Artificial Intelligence

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Introduction: aims, history, rational action, and agents

Reading: AIMA chapters 1, 2, 26 and 27.

Introduction: what are our aims?

Artificial Intelligence (AI) is currently at the top of its *periodic hype-cycle*.

Much of this has been driven by *philosophers* and *people with something to sell*.

Introduction: what are our aims?

What is the purpose of Artificial Intelligence (AI)? If you're a *philosopher* or a *psychologist* then perhaps it's:

- To *understand intelligence*.
- To understand *ourselves*.

Philosophers have worked on this for at least 2000 years. They've also wondered about:

- *Can* we do AI? *Should* we do AI? What are the *ethical implications*?
- Is AI *impossible*? (Note: I didn't write *possible* here, for a good reason...)

Despite 2000 years of work by philosophers, there's essentially *nothing* in the way of results.

Introduction: what are our aims?

Luckily, we were sensible enough not to pursue degrees in philosophy—we're scientists/engineers, so while we might have *some* interest in such pursuits, our perspective is different:

- Brains are small (true) and apparently slow (not quite so clear-cut), but incredibly good at some tasks—we want to understand a specific form of *computation*.
- It would be nice to be able to *construct* intelligent systems.
- It is also nice to *make and sell cool stuff*.

Historically speaking, this view *seems to be the more successful...*

AI has been entering our lives for decades, almost without us being aware of it.

But be careful: brains are *much more complex than you think*.

Introduction: now is a fantastic time to investigate AI

In many ways this is a young field, having only really got under way in 1956 with the *Dartmouth Conference*.

`www-formal.stanford.edu/jmc/history/dartmouth/dartmouth.html`

- This means we can actually *do* things. It's as if we were physicists before anyone thought about atoms, or gravity, or....
- Also, we know what we're trying to do is *possible*. (Unless we think humans don't exist. *NOW STEP AWAY FROM THE PHILOSOPHY* before *SOMEONE GETS HURT!!!!*)

Perhaps I'm being too hard on them; there was some good groundwork: *Socrates* wanted an algorithm for *"piety"*, leading to *Syllogisms*. Ramon Lull's *concept wheels* and other attempts at mechanical calculators. Rene Descartes' *Dualism* and the idea of mind as a *physical system*. Wilhelm Leibnitz's opposing position of *Materialism*. (The intermediate position: mind is *physical* but *unknowable*.) The origin of *knowledge*: Francis Bacon's *Empiricism*, John Locke: *"Nothing is in the understanding, which was not first in the senses"*. David Hume: we obtain rules by repeated exposure: *Induction*. Further developed by Bertrand Russell and in the *Confirmation Theory* of Carnap and Hempel.

More recently: the connection between *knowledge* and *action*? How are actions *justified*? If to achieve the end you need to achieve something intermediate, consider how to achieve that, and so on. This approach was implemented in Newell and Simon's 1957 *General Problem Solver (GPS)*.

What has been achieved?

Artificial Intelligence (AI) is currently at the top of its *periodic hype-cycle*.

As a result, it's important to maintain some sense of perspective.

Notable successes:

- Perception: vision, speech processing, inference of emotion from video, scene labelling, touch sensing, artificial noses...
- Logical reasoning: prolog, expert systems, CYC, Bayesian reasoning, Watson...
- Playing games: chess, backgammon, go, robot football...
- Diagnosis of illness in various contexts...
- Theorem proving: Robbin's conjecture, formalization of the Kepler conjecture...
- Literature and music: automated writing and composition...
- And many more... (most of which don't include the word *'DEEP'*!)

What has been achieved?

Artificial Intelligence (AI) is currently at the top of its *periodic hype-cycle*.

As a result, it's important to maintain some sense of perspective.

There are equally many areas in which we currently *can't do things very well*:

"Sleep that knits up the ragged sleeve of care"

is a line from Shakespeare's Macbeth.

On the other hand...

When AI has a success, the ideas in question tend to *stop being called AI*.

Do you consider the fact that *your phone can do speech recognition* to be a form of AI?

The nature of the pursuit

What is AI? This is not necessarily a straightforward question.

It depends on who you ask...

We can find many definitions and a rough categorisation can be made depending on whether we are interested in:

- The way in which a system *acts* or the way in which it *thinks*.
- Whether we want it to do this in a *human* way or a *rational* way.

Here, the word *rational* has a special meaning: it means *doing the correct thing in given circumstances*.

What is AI, version one: acting like a human

Alan Turing proposed what is now known as the *Turing Test*.

- A human judge is allowed to interact with an AI program via a terminal.
- This is the *only* method of interaction.
- If the judge can't decide whether the interaction is produced by a machine or another human then the program passes the test.

In the *unrestricted* Turing test the AI program may also have a camera attached, so that objects can be shown to it, and so on.

The Turing test is informative, and (very!) hard to pass. (See the *Loebner Prize*...)

- It requires many abilities that seem necessary for AI, such as learning. *BUT*: a human child would probably not pass the test.
- Sometimes an AI system needs human-like acting abilities—for example *expert systems* often have to produce explanations—but *not always*.

What is AI, version two: thinking like a human

There is always the possibility that a machine *acting* like a human does not actually *think*. The *cognitive modelling* approach to AI has tried to:

- Deduce *how humans think*—for example by *introspection* or *psychological experiments*.
- Copy the process by mimicking it within a program.

An early example of this approach is the *General Problem Solver* produced by Newell and Simon in 1957. They were concerned with whether or not the program reasoned in the same manner that a human did.

Computer Science + Psychology = *Cognitive Science*

What is AI, version three: thinking rationally and the "laws of thought"

The idea that intelligence reduces to *rational thinking* is a very old one, going at least as far back as Aristotle as we've already seen.

The general field of *logic* made major progress in the 19th and 20th centuries, allowing it to be applied to AI.

- We can *represent* and *reason* about many different things.
- The *logicist* approach to AI.

This is a very appealing idea, but there are obstacles. It is hard to:

- Represent *commonsense knowledge*.
- Deal with *uncertainty*.
- Reason without being tripped up by *computational complexity*.
- Sometimes it's necessary to act when there's *no* logical course of action.
- Sometimes inference is *unnecessary* (reflex actions).

These will be recurring themes in this course, and in *Machine Learning and Bayesian Inference* next year.

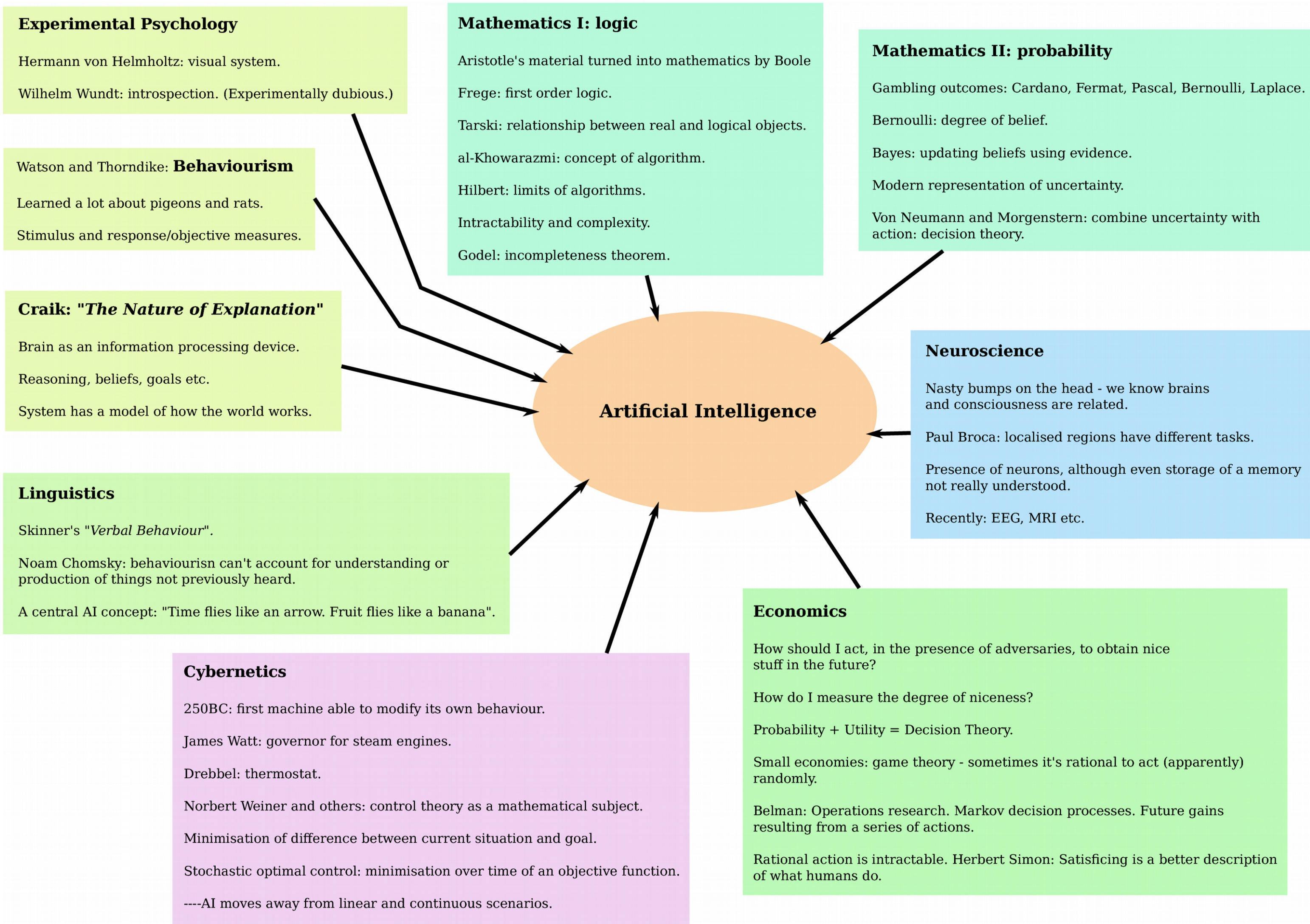
What is AI, version four: acting rationally

Basing AI on the idea of *acting rationally* means attempting to design systems that act to *achieve their goals* given their *beliefs*.

- Thinking about this in engineering terms, it seems *almost inevitably* to lead us towards the usual subfields of AI. What might be needed?
- The concepts of *action*, *goal* and *belief* can be defined precisely making the field suitable for scientific study.
- This is important: if we try to model AI systems on humans, we can't even propose *any* sensible definition of *what a belief or goal is*.
- In addition, humans are a system that is still changing and adapted to a very specific environment.
- All of the things needed to pass a Turing test seem necessary for rational acting, so this seems preferable to the *acting like a human* approach.
- The logicist approach can clearly form *part* of what's required to act rationally, so this seems preferable to the *thinking rationally* approach alone.

As a result, we will focus on the idea of designing systems that *act rationally*.

Other fields that have contributed to AI



What's in this course?

This course introduces some of the fundamental areas that make up AI:

- An outline of the background to the subject.
- An introduction to the idea of an *agent*.
- Solving problems in an intelligent way by *search*.
- Solving problems represented as *constraint satisfaction* problems.
- Playing *games*.
- *Knowledge representation, and reasoning*.
- *Planning*.
- *Learning* using *neural networks*.

Strictly speaking, this course covers what is often referred to as *"Good Old-Fashioned AI"*. (Although "Old-Fashioned" is a misleading term.)

The nature of the subject changed when the importance of *uncertainty* was fully appreciated. *Machine Learning and Bayesian Inference* covers this more recent material.

What's *not* in this course?

- The classical AI programming languages *Prolog* and *Lisp*.
- A great deal of all the areas on the last slide!
- Perception: *vision*, *hearing* and *speech processing*, *touch* (force sensing, knowing where your limbs are, knowing when something is bad), *taste*, *smell*.
- Natural language processing.
- Acting on and in the world: *robotics* (effectors, locomotion, manipulation), *control engineering*, *mechanical engineering*, *navigation*.
- Areas such as *genetic algorithms/programming*, *swarm intelligence*, *artificial immune systems* and *fuzzy logic*, for reasons that I will expand upon during the lectures.
- *Uncertainty* and much further probabilistic material. (You'll have to wait until next year.)

Introductory reading that *isn't nonsense*

- Francis Crick, *"The recent excitement about neural networks"*, Nature (1989) is still entirely relevant:

  `www.nature.com/nature/journal/v337/n6203/abs/337129a0.html`

- The *Loebner Prize in Artificial Intelligence*:

  `aisb.org.uk/aisb-events/`

  provides a good illustration of how far we are from passing the Turing test.

- Marvin Minsky, *"Why people think computers can't"*, AI Magazine (1982) is an excellent response to nay-saying philosophers.

  `http://web.media.mit.edu/~minsky/`

- Go: `www.nature.com/nature/journal/v529/n7587/full/nature16961.html`
- The Cyc project: `www.cyc.com`
- AI at Nasa Ames:

  `www.nasa.gov/centers/ames/research/areas-of-ames-ingenuity-autonomy-and-robotics`

Introductory reading that *isn't nonsense*

- *AI in the UK: ready, willing and able?*

  House of Lords, Select Committee on Artificial Intelligence

  `https://publications.parliament.uk/pa/ld201719/ldselect/ldai/100/100.pdf`

- *Machine learning: the power and promise of computers that learn by example*

  The Royal Society

  `https://royalsociety.org/topics-policy/projects/machine-learning/`

- *Building machines that learn and think like people*

  Brenden M. Lake *et al*, Behavioral and Brain Sciences, Cambridge University Press, 2017.

