Planning algorithms

Reading: AIMA, chapter 11.

Problem solving is different to planning

In *search problems* we:

- *Represent states*: and a state representation contains *everything* that's relevant about the environment.
- *Represent actions*: by describing a new state obtained from a current state.
- *Represent goals*: all we know is how to test a state either to see if it's a goal, or using a heuristic.
- *A sequence of actions is a 'plan'*: but we only consider *sequences of consecutive actions*.

Search algorithms are good for solving problems that fit this framework. However for more complex problems they may fail completely...

Problem solving is different to planning

Representing a problem such as: *'go out and buy some pies'* is hopeless:

- There are *too many possible actions* at each step.
- A heuristic can only help you rank states. In particular it does not help you *ignore* useless actions.
- We are forced to start at the initial state, but you have to work out *how to get the pies*—that is, go to town and buy them, get online and find a web site that sells pies *etc—before you can start to do it*.

Knowledge representation and reasoning might not help either: although we end up with a sequence of actions—a plan—there is so much flexibility that complexity might well become an issue.

Our aim now is to look at how an agent might *construct a plan* enabling it to achieve a goal.

- We look at how we might update our concept of *knowledge representation and reasoning* to apply more specifically to planning tasks.
- We look in detail at the *partial-order planning algorithm.*

Planning algorithms work differently

Difference 1:

- Planning algorithms use a *special purpose language*—often based on FOL or a subset— to represent states, goals, and actions.
- States and goals are described by sentences, as might be expected, but...
- ...actions are described by stating their *preconditions* and their *effects*.

So if you know the goal includes (maybe among other things)

$$\texttt{Have}(\texttt{pie})$$

and action $\texttt{Buy}(x)$ has an effect $\texttt{Have}(x)$ then you know that a plan *including*

$$\texttt{Buy}(\texttt{pie})$$

might be reasonable.

Planning algorithms work differently

Difference 2:

- Planners can add actions at *any relevant point at all between the start and the goal*, not just at the end of a sequence starting at the start state.
- This makes sense: I may determine that $\texttt{Have}(\texttt{carKeys})$ is a good state to be in without worrying about what happens before or after finding them.
- By making an important decision like requiring $\texttt{Have}(\texttt{carKeys})$ early on we may reduce branching and backtracking.
- State descriptions are not complete—$\texttt{Have}(\texttt{carKeys})$ describes a *class of states*—and this adds flexibility.

So: you have the potential to search both *forwards* and *backwards* within the same problem.

Planning algorithms work differently

Difference 3:

It is assumed that most elements of the environment are *independent of most other elements.*

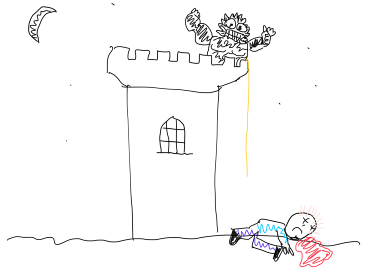
- A goal including several requirements can be attacked with a divide-and-conquer approach.
- Each individual requirement can be fulfilled using a subplan…
- …and the subplans then combined.

This works provided there is not significant interaction between the subplans.

Remember: the *frame problem.*

Running example: gorilla-based mischief

We will use a simple example, based on one from Russell and Norvig.



The intrepid little scamps in the *Cambridge University Roof-Climbing Society* wish to attach an *inflatable gorilla* to the spire of a *Famous College*. To do this they need to leave home and obtain:

- *An inflatable gorilla*: these can be purchased from all good joke shops.
- *Some rope*: available from a hardware store.
- *A first-aid kit*: also available from a hardware store.

They need to return home after they've finished their shopping. How do they go about planning their *jolly escapade*?

The STRIPS language

STRIPS: *"Stanford Research Institute Problem Solver"* (1970).

States: are *conjunctions* of *ground literals*. They must not include *function symbols*.

$$\begin{aligned}\texttt{At}(\texttt{home}) &\wedge \neg\texttt{Have}(\texttt{gorilla}) \\ &\wedge \neg\texttt{Have}(\texttt{rope}) \\ &\wedge \neg\texttt{Have}(\texttt{kit})\end{aligned}$$

Goals: are *conjunctions* of *literals* where variables are assumed *existentially quantified*.

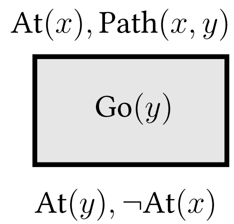
$$\texttt{At}(x) \wedge \text{Sells}(x, \texttt{gorilla})$$

A planner finds a sequence of actions that when performed makes the goal true.

We are no longer employing a full theorem-prover.

The STRIPS language

STRIPS represents actions using *operators*. For example



$$\text{Op}(\text{Action: Go}(y), \text{Pre: At}(x) \wedge \text{Path}(x, y), \text{Effect: At}(y) \wedge \neg\text{At}(x))$$

All variables are implicitly universally quantified. An operator has:

- An *action description*: what the action does.
- A *precondition*: what must be true before the operator can be used. A *conjunction of positive literals*.
- An *effect*: what is true after the operator has been used. A *conjunction of literals*.

