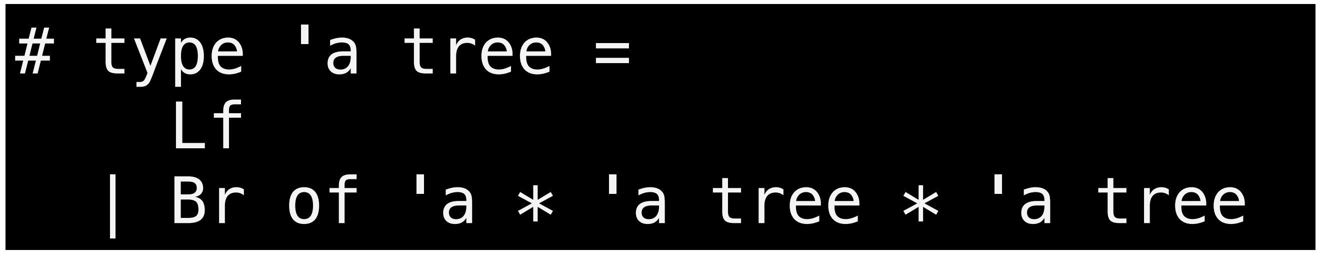
```
change [5; 2] 6
  ⇒ 5::change [5; 2] 1 with C -> change [2] 6
  ⇒ 5::(5::change [5; 2] -4) with C -> change [2] 1
                             with C -> change [2] 6
  ⇒ 5::(change [2] 1) with C -> change [2] 6
  ⇒ 5::(2::change [2] -1) with Chang -> change [] 1
                          with C -> change [2] 6
  ⇒ 5::(change [] 1) with C -> change [2] 6
  ⇒ change [2] 6
  ⇒ 2::(change [2] 4) with C -> change [] 6
  ⇒ 2::(2::change [2] 2) with C -> change [] 4
                         with C -> change [] 6
  ⇒ 2::(2::(2::change [2] 0)) with C -> change [] 2
                              with C -> change [] 4
                              with C -> change [] 6
  ⇒ 2::(2::[2]) with C -> change [] 4
                with C -> change [] 6
  ⇒ 2::[2; 2] with C -> change [] 6
  ⇒ [2; 2; 2]
```

Recursive Types

Binary Trees

```
# type 'a tree =
    Lf
  | Br of 'a * 'a tree * 'a tree
```

Binary Trees



```
# type 'a tree =
    Lf
  | Br of 'a * 'a tree * 'a tree
```

- A data structure with multiple branching is called a **tree**.
- Trees are nearly as fundamental a structure as lists.
- Each node is either a **leaf** (empty) or a **branch** with a label and two subtrees.

Binary Trees

```
# type 'a tree =
    Lf
  | Br of 'a * 'a tree * 'a tree
```

“Polymorphic” type

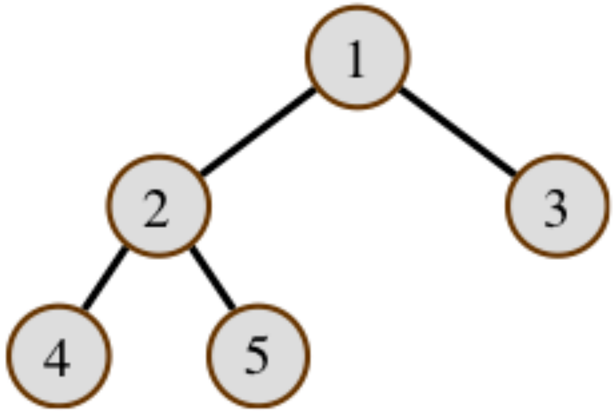
Binary Trees

“Polymorphic” type

```
# type 'a tree =
    Lf
  | Br of 'a * 'a tree * 'a tree
```

`int tree`

```
# Br(1, Br(2, Br(4, Lf, Lf),
            Br(5, Lf, Lf)),
        Br(3, Lf, Lf))
```



Binary Trees & Lists

```
# type 'a tree =
    Lf
  | Br of 'a * 'a tree * 'a tree
```

```
# type 'a mylist =
| Nil
| Cons of 'a * 'a mylist

# Cons (1, Cons (2, Cons (3, Nil)))
– : int mylist
```

Polymorphism & Recursion

Polymorphic and Recursive

```
# type 'a tree =
  Lf
 | Br of 'a * 'a tree * 'a tree
```

Recursive

```
type shape =
| Null
| Join of shape * shape
```

Polymorphic

```
type 'a option =
| None
| Some of 'a
```

Simple Operations on Trees

```
(* number of branch nodes *)
# let rec count = function
  | Lf -> 0
  | Br (v, t1, t2) -> 1 + count t1 + count t2
val count : 'a tree -> int = <fun>

(* length of longest path *)
# let rec depth = function
  | Lf -> 0
  | Br (v, t1, t2) -> 1 + max (depth t1) (depth t2)
val depth : 'a tree -> int = <fun>
```

- Use pattern matching to build expressions over trees
- The invariant $\text{count}(t) \leq 2^{\text{depth}(t)} - 1$ holds above