Text book

The course is based on the relevant parts of:

Artificial Intelligence: A Modern Approach, Third Edition (2010). Stuart Russell and Peter Norvig, Prentice Hall International Editions.

and an alternative source is:

Artificial Intelligence: Foundations of Computational Agents, Second Edition (2017). David L. Poole and Alan K. Mackworth, Cambridge University Press.

For more depth on specific areas see:

Dechter, R. (2003). *Constraint processing*. Morgan Kaufmann.

Cawsey, A. (1998). *The essence of artificial intelligence*. Prentice Hall.

Ghallab, M., Nau, D. and Traverso, P. (2004). *Automated planning: theory and practice*. Morgan Kaufmann.

Bishop, C.M. (2006). *Pattern recognition and machine learning*. Springer.

Brachman, R. J. and Levesque, H. J. (2004). *Knowledge Representation and Reasoning*. Morgan Kaufmann.

Prerequisites

The prerequisites for the course are: first order logic, some algorithms and data structures, discrete and continuous mathematics, and basic computational complexity.

DIRE WARNING:

No doubt you want to know something about *machine learning*, given the recent peek in interest.

In the lectures on *machine learning* I will be talking about *neural networks*.

I will introduce the *backpropagation algorithm*, which is the foundation for both *classical neural networks* and the more fashionable *deep learning* methods.

This means you will need to be able to *differentiate* and also handle *vectors and matrices*.

If you've forgotten how to do this *you WILL get lost—I guarantee it!!!*

Self test:

1. Let

$$f(x_1, \ldots, x_n) = \sum_{i=1}^{n} a_i x_i^2$$

   where the a_i are constants. Can you compute $\partial f / \partial x_j$ where $1 \leq j \leq n$?

2. Let $f(x_1, \ldots, x_n)$ be a function. Now assume $x_i = g_i(y_1, \ldots, y_m)$ for each x_i and some collection of functions g_i. Assuming all requirements for differentiability and so on are met, can you write down an expression for $\partial f / \partial y_j$ where $1 \leq j \leq m$?

If the answer to either of these questions is “no” then it’s time for some revision. (You have about three weeks notice, so I’ll assume you know it!)

And finally…

There are some important points to be made regarding *computational complexity*.

First, you might well hear the term *AI-complete* being used a lot. What does it mean?

AI-complete: only solvable if you can solve AI in its entirety.

For example: high-quality automatic translation from one language to another.

To produce a genuinely good translation of *Moby Dick* from English to Cantonese is likely to be AI-complete.

And finally...

More practically, you will often hear me make the claim that *everything that's at all interesting in AI is at least NP-complete*.

There are two ways to interpret this:

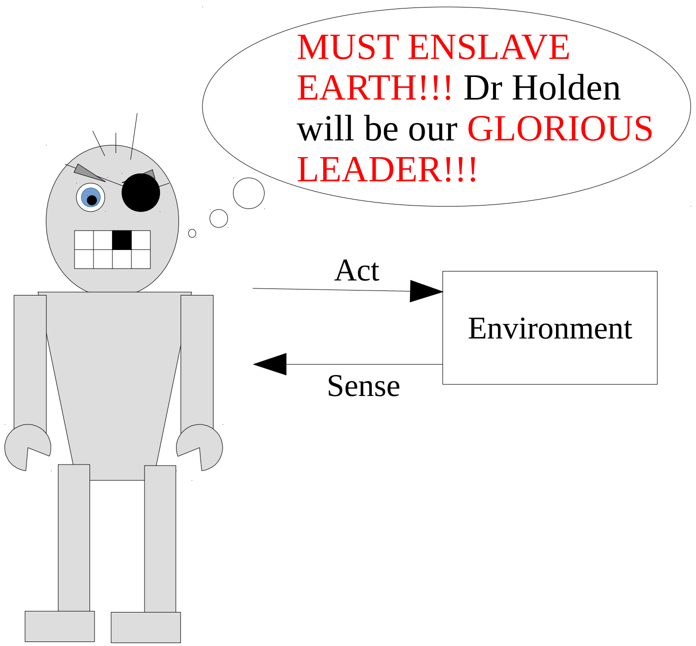
1. The wrong way: "It's all a waste of time.[1]" OK, so it's a partly understandable interpretation. *BUT* the fact that Boolean satisfiability is intractable *does not* mean we can't solve large instances in practice...
2. The right way: "It's an opportunity to design nice approximation algorithms." In reality, the algorithms that are *good in practice* are ones that try to *often* find a *good* but not necessarily *optimal* solution, in a *reasonable* amount of time and memory.

[1] In essence, a comment on a course assessment a couple of years back to the effect of: "Why do you teach us this stuff if it's all futile?"

Agents

There are many different definitions for the term *agent* within AI.

Allow me to introduce EVIL ROBOT.



We will use the following simple definition: *an agent is any device that can sense and act upon its environment*.

Agents

This definition can be very widely applied: to humans, robots, pieces of software, and so on.

We are taking quite an *applied* perspective. We want to *make things* rather than *copy humans*. So:

1. How can we judge an agent's performance?
2. How can an agent's *environment* affect its design?
3. Are there sensible ways in which to think about the *structure* of an agent?

Recall that we are interested in devices that *act rationally*, where 'rational' means doing the *correct thing* under *given circumstances*.

Measuring performance

Item 1: How can we judge an agent's performance?

- Any measure of performance is likely to be *problem-specific*.
  - Even a simple email filter is an agent—it can sense and act. Here the performance measure is straightforward.
  - For a self-driving car, it is more complicated!
- We're usually interested in *expected, long-term performance*.
  - *Expected* performance because usually agents are not *omniscient*—they don't *infallibly* know the outcome of their actions.

    (It is *rational* for you to enter this lecture theatre even if the roof falls in today. An agent capable of detecting and protecting itself from a falling roof might be more *successful* than you, but *not* more *rational*.
  - *Long-term performance* because it tends to lead to better approximations to what we'd consider rational behaviour.

Environments

Item 2: How can an agent's *environment* affect its design?

Some common attributes of an environment have a considerable influence on agent design.

- *Accessible/inaccessible:* do percepts tell you *everything* you need to know about the world?
- *Deterministic/non-deterministic:* does the future depend *predictably* on the present and your actions?
- *Episodic/non-episodic* is the agent run in independent episodes.
- *Static/dynamic:* can the world change while the agent is deciding what to do?
- *Discrete/continuous:* an environment is discrete if the sets of allowable percepts and actions are finite.
- *For multiple agents:* whether the situation is *competitive* or *cooperative*, and whether *communication* is required.

Item 3: Are there sensible ways in which to think about the *structure* of an agent?

A basic agent can be thought of as working according to a straightforward underlying process. To achieve some *goal*:

- *Gather perceptions*.
- Update *working memory* to take account of them.
- On the basis of what's in the working memory, *choose an action* to perform.
- *Update* the working memory to take account of this action.
- *Do* the chosen action.

Obviously, this hides a great deal of complexity:

- A percept might arrive *while an action is being chosen*.
- The world may change *while an action is being chosen*.
- Actions may affect the world in *unexpected ways*.
- We might have *multiple goals*, which *interact* with each other.
- And so on...

Keeping track of the environment, and having a goal

It seems reasonable that an agent should maintain:

- A *description of the current state of its environment*.
- Knowledge of how the environment *changes independently of the agent*.
- Knowledge of how the agent's *actions affect its environment*.

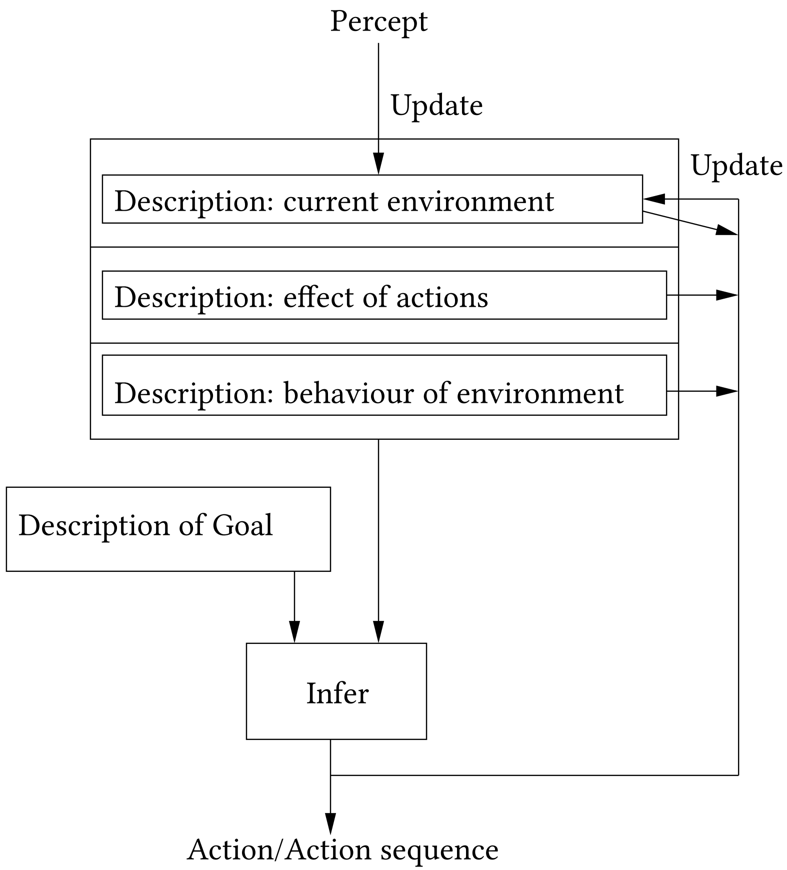
This requires us to do *knowledge representation* and *reasoning*.

It also seems reasonable that an agent should choose a rational course of action depending on its *goal*.

- If an agent has knowledge of how its actions affect the environment, then it has a basis for choosing actions to achieve goals.
- To obtain a *sequence* of actions we need to be able to *search* and to *plan*.

Goal-based agents

We now have a basic design that looks something like this:



Utility-based agents

Introducing goals is still not the end of the story.

- There may be *many* sequences of actions that lead to a given goal, and *some may be preferable to others*.
- We might need to trade-off *conflicting goals*, for example speed and safety.
- An agent may have several goals, but not be certain of achieving any of them. Can it trade-off the likelihood of reaching a goal against the desirability of getting there?

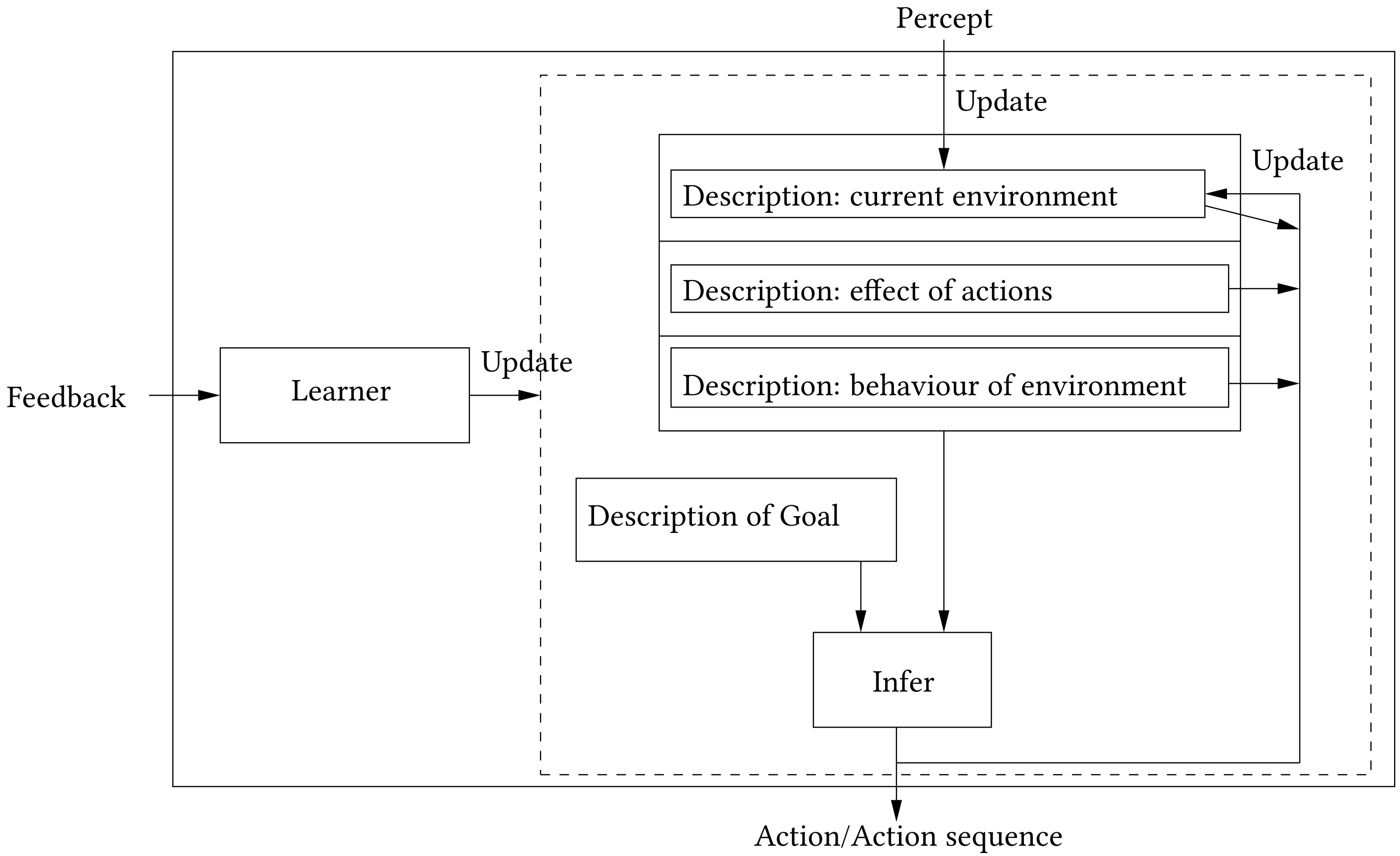
A *utility function* maps a state to a number representing the desirability of that state.