The space of plans

We now make a change in perspective—we search in *plan space*:

- Start with an *empty plan*.
- *Operate on it* to obtain new plans. Incomplete plans are called *partial plans*. *Refinement operators* add constraints to a partial plan. All other operators are called *modification operators*.
- Continue until we obtain a plan that solves the problem.

Operations on plans can be:

- *Adding a step*.
- *Instantiating a variable*.
- *Imposing an ordering* that places a step in front of another.
- and so on...

Representing a plan: partial order planners

When putting on your shoes and socks:

- It *does not matter* whether you deal with your left or right foot first.
- It *does matter* that you place a sock on *before* a shoe, for any given foot.

It makes sense in constructing a plan *not* to make any *commitment* to which side is done first *if you don't have to.*

Principle of least commitment: do not commit to any specific choices until you have to. This can be applied both to ordering and to instantiation of variables.

A *partial order planner* allows plans to specify that some steps must come before others but others have no ordering.

A *linearisation* of such a plan imposes a specific sequence on the actions therein.

Representing a plan: partial order planners

A plan consists of:

1. A set $\{S_1, S_2, \ldots, S_n\}$ of *steps*. Each of these is one of the available *operators*.
2. A set of *ordering constraints*. An ordering constraint $S_i < S_j$ denotes the fact that step S_i must happen before step S_j. $S_i < S_j < S_k$ and so on has the obvious meaning. $S_i < S_j$ does *not* mean that S_i must *immediately* precede S_j.
3. A set of variable bindings $v = x$ where v is a variable and x is either a variable or a constant.
4. A set of *causal links* or *protection intervals* $S_i \xrightarrow{c} S_j$. This denotes the fact that the purpose of S_i is to achieve the precondition c for S_j.

A causal link is *always* paired with an equivalent ordering constraint.

Representing a plan: partial order planners

The *initial plan* has:

- Two steps, called Start and Finish.
- A single ordering constraint Start $<$ Finish.
- No *variable bindings*.
- No *causal links*.

In addition to this:

- The step Start has no preconditions, and its effect is the start state for the problem.
- The step Finish has no effect, and its precondition is the goal.
- Neither Start or Finish has an associated action.

We now need to consider what constitutes a *solution*...

Solutions to planning problems

A solution to a planning problem is any *complete* and *consistent* partially ordered plan.

Complete: each precondition of each step is *achieved* by another step in the solution.

A precondition c for S is achieved by a step S' if:

1. The precondition is an effect of the step

$$S' < S \text{ and } c \in \text{Effects}(S')$$

   and...

2. ... there is *no other* step that *could* cancel the precondition. That is, no S'' exists where:
   - The existing ordering constraints allow S'' to occur *after* S' but *before* S.
   - $\neg c \in \text{Effects}(S'')$.

Consistent: no contradictions exist in the binding constraints or in the proposed ordering. That is:

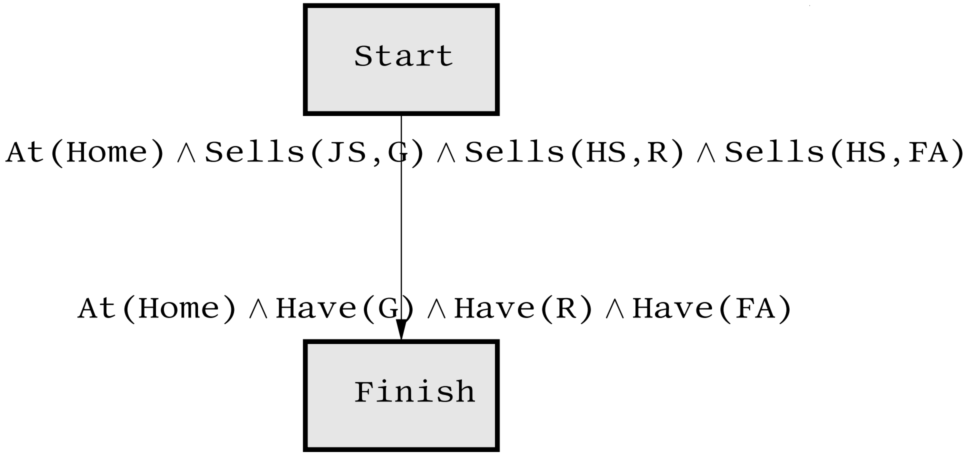
1. For binding constraints, we never have $v = X$ and $v = Y$ for distinct constants X and Y.
2. For the ordering, we never have $S < S'$ and $S' < S$.

Returning to the roof-climbers' shopping expedition, here is the basic approach:

- Begin with only the `Start` and `Finish` steps in the plan.
- At each stage add a new step.
- Always add a new step such that a *currently non-achieved precondition is achieved*.
- Backtrack when necessary.

An example of partial-order planning

Here is the *initial plan*:



Thin arrows denote ordering.

An example of partial-order planning

There are *two actions available*:

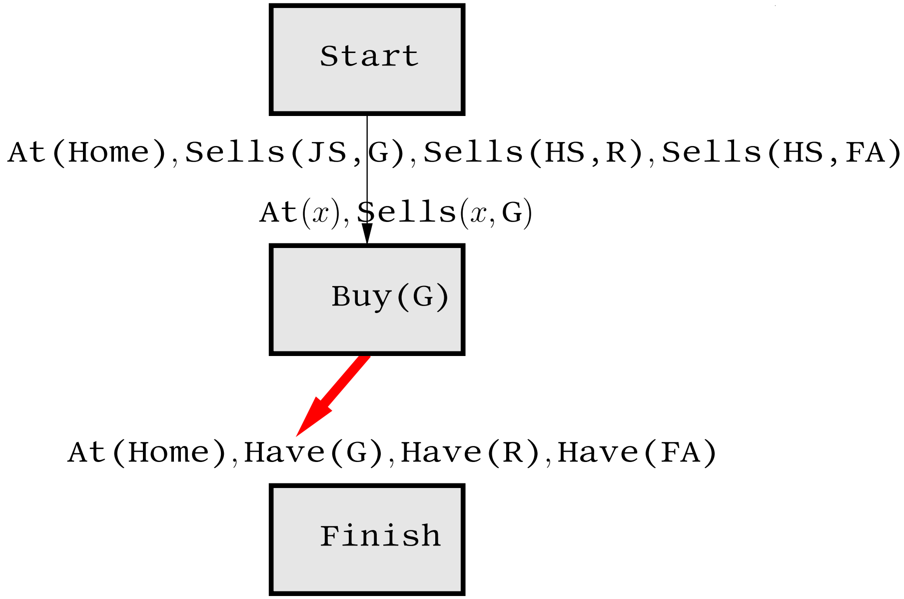
A planner might begin, for example, by adding a Buy(G) action in order to achieve the Have(G) precondition of Finish.

Note: the following order of events is by no means the only one available to a planner.

It has been chosen for illustrative purposes.

An example of partial-order planning

Incorporating the suggested step into the plan:

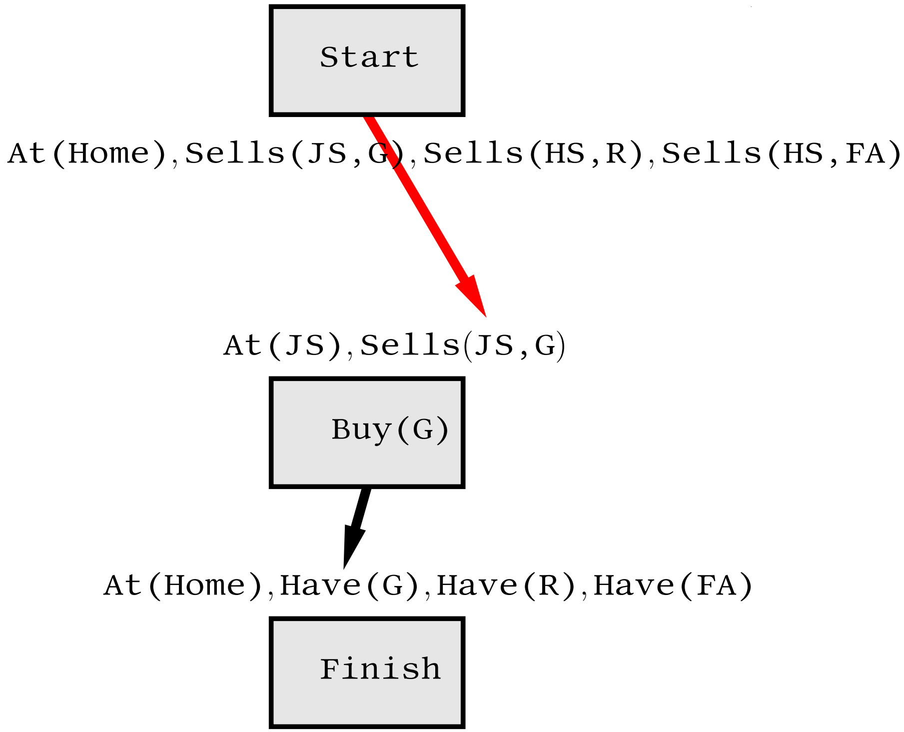


Thick arrows denote causal links. They always have a thin arrow underneath.

Here the new `Buy` step achieves the `Have`(`G`) precondition of Finish.