Maximising expected utility over time forms a fundamental model for the design of agents.

Unfortunately, there is insufficient time in this course to properly explore agents based on utility.

Learning agents

It seems reasonable that an agent should *learn from experience*:



What might this entail?

Learning mainly requires two additions:

1. The learner needs some form of *feedback* on the agent's performance. This can come in several different forms.
2. The learner needs a means of *generating new behaviour* in order to find out about the world.

The second point leads to an important trade-off:

1. Should the agent spend time *exploiting* what it's learned so far, if it's achieving a level of success, or...
2. ...should the agent try new things, *exploring* the environment on the basis that it might learn something *really useful* even if it performs *worse in the short term*?

Problem solving by search

Reading: AIMA chapters 3 and 4.

We begin with what is perhaps the simplest collection of AI techniques: those allowing an *agent* existing within an *environment* to *search* for a *sequence of actions* that *achieves a goal*.

Search algorithms apply to a particularly simple class of problems—we need to identify:

- *An initial state* s_0 from a set S of possible states.

  This models the agent's situation before anything else happens.

- *A set of actions*, denoted A.

  These are modelled by specifying what state will result on performing any available action in any state.

  We can model this using a function $\texttt{action} : A \times S \rightarrow S$: if the agent is in state s and performs action a then its new state is $\texttt{action}(a, s)$.

- *A goal test*: we can tell whether or not the state we're in corresponds to a goal.

  We can model this using a function $\texttt{goal} : S \rightarrow \{\texttt{true}, \texttt{false}\}$.

We also need the idea of *path cost*.

We need another function $\texttt{cost} : A \times S \to \mathbb{R}$. This denotes the *cost* of *performing an action a* in *state s*.

If the agent starts in state s_0 and takes a sequence of actions $a_0, a_1, \ldots, a_n$ then it moves through a sequence of states

$$s_0 \xrightarrow{\texttt{cost}(a_0,s_0)} s_1 \xrightarrow{\texttt{cost}(a_1,s_1)} s_2 \xrightarrow{\texttt{cost}(a_2,s_2)} \cdots \xrightarrow{\texttt{cost}(a_n,s_n)} s_{n+1}$$

with $s_{i+1} = \texttt{action}(a_i, s_i)$. We then define the *path cost* of this path as

$$p(s_{n+1}) = \sum_{i=0}^{n} \texttt{cost}(a_i, s_i).$$

We generally want a path to a *goal* that has *minimim path cost*.

Note that you have *already seen* problems like this...

Problem solving by search

You have *already seen* problems like this...

- *Foundations of Computer Science*: talks about searching in *trees*.

  It covers *depth-first*, *breadth-first* and *iterative deepening* search.

- *Algorithms*: talks about searching in *graphs*.

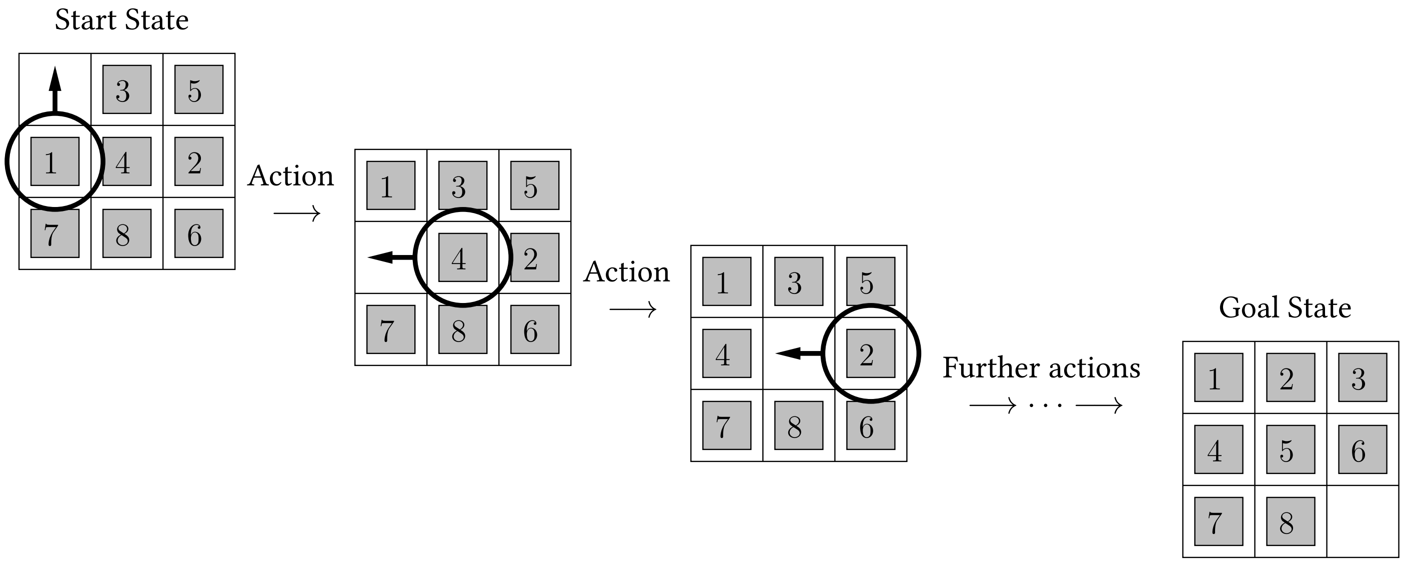
  It also covers *depth-first* and *breadth-first* search, from a more formal perspective.

This is all important stuff, but there's a problem: *none of these methods works in practice for typical AI problems!*

Essentially, the problem is that they are too naïve in the way that they *choose a state to explore* at each step.

I'm going to assume that you know this material and move on...

A simple example: *the 8-puzzle*.



From the *pre-PC dark ages*. Christmas was grim...

Here we have:

- *Start state:* a randomly-selected configuration of the numbers 1 to 8 arranged on a 3×3 square grid, with one square empty.
- *Goal state:* the numbers in ascending order with the bottom right square empty.
- *Actions:* `left`, `right`, `up`, `down`. We can move any square adjacent to the empty square into the empty square. (It's not always possible to choose from all four actions.)
- *Path cost:* 1 per move.

The 8-puzzle is very simple. However general sliding block puzzles are a good test case. The general problem is NP-complete. The 5×5 version has about 10^{25} states, and a random instance is in fact quite a challenge.

Problems of this kind are very simple, but a surprisingly large number of applications have appeared:

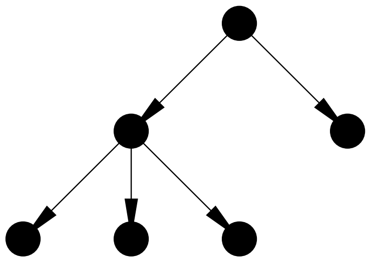
- Route-finding/tour-finding.
- Layout of VLSI systems.
- Navigation systems for robots.
- Sequencing for automatic assembly.
- Searching the internet.
- Design of proteins.

and many others...

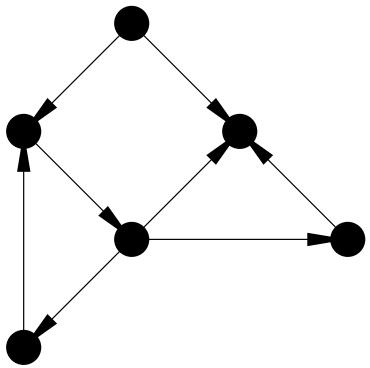
Problems of this kind continue to form an active research area.

Search trees versus search graphs

We need to make an important distinction between *search trees* and *search graphs*.



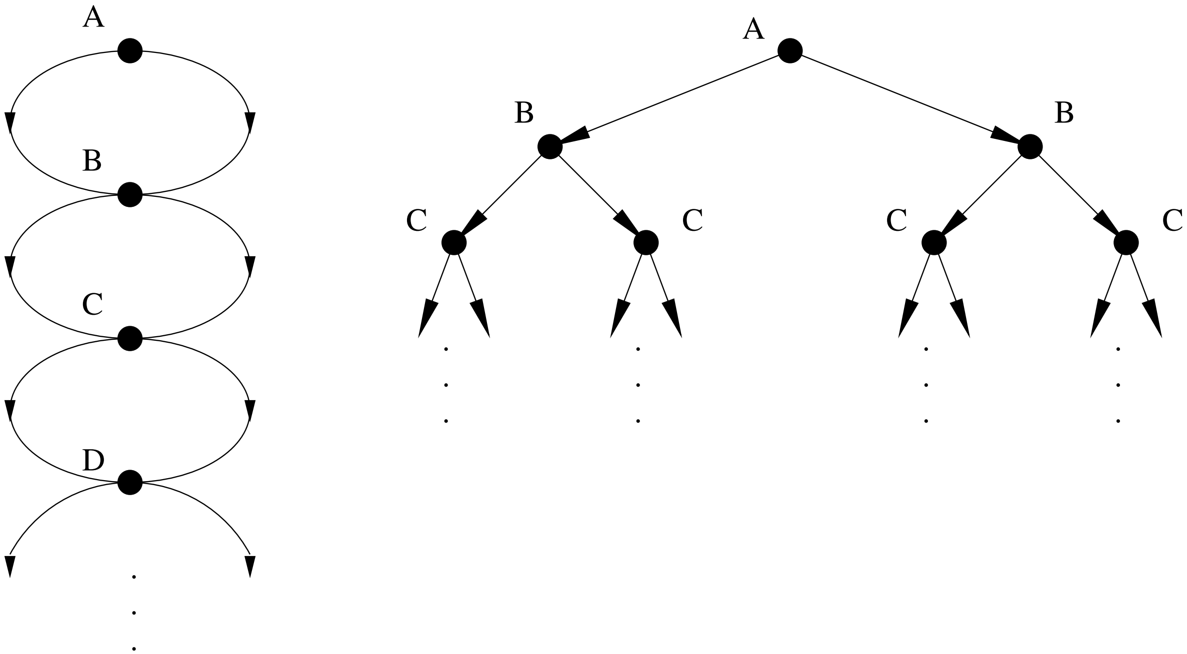
as opposed to



- In a *tree* only *one path* can lead to a given state.
- In a *graph* a *state* can be reached via possibly *multiple paths*.
- In a *graph* we may also encounter *cycles*.

Search trees versus search graphs

Graphs can lead to *problems*:



The *sliding blocks puzzle* for example suffers this way.

So: we start by assuming the search is taking place on a *tree*.

The basic tree-search algorithm

We need to define one more function: expand takes any *state* s. It applies all *actions* that can be applied in s and returns the *set of the resulting states*:

$$\texttt{expand}(s) = \{s' | s' = \texttt{action}(a, s) \text{ where } a \text{ is an action possible in } s\}.$$

The algorithm for searching in a tree then looks like this:

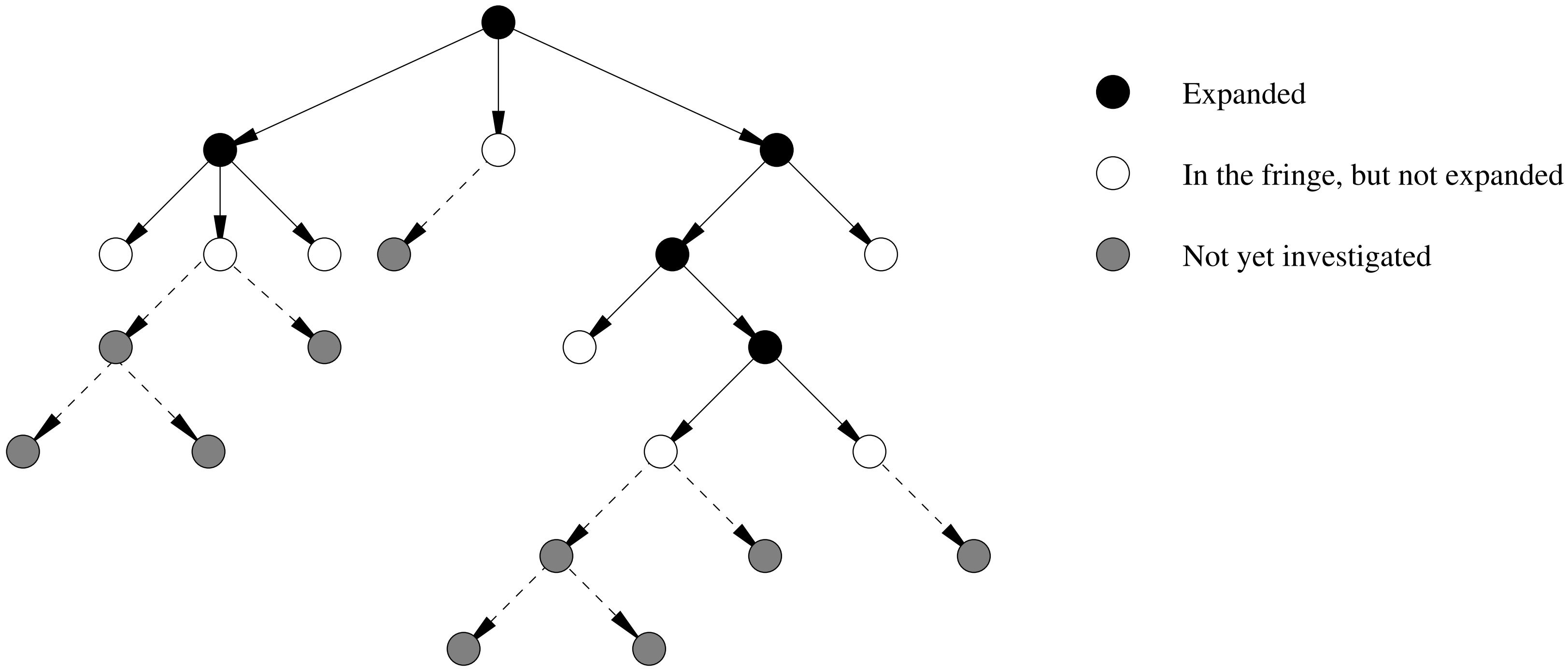
```
fringe = [s_0];
while true do
    if fringe.empty() then
        return NONE;
    s = fringe.remove();
    if goal(s) then
        return (SOME s);
    fringe.addAll(expand(s));
```

The *search strategy* is set by using a *priority queue* to implement the fringe.