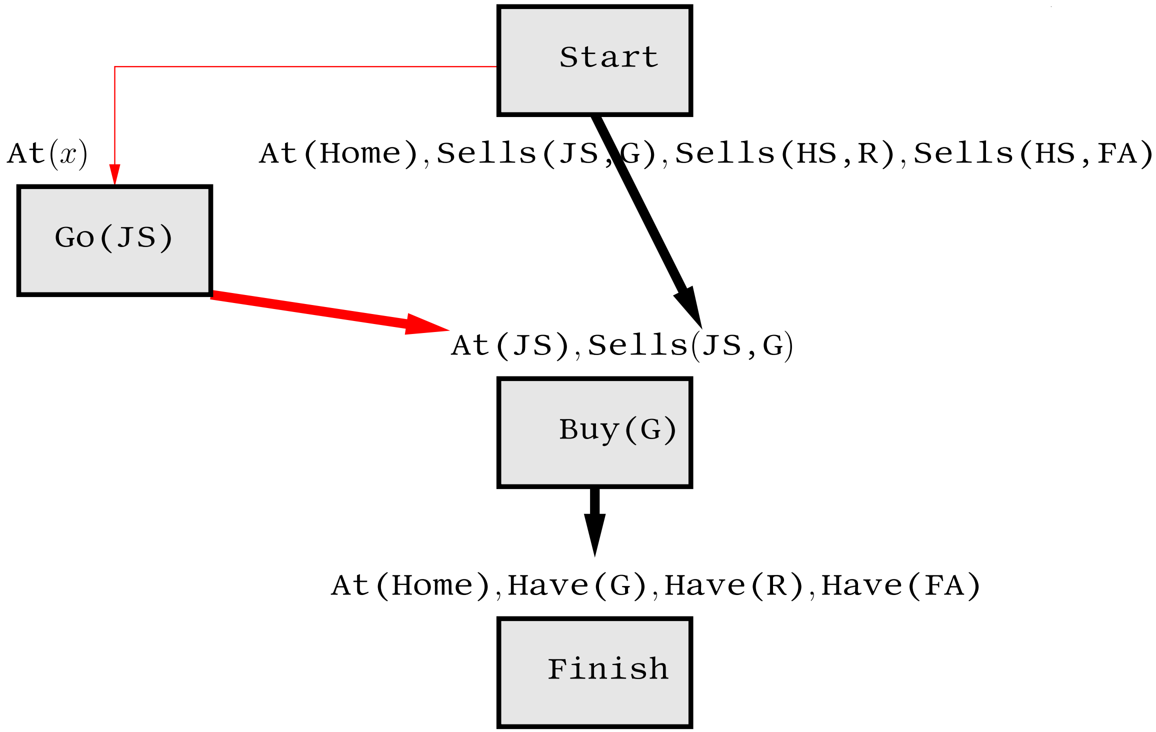
An example of partial-order planning

The planner can now introduce a second causal link from Start to achieve the Sells$(x, \mathtt{G})$ precondition of $\mathtt{Buy}(\mathtt{G})$.



An example of partial-order planning

The planner's next obvious move is to introduce a `Go` step to achieve the `At`(`JS`) precondition of `Buy`(`G`).



And we continue...

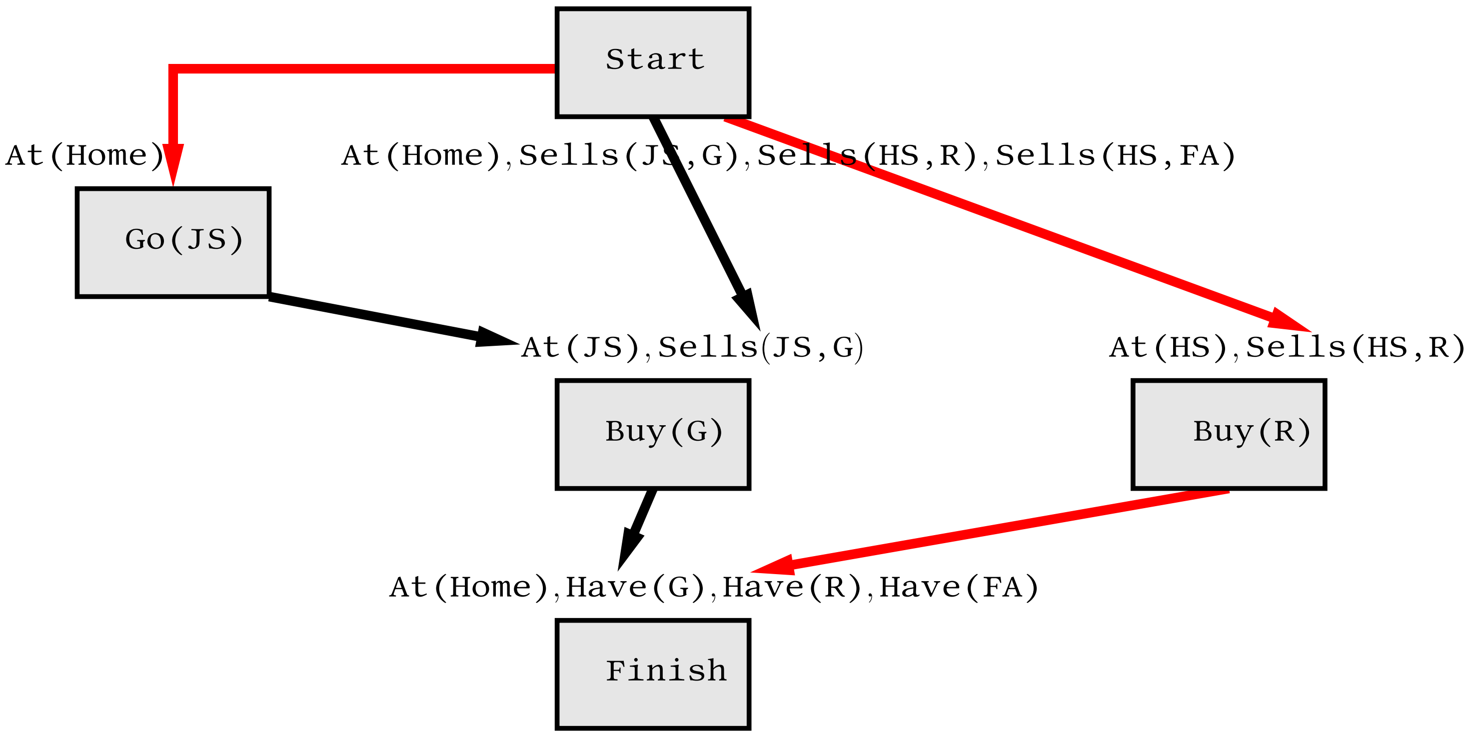
An example of partial-order planning

Initially the planner can continue quite easily in this manner:

- Add a causal link from $\texttt{Start}$ to $\texttt{Go}(\texttt{JS})$ to achieve the $\texttt{At}(x)$ precondition.
- Add the step $\texttt{Buy}(\texttt{R})$ with an associated causal link to the $\texttt{Have}(\texttt{R})$ precondition of $\texttt{Finish}$.
- Add a causal link from $\texttt{Start}$ to $\texttt{Buy}(\texttt{R})$ to achieve the $\texttt{Sells}(\texttt{HS}, \texttt{R})$ precondition.

But then things get more interesting...

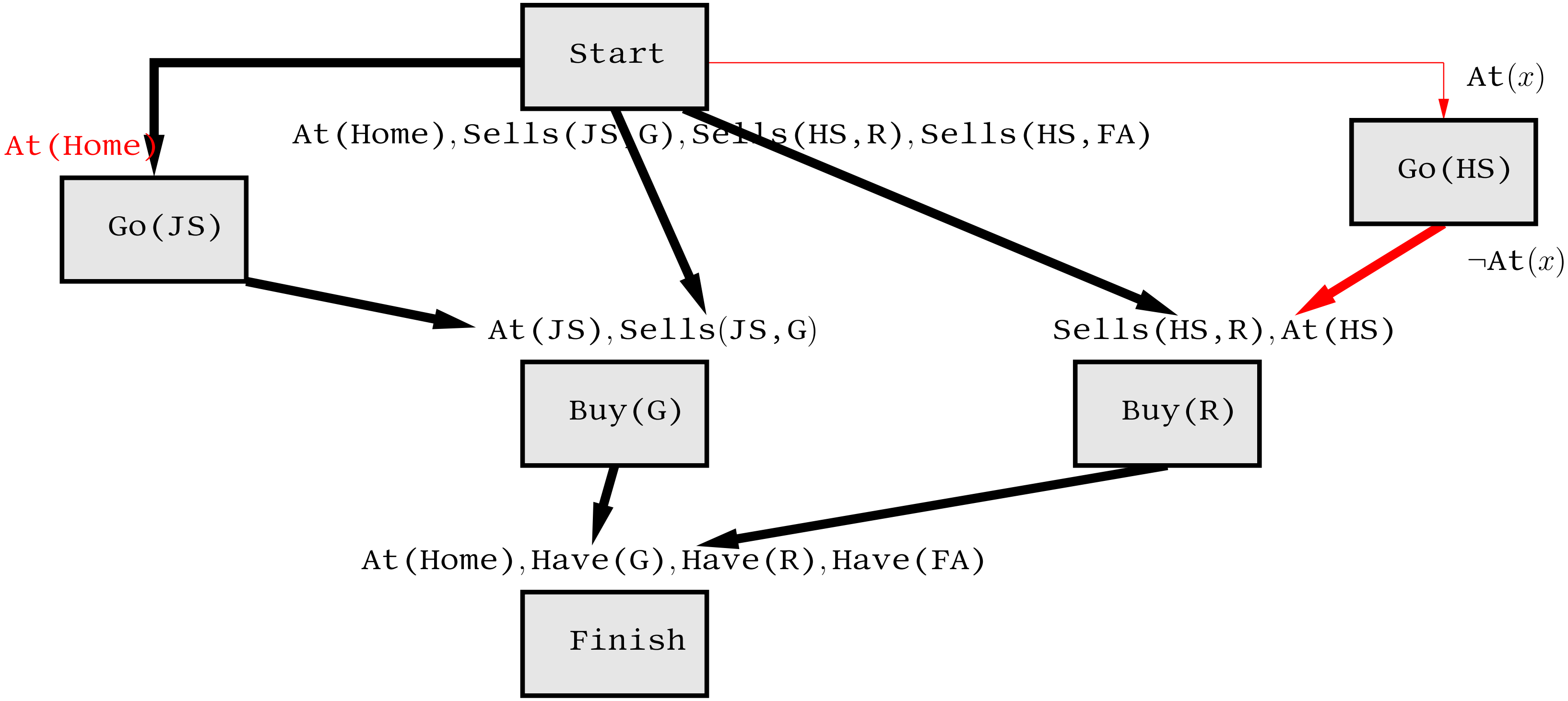
An example of partial-order planning



At this point it starts to get tricky...