The definition of *priority* then sets the way in which the tree is searched.

The basic tree-search algorithm

The process looks like this:



At each iteration, one node from the fringe is expanded. In general, if the *branching factor* is b then the *layer* at *depth* d can have b^d states.

The *entire tree* to depth d can have $\sum_{i=0}^{d} b^d = \frac{b^{d+1}-1}{b-1}$ states.

The performance of search techniques

How might we judge the performance of a search technique?

We are interested in:

- Whether a solution is found.
- Whether the solution found is a good one in terms of path cost.
- The cost of the search in terms of time and memory.

So

$$\text{the total cost} = \text{path cost} + \text{search cost}$$

If a problem is highly complex it may be worth settling for a *sub-optimal solution* obtained in a *short time*.

And we are interested in:

Completeness: does the strategy *guarantee* a solution is found?

Optimality: does the strategy guarantee that the *best* solution is found?

Once we start to consider these, things get a lot more interesting...

Basic search algorithms

We can immediately define some familiar tree search algorithms:

- New nodes are added to the *head of the queue*. This is *depth-first search*.
- New nodes are added to the *tail of the queue*. This is *breadth-first search*.

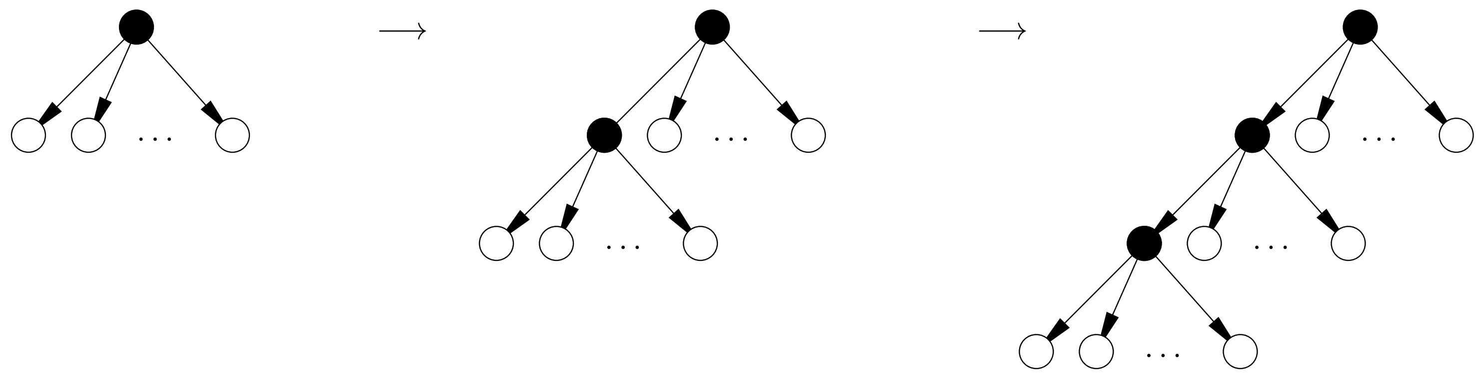
We will not dwell on these, as they are both *completely hopeless* in practice.

Why is breadth-first search hopeless?

- The procedure is *complete*: it is guaranteed to find a solution if one exists.
- The procedure is *optimal* if the path cost is a non-decreasing function of node-depth.
- The procedure has *exponential complexity for both memory and time*.

In practice it is the *memory* requirement that is problematic.

With depth-first search: for a given branching factor b and depth d the memory requirement is $O(bd)$.



This is because we need to store *nodes on the current path* and *the other unexpanded nodes*.

The time complexity is still $O(b^d)$ (if you know you only have to go to depth d).

The search is *no longer optimal*, and may not be *complete*.

Iterative-deepening combines the two, but *we can do better*.

Uniform-cost search

How might we change tree search to try to get to an *optimal solution* while limiting the *time and memory* needed?

The key point: so far we only distinguish *goal states* from *non-goal states*!

None of the searches you've seen so far tries to prioritize the exploration of good states!!!

What is a *good state*?

- Well, at any point in the search we can work out the *path cost* $p(s)$ of whatever state s we've got to.
- How about using the $p(s)$ as the priority for the priority queue?

This is called *Uniform-Cost Search*.

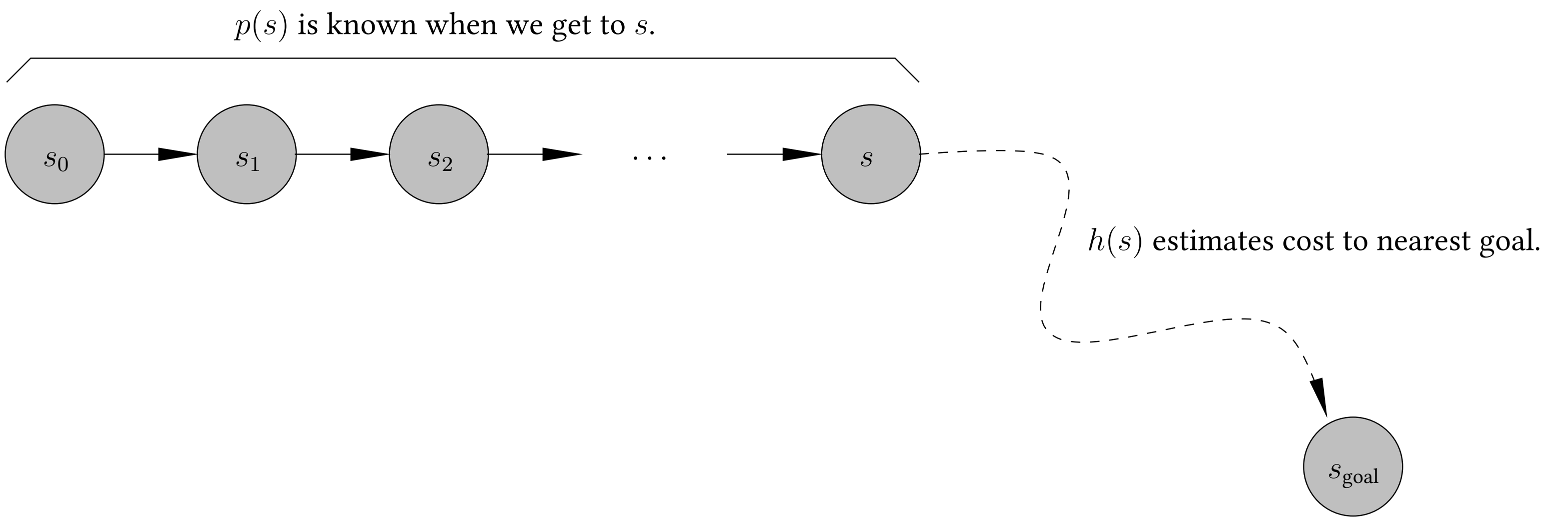
In practice it doesn't work very well: we need *something more subtle*.

But it does suggest the idea of an *evaluation function*: a function that attempts to measure the *desirability of each state*.

Heuristics

Why is *path cost* not a good evaluation function? It is not *directed* in any sense *toward the goal*.

A *heuristic function*, usually denoted $h(s)$, is one that *estimates* the cost of the best path from any state s to a goal. If s is a goal then $h(s) = 0$.



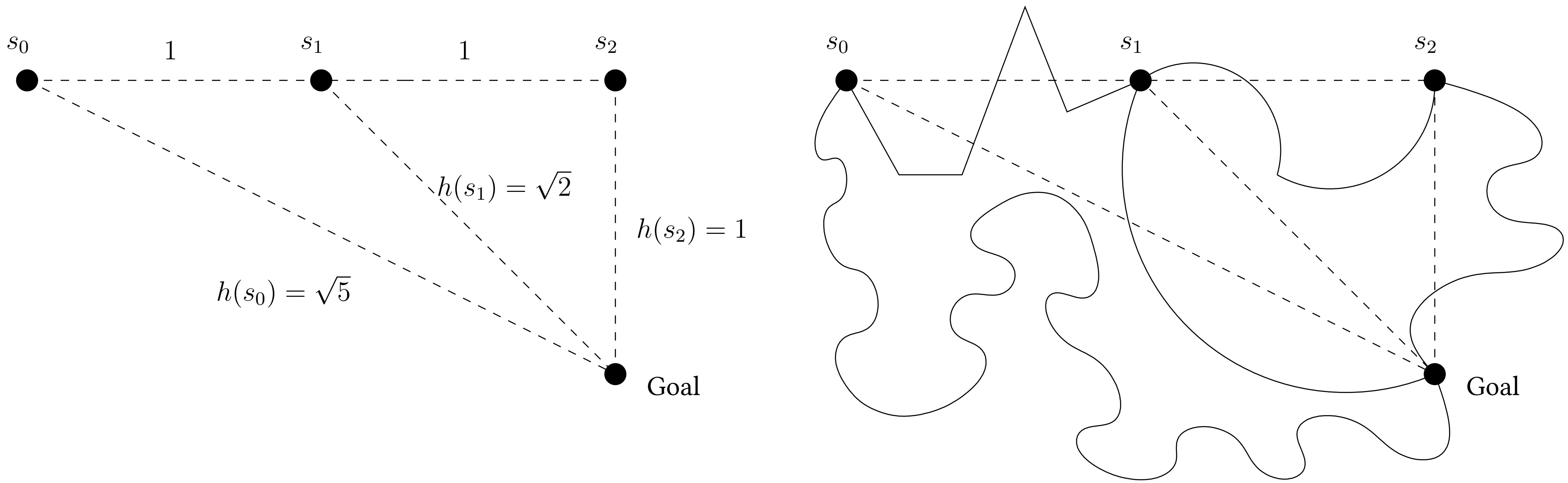
This is a *problem-dependent* measure. We are required either to *design it* using our *knowledge of the problem*, or by some other means.

The last point is critical: *AI is a long way from being independent of human ingenuity*.

Example: route-finding

Example: for route finding a reasonable heuristic function is

$$h(s) = \text{straight line distance from } s \text{ to the nearest goal}$$



Accuracy here obviously depends on what the roads are really like.

Can we use $h(s)$ in choosing a state to explore? If it's *really good* it can work well, but *we can still do better*!

$A^\star$ search

$A^\star$ search is the classical *AI-oriented search algorithm*.

$A^\star$ search combines the good points of:

- Using $p(s)$ to know how far we've come.
- Using $h(s)$ to estimate how far we have to go.

It does this in a very simple manner: it uses path cost $p(s)$ and also the heuristic function $h(s)$ by forming

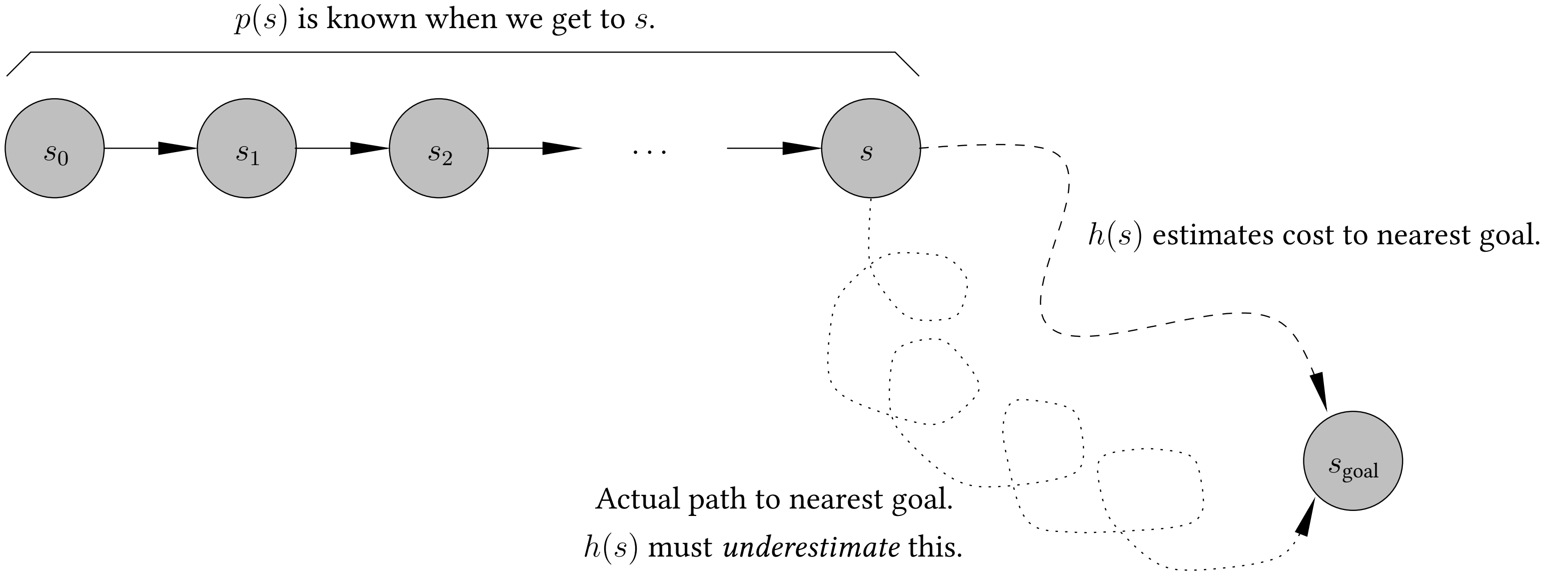
$$f(s) = p(s) + h(s).$$

So: $f(s)$ is the *estimated cost* of a path *through* s.