The `At(HS)` precondition in `Buy(R)` is not achieved.

An example of partial-order planning

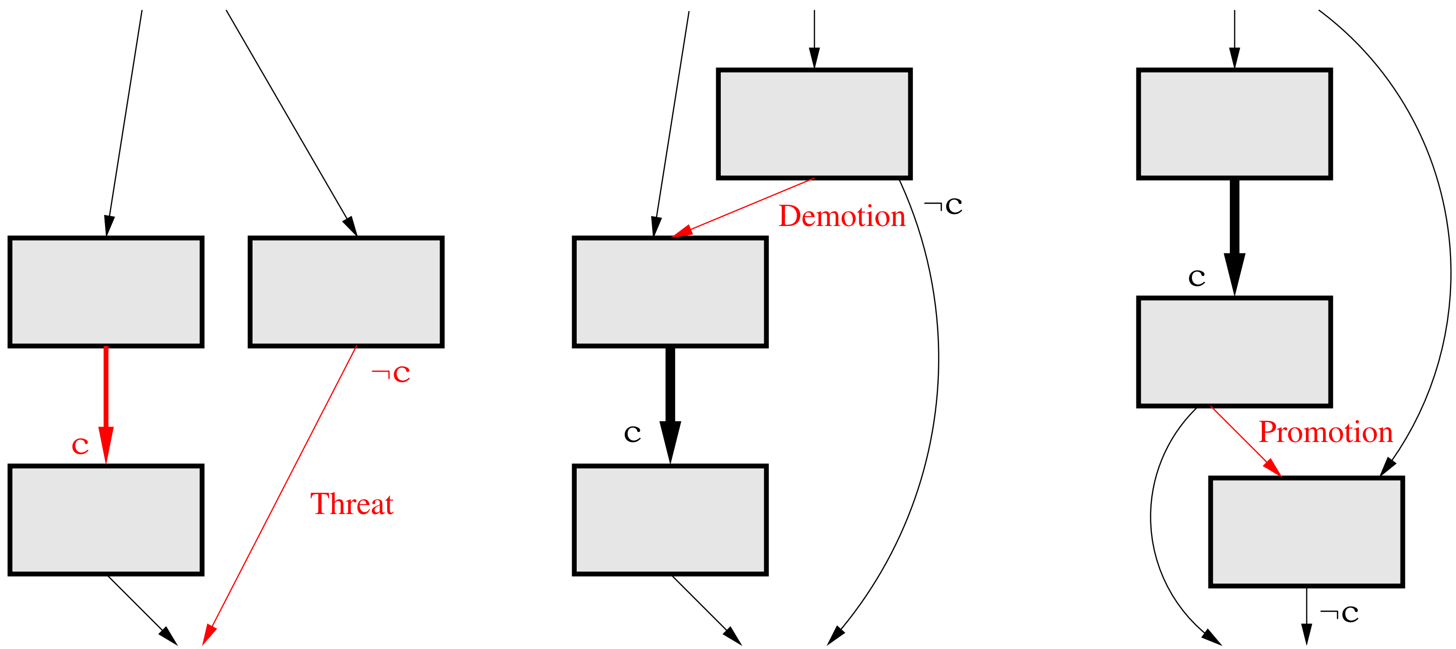


The At(HS) precondition is easy to achieve.

But if we introduce a causal link from Start *to* Go(HS) *then we risk invalidating the precondition for* Go(JS).