By using this as a priority for exploring states we get a search algorithm that is *optimal* and *complete* under simple conditions, and can be *vastly superior* to the more naïve approaches.

$A^\star$ search

Definition: an *admissible heuristic* $h(s)$ is one that *never overestimates* the cost of the best path from s to a goal.



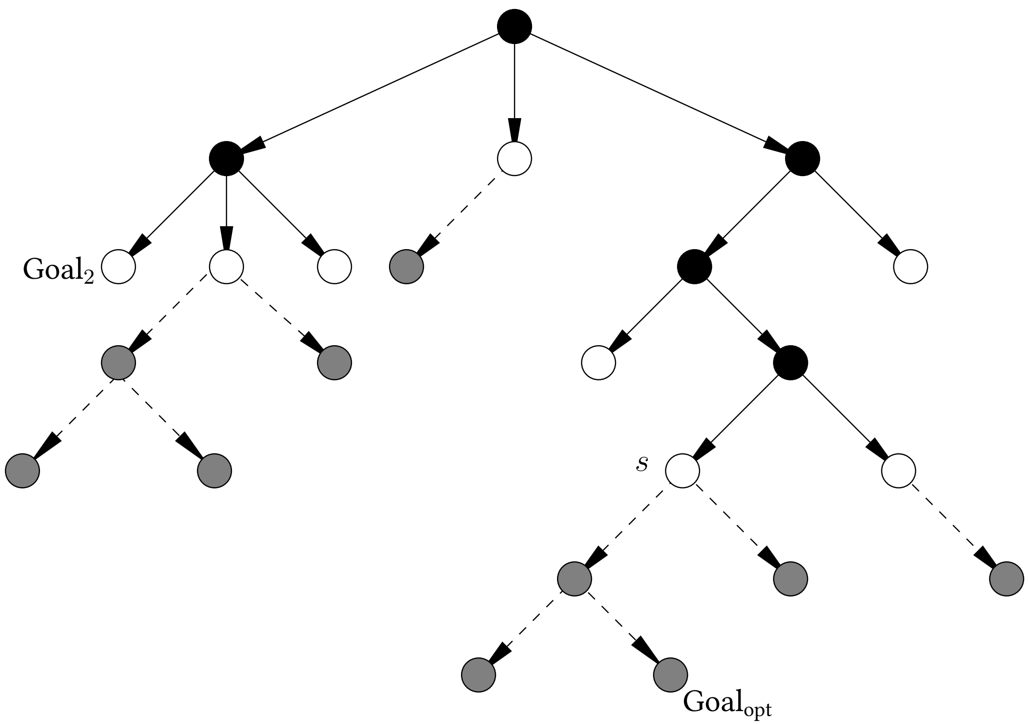
So if $h'(s)$ denotes the *actual* distance from s to the goal we have

$$\forall s . h(s) \leq h'(s).$$

If $h(s)$ is *admissible* then *tree-search $A^\star$ is optimal*.

$A^\star$ tree-search is optimal for admissible $h(s)$

To see that *tree-search $A^\star$ is optimal* we reason as follows. Let Goal_{opt} be an optimal goal state with $f(\text{Goal}_{\text{opt}}) = p(\text{Goal}_{\text{opt}}) = f_{\text{opt}}$ (because $h(\text{Goal}_{\text{opt}}) = 0$).



At some point Goal_2 is in the fringe.

Can it be selected before s?

Let Goal_2 be a suboptimal goal state with $f(\text{Goal}_2) = p(\text{Goal}_2) = f_2 > f_{\text{opt}}$. We need to demonstrate that *the search can never select* Goal_2.

$A^\star$ tree-search is optimal for admissible $h(s)$

Let s be a state in the fringe on an optimal path to Goal_{opt}. So

$$f_{\text{opt}} \geq p(s) + h(s) = f(s)$$

because h is admissible.

Now say Goal_2 is chosen for expansion *before* s. This means that

$$f(s) \geq f_2$$

so we've established that

$$f_{\text{opt}} \geq f_2 = p(\text{Goal}_2).$$

But this means that Goal_{opt} is not optimal: a contradiction.

And that's all that's needed for trees. *But for searching on graphs we need a little more...*

Graph search

To search in *graphs* we need a way to make sure no state gets visited *more than once*.

We need to add a *closed list*, and add a state to it when the state is *first seen*:

```
closed = [];
fringe = [s_0];
while true do
    if fringe.empty() then
        return NONE;
    s = fringe.remove();
    if goal(s) then
        return (SOME s);
    if !closed.contains(s) then
        closed.add(s);
        fringe.addAll(expand(s));
```

Graph search

There are several points to note regarding graph search:

1. The *closed list* contains all the expanded states.
2. The closed list can be implemented using a *hash table*. So the time taken to *add* or *check membership* can be managable.
3. Both worst case time and space are now *proportional to the size of the state space*. (Which is BIG!!!!)
4. *Memory:* depth first and iterative deepening search are no longer linear space as we need to store the closed list.
5. *Optimality:* when a repeat is found we are *discarding the new possibility even if it is better than the first one*. We may need to check which solution is better and if necessary modify path costs and depths for descendants of the repeated state.

Unfortunately last point breaks the proof...

$A^\star$ graph search

Unfortunately last point breaks the proof...

- Graph search can *discard an optimal* route if that route is not the first one generated.
- We could keep *only the least expensive path*. This means updating, which is extra work, not to mention messy, but sufficient to insure optimality.
- Alternatively, we can impose a further condition on $h(s)$ which *forces the best path to a repeated state to be generated first*.

The required condition is called *monotonicity*. As

$$\text{monotonicity} \longrightarrow \text{admissibility}$$

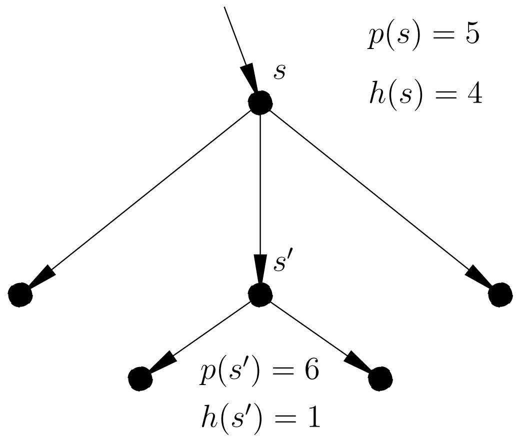
this is an important property.

Monotonicity

Assume h is admissible. Remember that $f(s) = p(s) + h(s)$ so if s' follows s

$$p(s') \geq p(s)$$

and we expect that $h(s') \leq h(s)$ although this does not have to be the case.



Here $f(s) = 9$ and $f(s') = 7$ so $f(s') < f(s)$.

Monotonicity

Monotonicity:

- If it is always the case that $f(s') \geq f(s)$ then $h(s)$ is called *monotonic*.
- $h(s)$ is monotonic if and only if it obeys the *triangle inequality*.

$$h(s) \leq \texttt{cost}(a, s) + h(s')$$

  where a is the action moving us from s to s'.

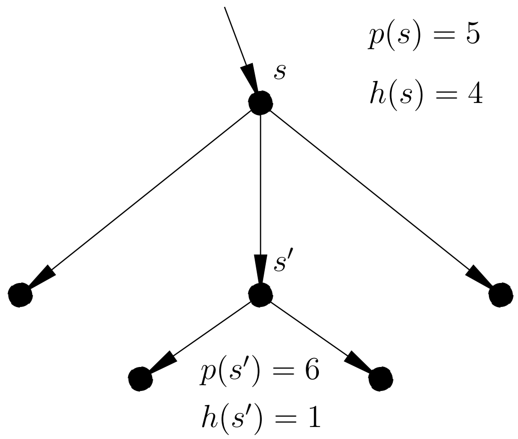
If $h(s)$ is *not* monotonic we can make a simple alteration and use

$$f(s') = \max\{f(s), p(s') + h(s')\}$$

This is called the *pathmax* equation.

The pathmax equation

Why does this make sense?



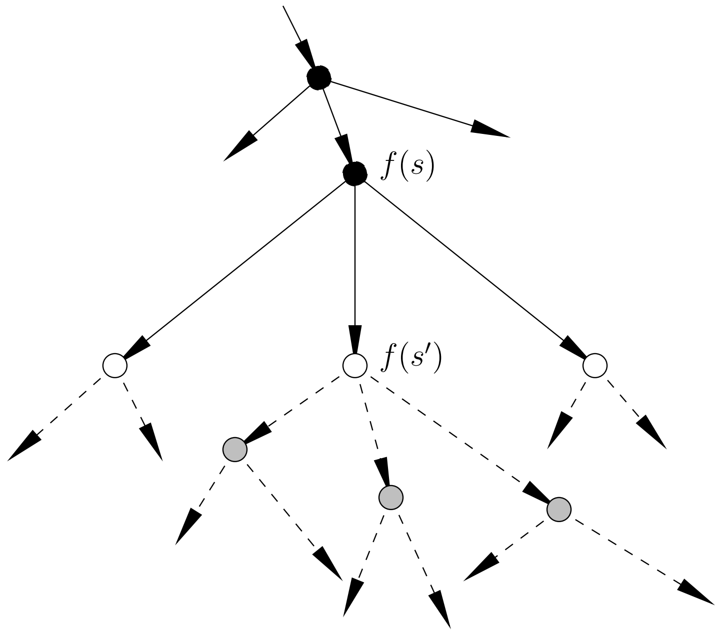
The fact that $f(s) = 9$ tells us the cost of a path through s is *at least* 9 (because $h(s)$ is admissible).

But s' is *on a path through* s. So to say that $f(s') = 7$ makes no sense.

$A^\star$ graph search is optimal for monotonic heuristics

The crucial fact from which optimality follows is that if $h(s)$ is monotonic then the values of $f(s)$ along any path are non-decreasing.

We therefore have the following situation:



You can't deal with s' until everything with $f(s'') < f(s')$ has been dealt with.

Consequently everything with $f(s'') < f_{\text{opt}}$ gets explored. Then one or more things with f_{opt} get found (not necessarily all goals).

$A^\star$ search is complete

$A^\star$ search is *complete* provided:

1. The graph has *finite branching factor*.
2. There is a *finite, positive constant* c such that *each action* has *cost at least* c.

Why is this? The search expands nodes according to increasing $f(s)$. So: the only way it can fail to find a goal is if there are *infinitely many nodes with* $f(s) < f(\text{Goal})$.

There are two ways this can happen:

1. There is a node with an *infinite number of descendants*.
2. There is a path with an *infinite number of nodes* but a *finite path cost*.

Complexity

We won't be *proving* the following, but they are *good things to know*:

- $A^\star$ search has a further desirable property: it is *optimally efficient*.
- This means that no other optimal algorithm that works by constructing paths from the root can *guarantee to examine fewer nodes*.
- *BUT*: despite its good properties we're not done yet...
- ...$A^\star$ search unfortunately still has *exponential time complexity in most cases* unless $h(s)$ satisfies a very stringent condition that is generally unrealistic:

  $$|h(s) - h'(s)| \leq O(\log h'(s))$$

  where $h'(s)$ denotes the *real* cost from s to the goal.
- As $A^\star$ search also stores all the nodes it generates: once again it is generally *memory that becomes a problem before time*.

IDA* - iterative deepening $A^\star$ search

How might we *improve* the way in which $A^\star$ search uses *memory*?

- Iterative deepening search used depth-first search with a *limit on depth* that is *gradually increased*.
- *IDA** does the same thing *with a limit on f cost*.

IDA* - iterative deepening $A^\star$ search

The function contour searches from a specified state s *as far as a specified limit* fLimit *on* f.

It returns either a path from s to a goal, or the *next biggest* value to try for the limit on f.