An example of partial-order planning

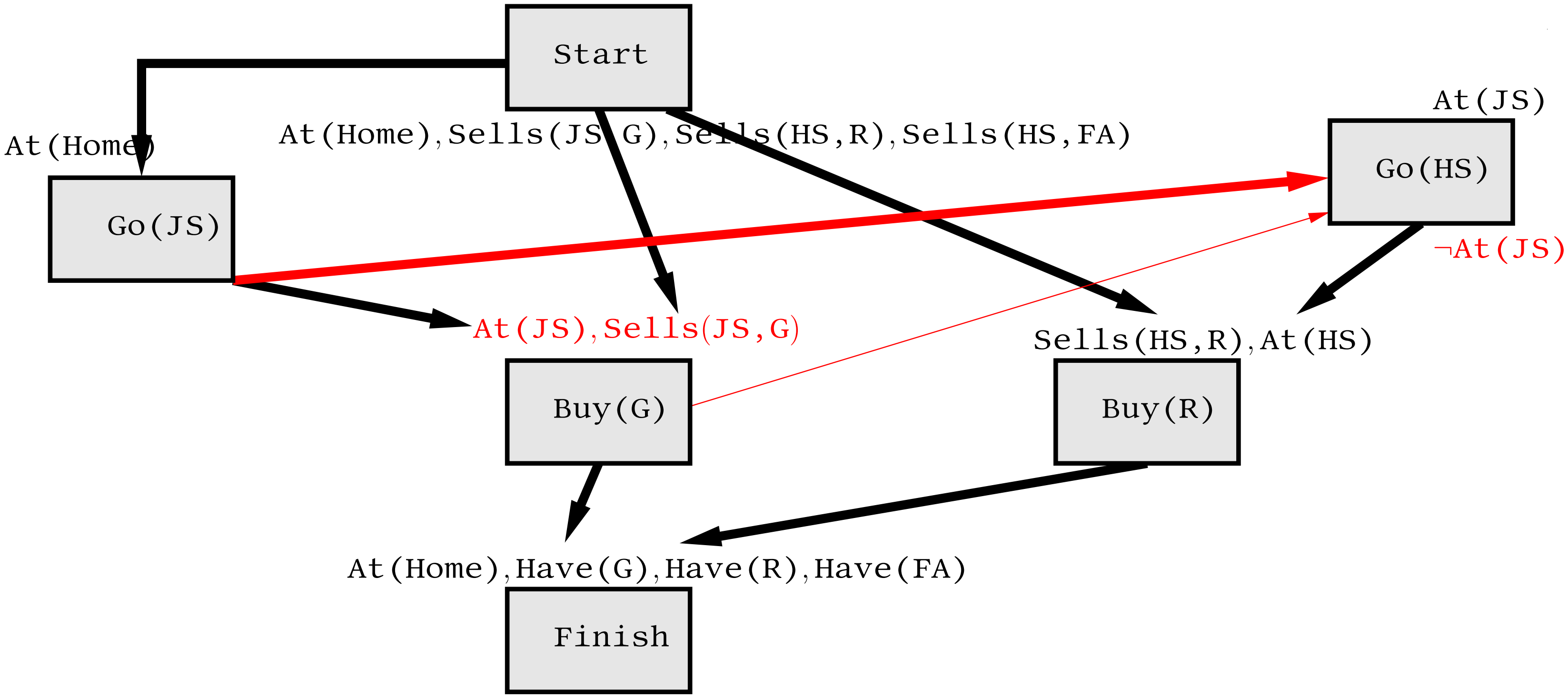
A step that might invalidate (sometimes the word *clobber* is employed) a previously achieved precondition is called a *threat*.



A planner can try to fix a threat by introducing an ordering constraint.

An example of partial-order planning

The planner could backtrack and try to achieve the $\texttt{At}(x)$ precondition using the existing $\texttt{Go}(\texttt{JS})$ step.



This involves a threat, but one that can be fixed using promotion.

The algorithm

Simplifying slightly to the case where there are *no variables*.

Say we have a partially completed plan and a set of the preconditions that have yet to be achieved.

- Select a precondition p that has not yet been achieved and is associated with an action B.
- At each stage *the partially complete plan is expanded into a new collection of plans.*
- To expand a plan, we can try to achieve p *either* by using an action that's already in the plan or by adding a new action to the plan. In either case, call the action A.

We then try to construct consistent plans where A achieves p.

The algorithm

This works as follows:

- For *each possible way of achieving* p:
  - Add Start $< A$, $A <$ Finish, $A < B$ and the causal link $A \xrightarrow{p} B$ to the plan.
  - If the resulting plan is consistent we're done, otherwise *generate all possible ways of removing inconsistencies* by promotion or demotion and *keep any resulting consistent plans.*

At this stage:

- If you have *no further preconditions that haven't been achieved* then *any plan obtained is valid.*

The algorithm

But how do we try to *enforce consistency*?

When you attempt to achieve p using A:

- Find all the existing causal links $A' \xrightarrow{\neg p} B'$ that are *clobbered* by A.
- For each of those you can try adding $A < A'$ or $B' < A$ to the plan.
- Find all existing actions C in the plan that clobber the *new* causal link $A \xrightarrow{p} B$.
- For each of those you can try adding $C < A$ or $B < C$ to the plan.
- Generate *every possible combination* in this way and retain any consistent plans that result.

Possible threats

What about dealing with *variables*?

If at any stage an effect $\neg\texttt{At}(x)$ appears, is it a threat to $\texttt{At}(\texttt{JS})$?

Such an occurrence is called a *possible threat* and we can deal with it by introducing *inequality constraints*: in this case $x \neq \texttt{JS}$.

- Each partially complete plan now has a set I of inequality constraints associated with it.
- An inequality constraint has the form $v \neq X$ where v is a variable and X is a variable or a constant.
- Whenever we try to make a substitution we check I to make sure we won't introduce a conflict.