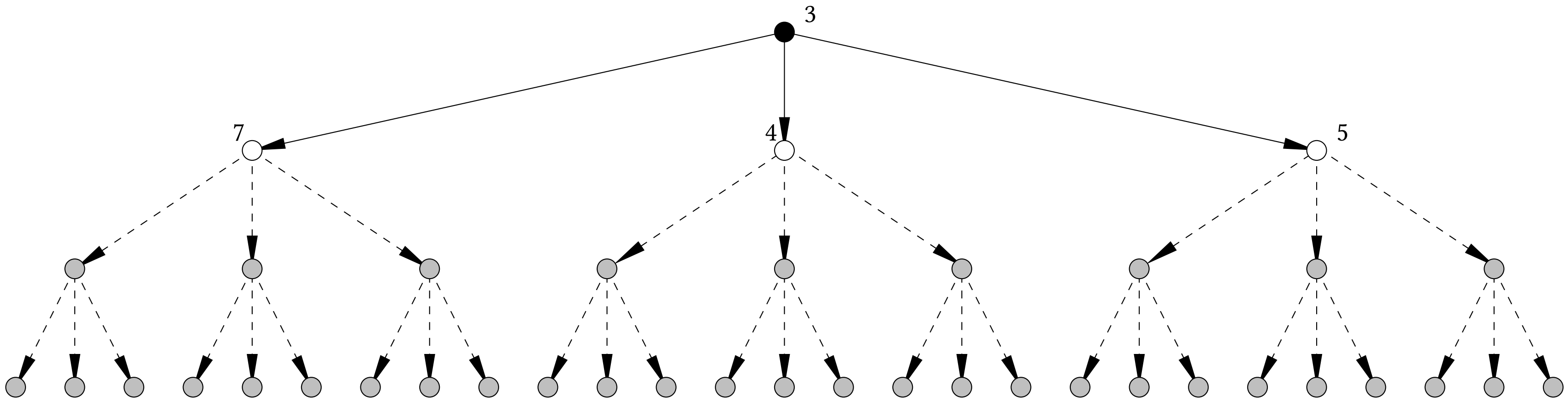
```
function contour(s, fLimit, path)
    nextF = ∞;
    if f(s) > fLimit then
        return ([], f(s));
    if goal(s) then
        return (s :: path, fLimit)
    for s' ∈ expand(s) do
        (newPath, newF) = contour(s', fLimit, s :: path);
        if newPath != [] then
            return (newPath, fLimit);
        nextF = min(nextF, newF);
    return ([], nextF);
```

IDA* - iterative deepening $A^\star$ search

```
function iterativeDeepeningAStar()
    fLimit = f(s_0);
    while true do
        (path, fLimit) = contour(s_0, fLimit, []);
        if path != [] then
            return path;
        if fLimit == ∞ then
            return [];
```

IDA* - iterative deepening $A^\star$ search

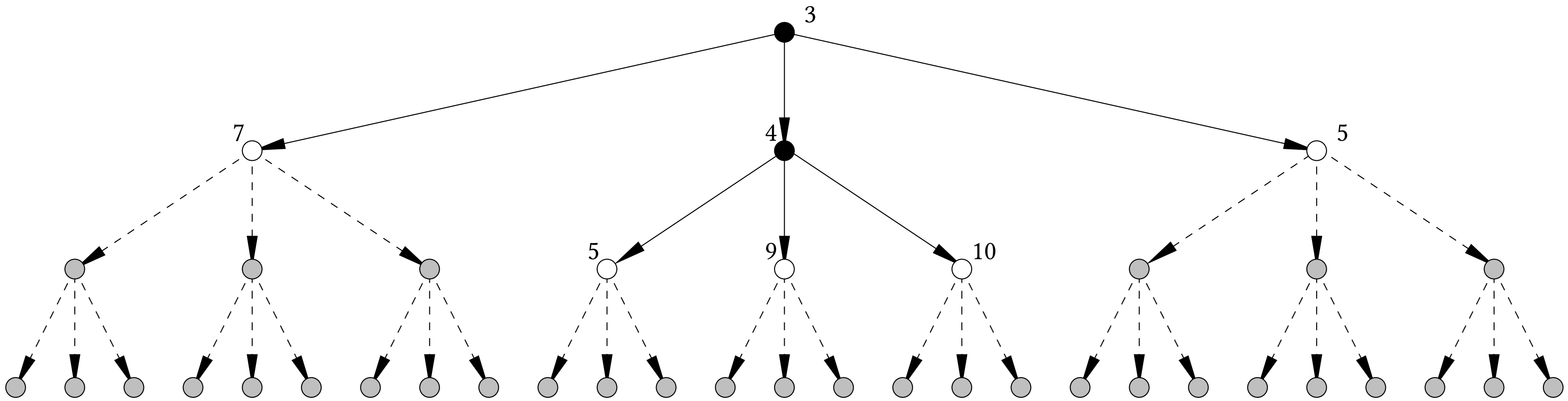
This is a little tricky to unravel, so here is an example:



Initially, the algorithm looks ahead and finds the *smallest* f cost that is *greater than* its current f cost limit. The new limit is 4.

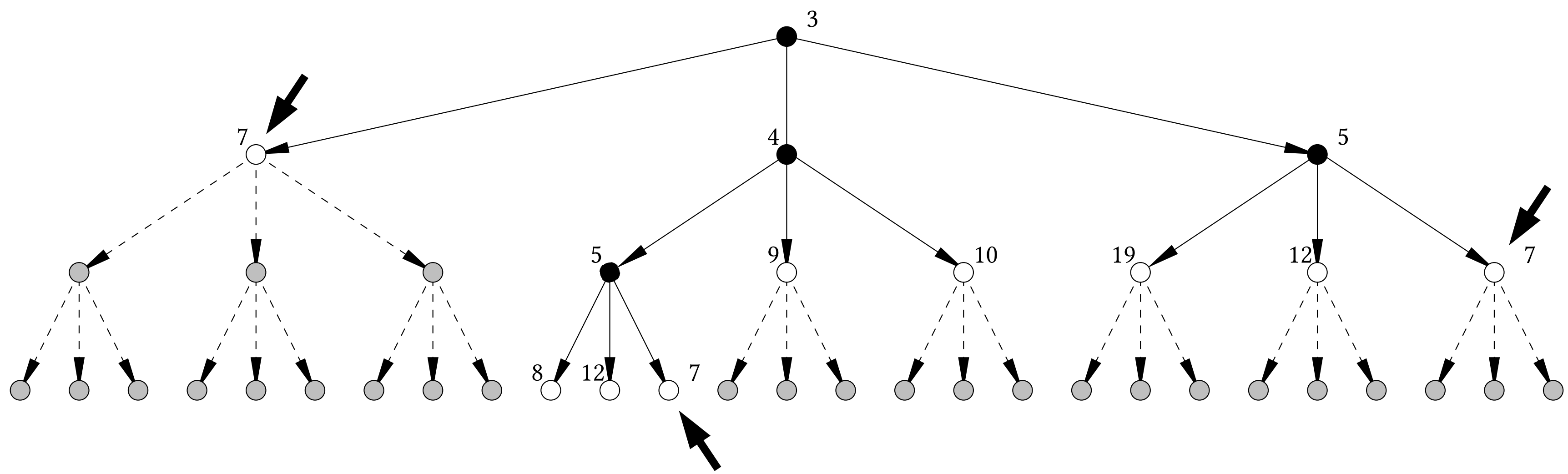
IDA* - iterative deepening $A^\star$ search

It now does the same again:



Anything with f cost *at most* equal to the current limit gets explored, and the algorithm keeps track of the *smallest* f cost that is *greater than* its current limit. The new limit is 5.

And again:



The new limit is 7, so at the next iteration the three arrowed nodes will be explored.

IDA* - iterative deepening $A^\star$ search

Properties of IDA*:

- It is complete and optimal under the same conditions as $A^\star$.
- It is often good if we have step costs equal to 1.
- It does not require us to maintain a sorted queue of nodes.
- It only requires *space proportional to the longest path*.
- The time taken depends on the number of values h can take.

If h takes enough values to be problematic we can increase the limit on f by a fixed ϵ at each stage, guaranteeing a solution at most ϵ worse than the optimum.

Another method by which we can attempt to overcome memory limitations is the *Recursive Best-First Search (RBFS)*.

Idea: try to use f, but only use *linear space* by doing a depth-first search with a few modifications:

1. We remember the $f(s')$ for the best alternative state s' we've seen so far on the way to the state s we're currently considering.
2. If s has $f(s) > f(s')$:
   - We go back and explore the best alternative...
   - ...and as we retrace our steps we replace the f cost of every state we've seen in the current path with $f(s)$.

The replacement of f values as we retrace our steps provides a means of remembering how good a discarded path might be, so that we can easily return to it later.

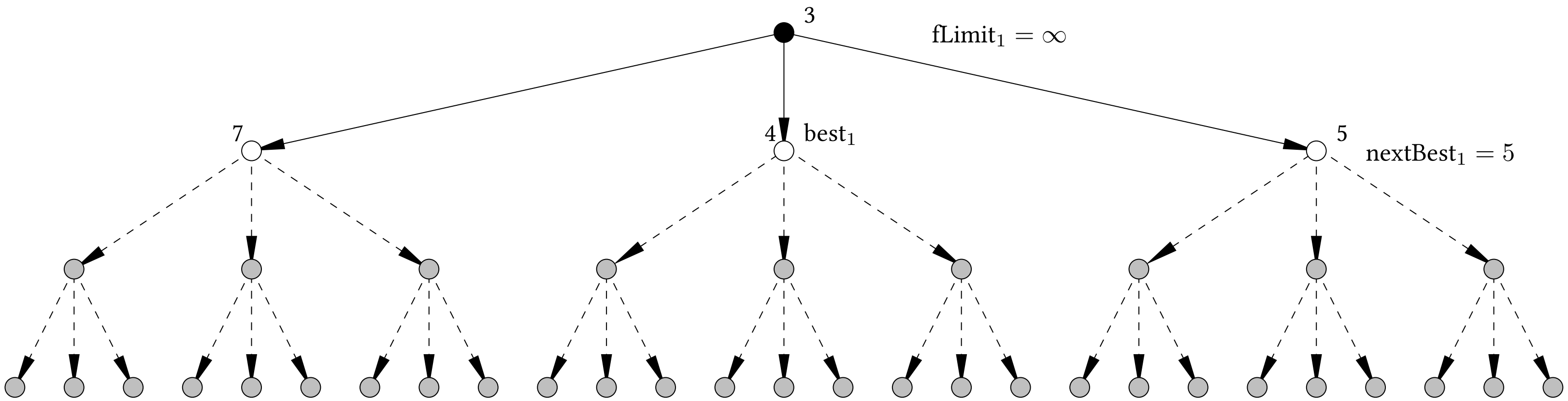
Recursive best-first search (RBFS)

```
function rbfs(s, fLimit)
    if goal(s) then
        return (SOME s, fLimit);
    if expand(s) = ∅ then
        return (NONE, ∞);
    for each s′ ∈ expand(s) do
        f(s′) = maximum(f(s′), f(s));
    while true do
        best = s′ ∈ expand(s) with smallest f(s′);
        if f(best) > fLimit then
            return (NONE, f(best));
        nextBest = s′ ∈ expand(s) with second smallest f(s′);
        (result, f′) = rbfs (best, minimum(fLimit, f(nextBest)));
        f(best) = f′;
        if result ! = NONE then
            return (result, f′);
```

Recursive best-first search (RBFS): an example

This function is called using $\texttt{rbfs}(s_0, \infty)$ to begin the process.

Function call number 1:

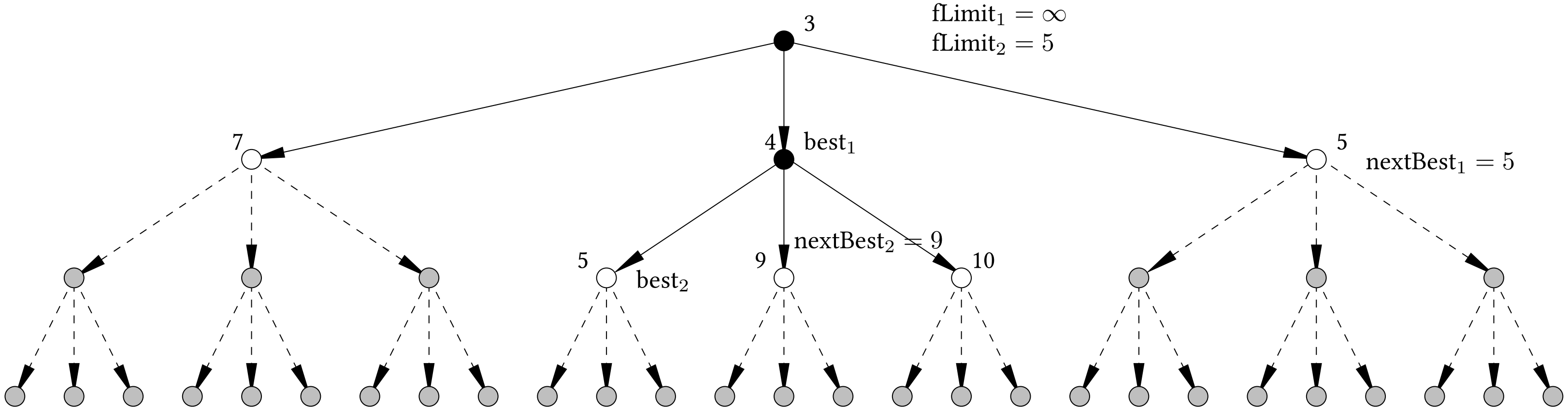


Now perform the recursive function call $(\text{result}_2, f') = \texttt{rbfs}(\text{best}_1, 5)$

so $f(\text{best}_1)$ takes the returned value f'

Recursive best-first search (RBFS): an example

Function call number 2:

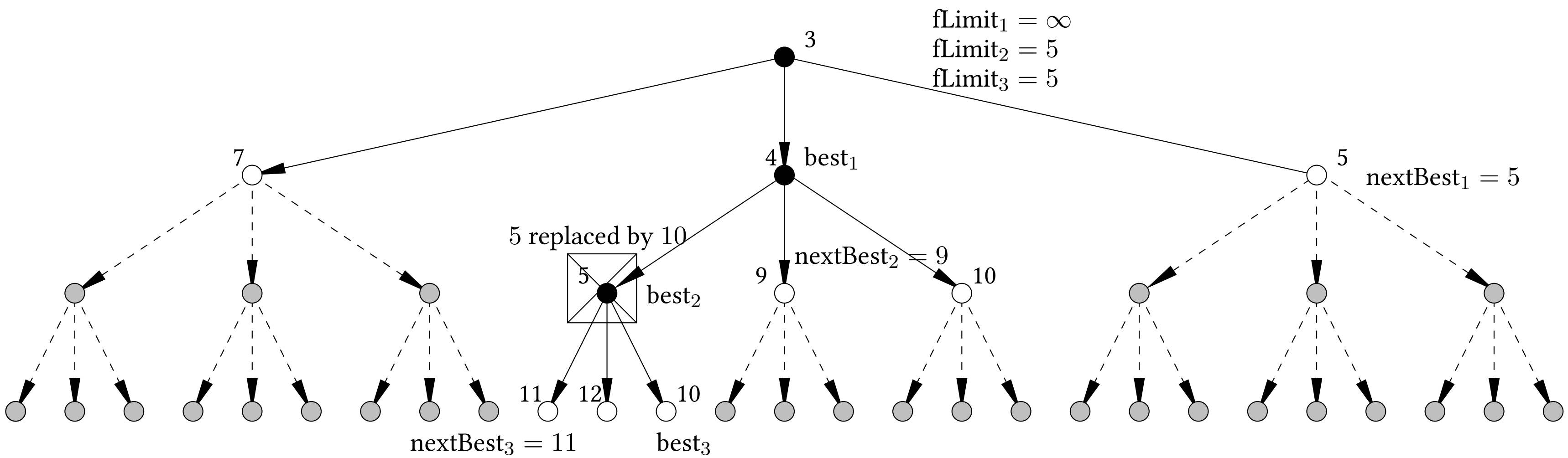


Now perform the recursive function call $(\text{result}_3, f') = \texttt{rbfs}(\text{best}_2, 5)$

so $f(\text{best}_2)$ takes the returned value f'

Recursive best-first search (RBFS): an example

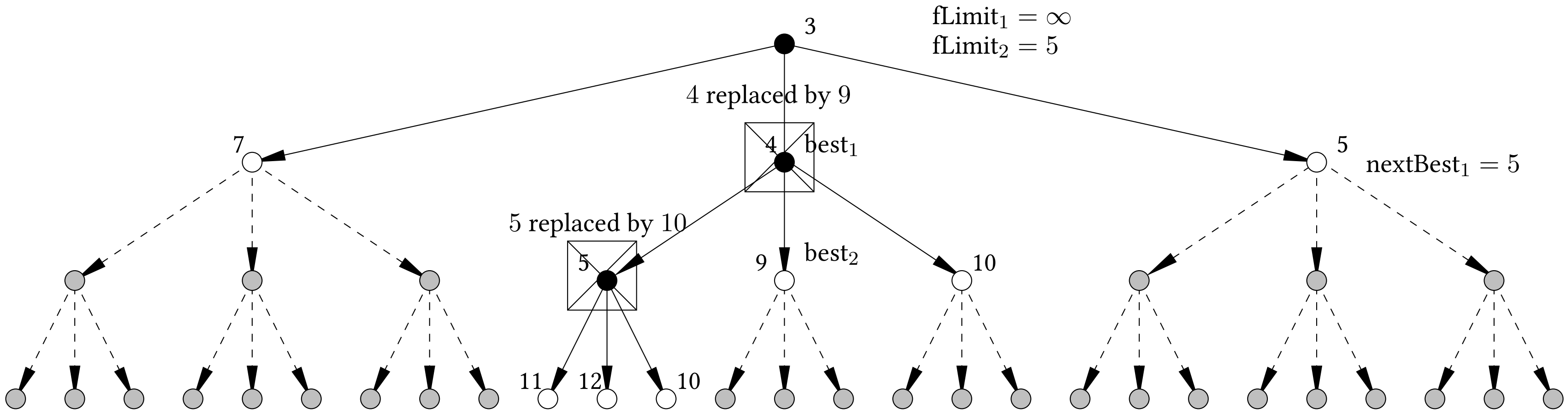
Function call number 3 :



Now $f(\text{best}_3) > \text{fLimit}_3$ so the function call returns $(\text{NONE}, 10)$ into (result_3, f') and $f(\text{best}_2) = 10$.

Recursive best-first search (RBFS): an example

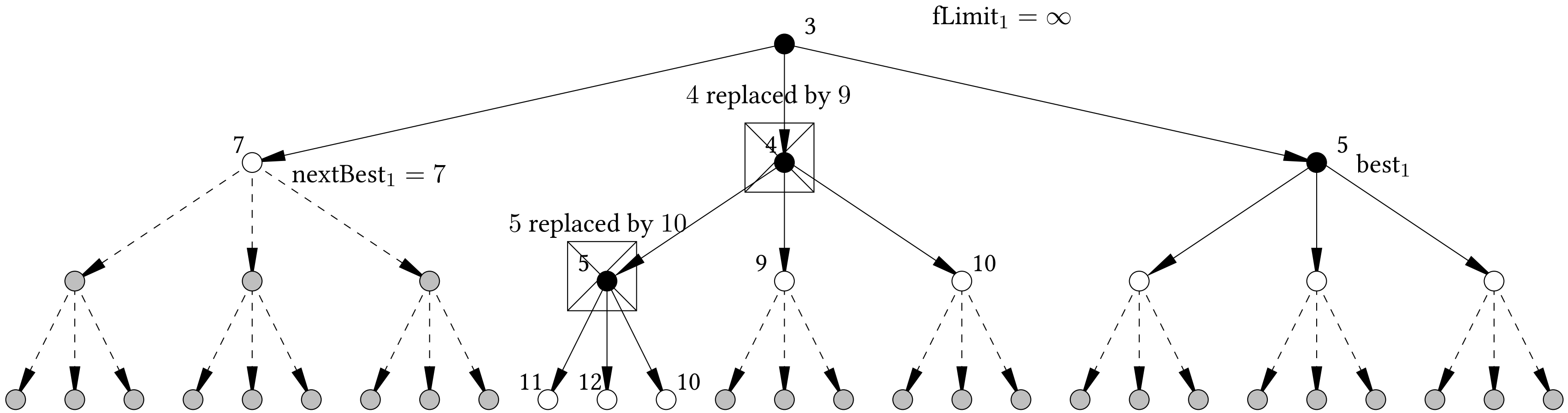
The while loop for function call 2 now repeats:



Now $f(\text{best}_2) > \text{fLimit}_2$ so the function call returns $(\text{NONE}, 9)$ into (result_2, f') and $f(\text{best}_1) = 9$.

Recursive best-first search (RBFS): an example

The while loop for function call 1 now repeats:



We do a further function call to expand the new best node, and so on...

Recursive best-first search (RBFS)

Some nice properties:

- If h is admissible then RBFS is optimal.
- Memory requirement is $O(bd)$
- Generally more efficient than IDA*.

And some less nice ones:

- Time complexity is hard to analyse, but can be exponential.
- Can spend a lot of time *re-generating nodes*.

To some extent IDA* and RBFS throw the baby out with the bathwater.

- They limit memory too harshly, so...
- ...we can try to use *all available memory*.

MA* and SMA* will not be covered in this course...

Local search

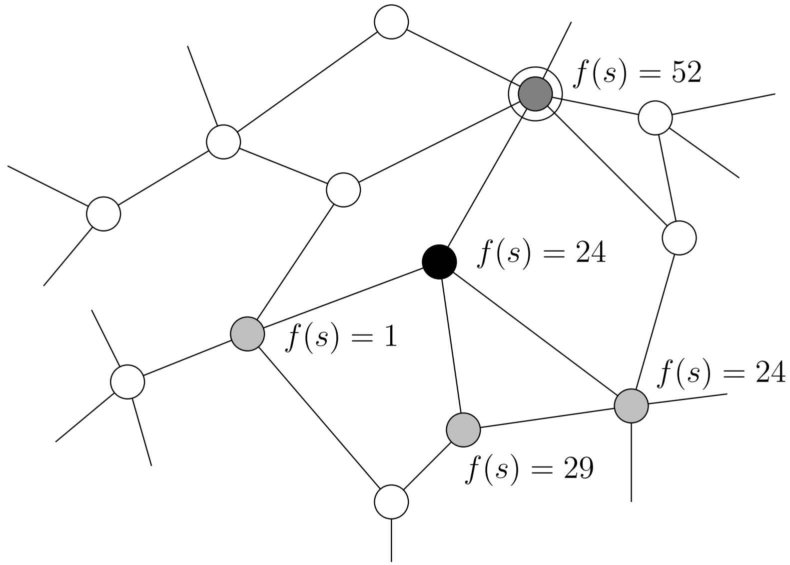
Sometimes, it's only the *goal* that we're interested in. The *path* needed to get there is irrelevant.

- For example: VLSI layout, factory design, automatic programming…
- We are now simply searching for a state that is in some sense *the best*.
- This is also known as *optimisation*.

This leads to the remarkably simple concept of *local search*.

Local search

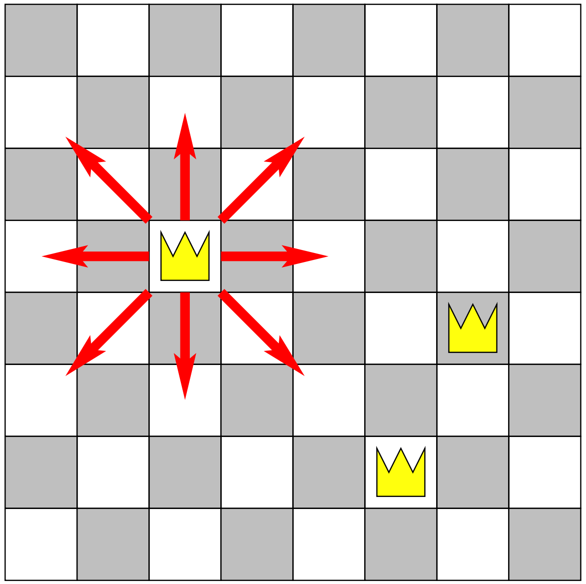
Instead of trying to find a path from start state to goal, we explore the *local area* of the graph, meaning those states one edge away from the one we're at:



We assume that we have a function $f(s)$ such that $f(s') > f(s)$ indicates s' is preferable to s.

The m-queens problem

You may be familiar with the *m-queens problem*.



Find an arrangement of m queens on an m by m board such that no queen is attacking another.

In the Prolog course you may have been tempted to generate permutations of row numbers and test for attacks.

This is a *hopeless strategy* for large m. (Imagine $m \simeq 1,000,000$.)

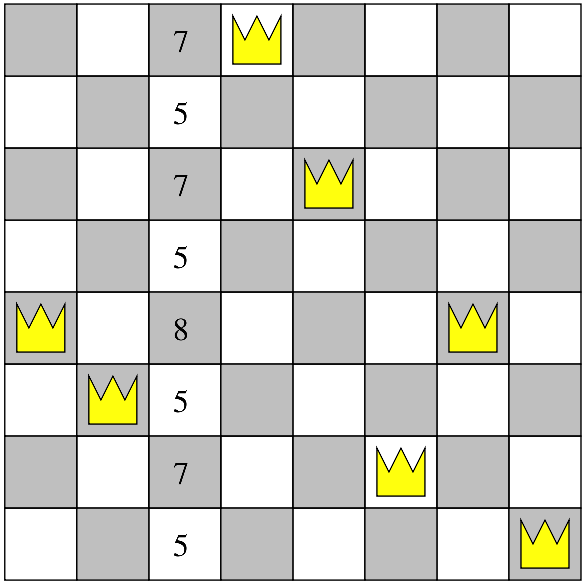
We might however consider the following:

- A state s for an m by m board is a sequence of m numbers drawn from the set $\{1, \ldots, m\}$, possibly including repeats.
- We move from one state to another by moving a *single queen* to *any* alternative row.
- We define $f(s)$ to be the number of pairs of queens attacking one-another in the new position[2]. (Regardless of whether or not the attack is direct.)

[2]Note that we actually want to *minimize* f here. This is equivalent to maximizing $-f$, and I will generally use whichever seems more appropriate.

The m-queens problem

Here, we have $\{4, 3, ?, 8, 6, 2, 4, 1\}$ and the f values for the undecided queen are shown.



As we can choose which queen to move, each state in fact has 56 neighbours in the graph.

Hill-climbing search

Hill-climbing search is remarkably simple:

Generate a start state s;
while `true` **do**
    Generate the neighbours $N = \{s_1, \dots, s_p\}$ of s;
    $N_f = \{f(s_i) | s_i \in N\}$;
    if $\max N_f \leq f(s)$ **then**
        return s;
    $s = s_i \in N$ with maximum $f(s_i)$;

In fact, that looks so simple that it's amazing the algorithm is at all useful.

In this version we stop when we get to a node with no better neighbour.

Hill-climbing search: the reality

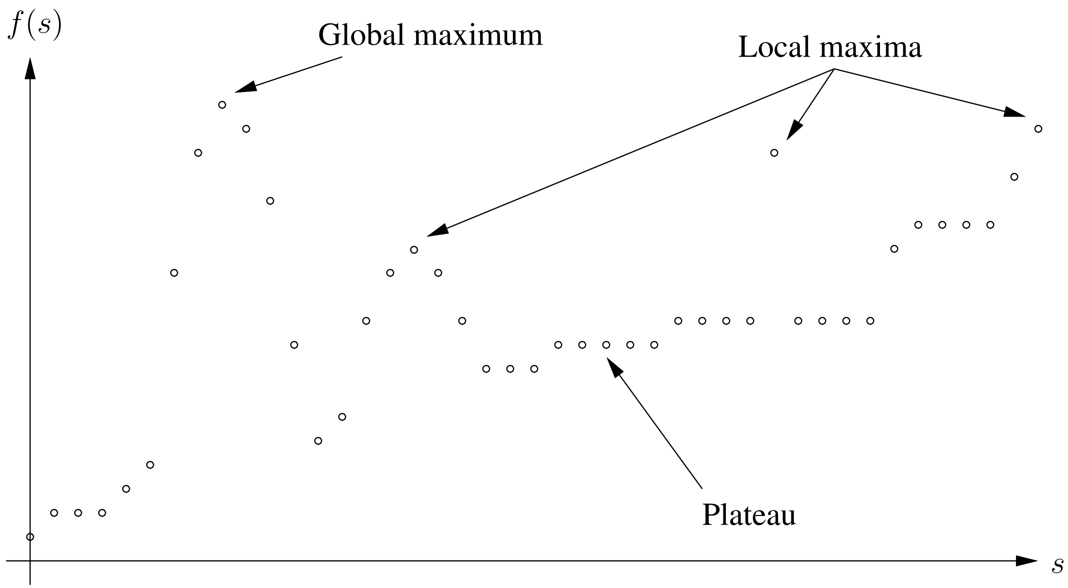
We might alternatively allow *sideways moves* by changing the stopping condition:

```
if max N_f < f(s) then
    return s;
```

Why would we consider doing this?

Hill-climbing search: the reality

In reality, nature has a number of ways of shaping f to complicate the search process.



Sideways moves allow us to move across *plateaus*.

However, should we ever find a *local maximum* then we'll return it: we won't keep searching to find a *global maximum*.

Hill-climbing search: the reality

Of course, the fact that we're dealing with a *general graph* means we need to think of something like the preceding figure, but in a *very large number of dimensions*, and this makes the problem *much harder*.

There is a body of techniques for trying to overcome such problems. For example:

- *Stochastic hill-climbing:* Choose a neighbour at random, perhaps with a probability depending on its f value. For example: let $N(s)$ denote the neighbours of s. Define

  $$N^{+}(s) = \{s' \in N(s) | f(s') \geq f(s)\}$$
  $$N^{-}(s) = \{s' \in N(s) | f(s') < f(s)\}.$$

  Then

  $$\Pr(s') = \begin{cases} 0 & \text{if } s' \in N^{-}(s) \\ \frac{1}{Z}(f(s') - f(s)) & \text{otherwise.} \end{cases}$$

Hill-climbing search: the reality

- *First choice:* Generate neighbours at random. Select the first one that is better than the current one. (Particularly good if nodes have *many neighbours*.)
- *Random restarts:* Run a procedure k times with a limit on the time allowed for each run.

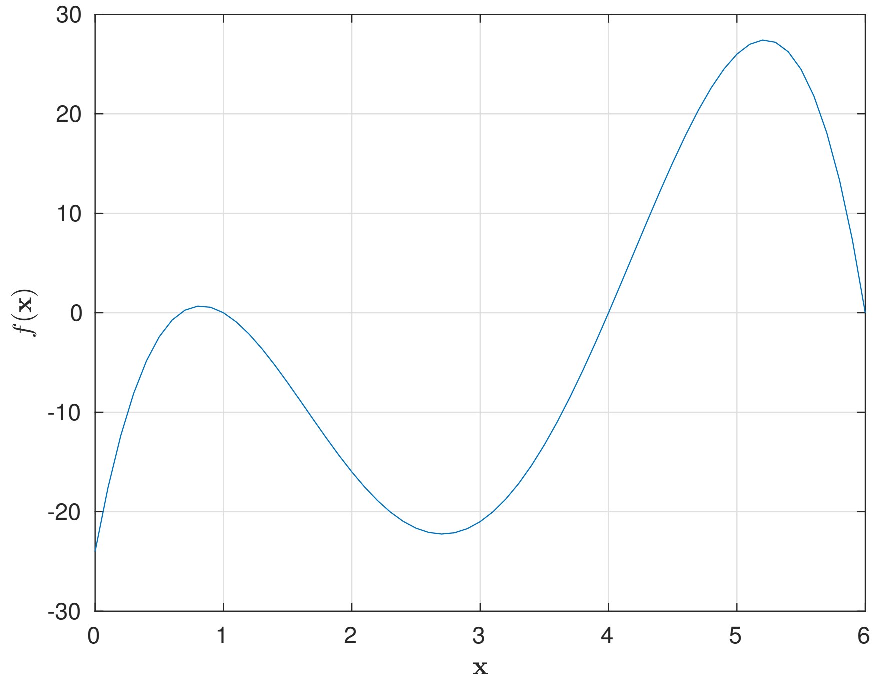
  Note: generating a start state at random may itself not be straightforward.
- *Simulated annealing:* Similar to stochastic hill-climbing, but start with lots of random variation and *reduce it over time*.

  Note: in some cases this is *provably* an effective procedure, although the time taken may be excessive if we want the proof to hold.
- *Beam search:* Maintain k states at any given time. At each search step, find the successors of each, and retain the best k from *all* the successors.

  Note: this is *not* the same as random restarts.

Gradient ascent and related methods

For some problems[3]—we do not have a search graph, but a *continuous search space*.



Typically, we have a function $f(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$ and we want to find

$$\mathbf{x}_{\text{opt}} = \underset{\mathbf{x}}{\operatorname{argmax}} f(\mathbf{x})$$

[3]For the purposes of this course, the *training of neural networks* is a notable example.

Gradient ascent and related methods

In a single dimension we can clearly try to solve

$$\frac{df(x)}{dx} = 0$$

to find the *stationary points*, and use

$$\frac{d^2 f(x)}{dx^2}$$

to find a global *maximum*. In *multiple dimensions* the equivalent is to solve

$$\nabla f(\mathbf{x}) = \frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} = \mathbf{0}$$

where

$$\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} = \left[\begin{array}{cccc} \frac{\partial f(\mathbf{x})}{\partial x_1} & \frac{\partial f(\mathbf{x})}{\partial x_2} & \cdots & \frac{\partial f(\mathbf{x})}{\partial x_n} \end{array} \right].$$

and the equivalent of the second derivative is the *Hessian* matrix

$$\mathbf{H} = \left[\begin{array}{cccc} \frac{\partial f^2(\mathbf{x})}{\partial x_1^2} & \frac{\partial f^2(\mathbf{x})}{\partial x_1 \partial x_2} & \cdots & \frac{\partial f^2(\mathbf{x})}{\partial x_1 \partial x_n} \\ \frac{\partial f^2(\mathbf{x})}{\partial x_2 \partial x_1} & \frac{\partial f^2(\mathbf{x})}{\partial x_2^2} & \cdots & \frac{\partial f^2(\mathbf{x})}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f^2(\mathbf{x})}{\partial x_n \partial x_1} & \frac{\partial f^2(\mathbf{x})}{\partial x_n \partial x_2} & \cdots & \frac{\partial f^2(\mathbf{x})}{\partial x_n^2} \end{array} \right].$$

Gradient ascent and related methods

However this approach is usually *not analytically tractable* regardless of dimensionality.

The simplest way around this is to employ *gradient ascent*:

- Start with a randomly chosen point $\mathbf{x}_0$.
- Using a small *step size* ϵ, iterate using the equation

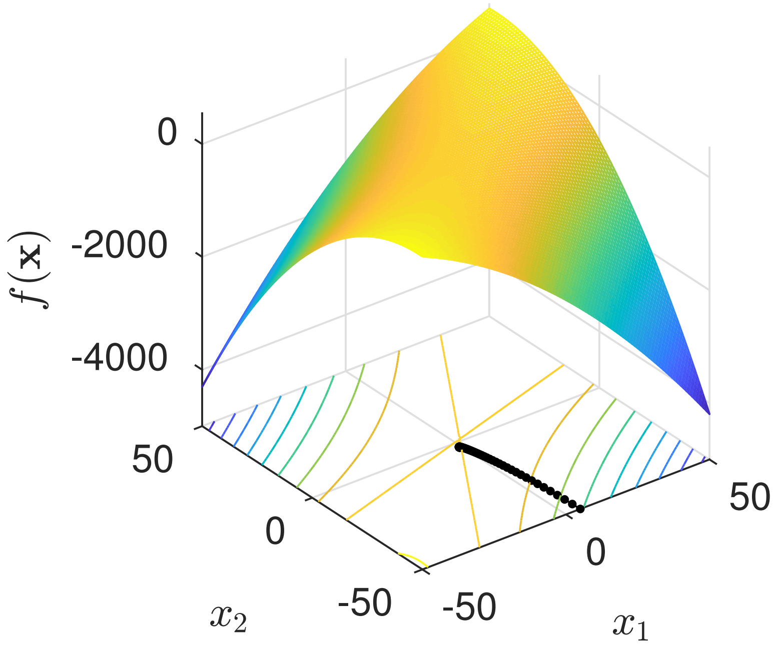
$$\mathbf{x}_{i+1} = \mathbf{x}_i + \epsilon \nabla f(\mathbf{x}_i).$$

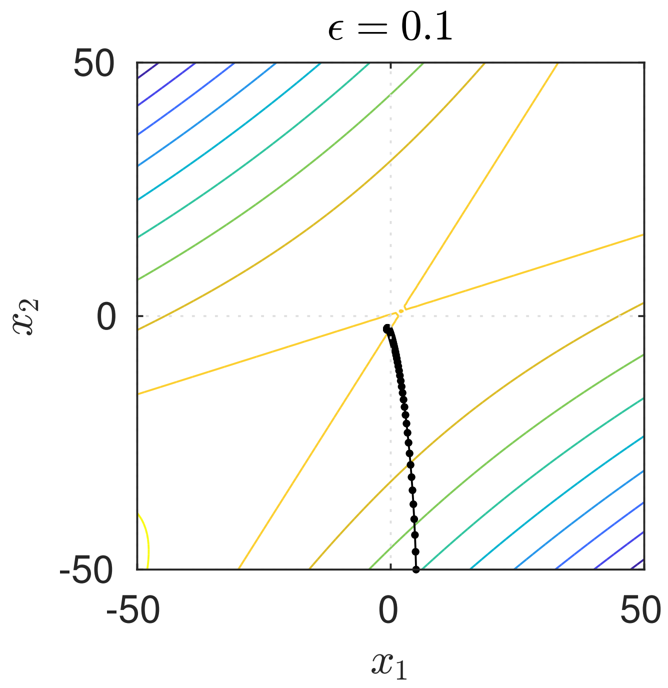
This can be understood as follows:

- At the current point $\mathbf{x}_i$ the gradient $\nabla f(\mathbf{x}_i)$ tells us the *direction* and *magnitude* of the slope at $\mathbf{x}_i$.
- Adding $\epsilon \nabla f(\mathbf{x}_i)$ therefore moves us a *small distance upward*.

This is perhaps more easily seen graphically…

Here we have a simple *parabolic surface*:

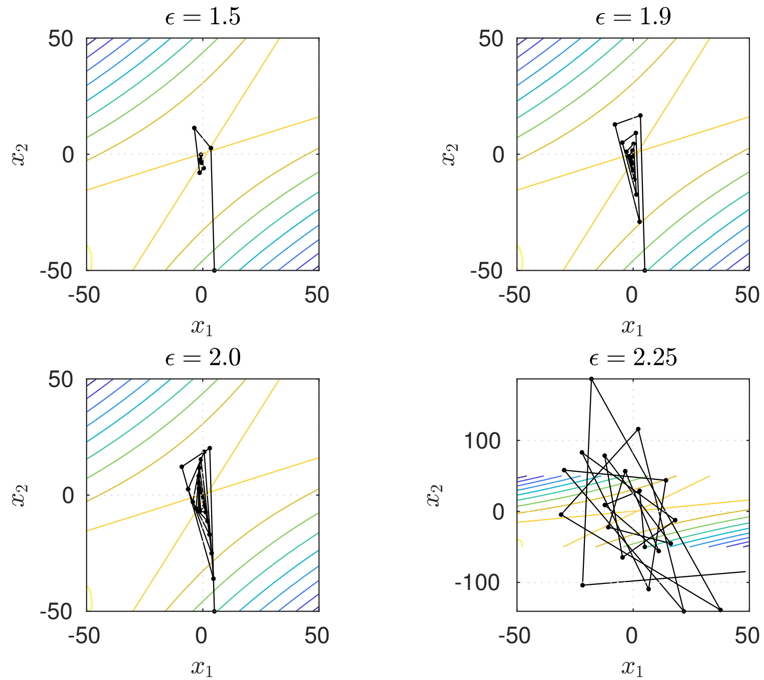




With $\epsilon = 0.1$ the procedure is clearly effective at finding the maximum.

Note however that *the steps are small*, and in a more realistic problem *it might take some time…*

Simply increasing the step size ϵ can lead to a different problem:



We can easily jump too far…

Gradient ascent and related methods

There is a large collection of more sophisticated methods. For example:

- *Line search:* increase ϵ until f *decreases* and maximise in the resulting interval. Then choose a new direction to move in. *Conjugate gradients*, the *Fletcher-Reeves* and *Polak-Ribiere* methods etc.
- Use $\mathbf{H}$ to exploit knowledge of the local shape of f. For example the *Newton-Raphson* and *Broyden-Fletcher-Goldfarb-Shanno (BFGS)* methods etc.