If we *would* introduce a conflict then we discard the partially completed plan as inconsistent.

Unsurprisingly, this process can become complex.

How might we improve matters?

One way would be to introduce *heuristics*. We now consider:

- The way in which *basic heuristics* might be defined for use in planning problems.
- The construction of *planning graphs* and their use in obtaining more sensible heuristics.
- Planning graphs as the basis of the *GraphPlan* algorithm.

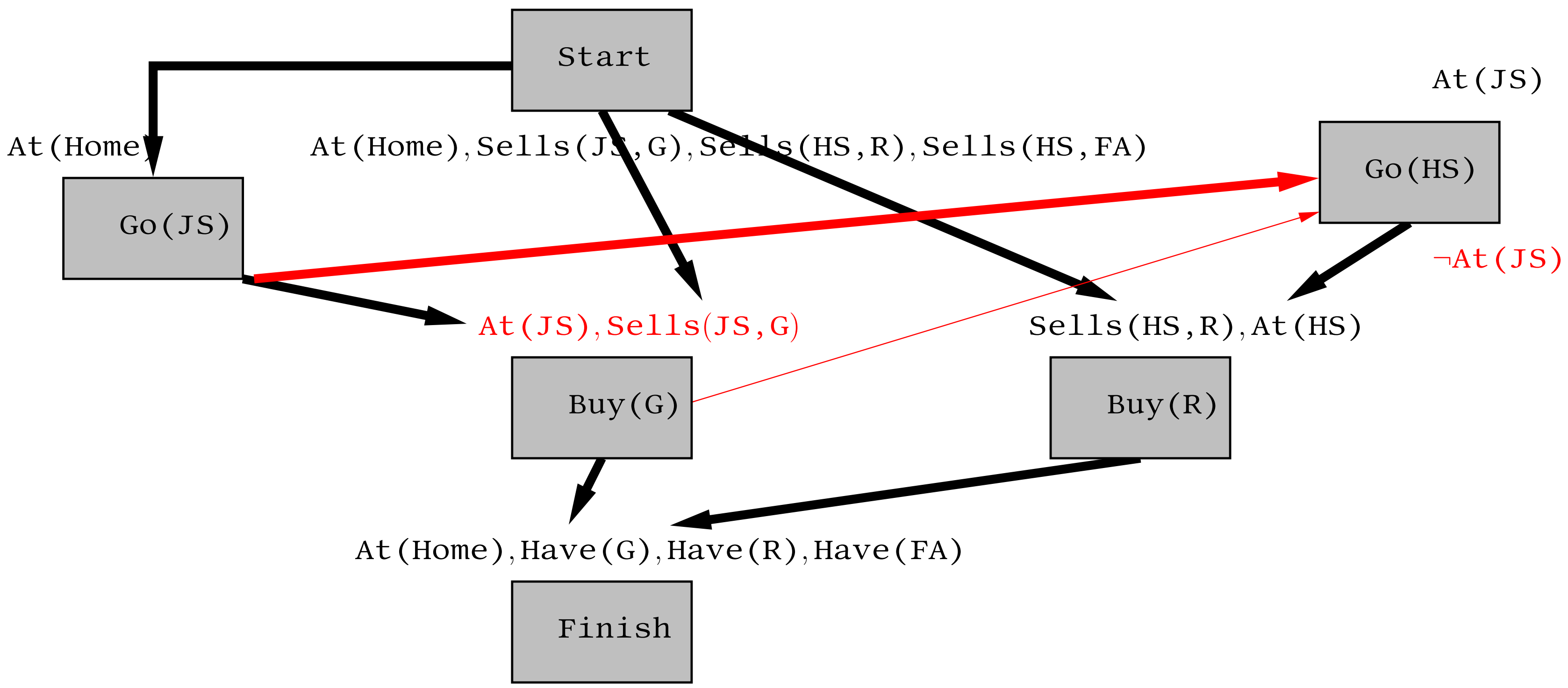
Another is to translate into the language of a *general-purpose* algorithm exploiting its own heuristics. We now consider:

- Planning using *propositional logic*.
- Planning using *constraint satisfaction*.

An example of partial-order planning

We left our example problem here:

The planner could backtrack and try to achieve the $\texttt{At}(x)$ precondition using the existing $\texttt{Go}(\texttt{JS})$ step.



This involves a threat, but one that can be fixed using promotion.

Using heuristics in planning

We found in looking at search problems that *heuristics* were a helpful thing to have.

Note that now there is no simple representation of a *state*, and consequently it is harder to measure the *distance to a goal*.

Defining heuristics for planning is therefore more difficult than it was for search problems. Simple possibilities:

$$h = \text{number of unsatisfied preconditions}$$

or

$$\begin{aligned} h = & \text{number of unsatisfied preconditions} \\ & - \text{number satisfied by the start state} \end{aligned}$$

These can lead to underestimates or overestimates:

- Underestimates if *actions can affect one another in undesirable ways.*
- Overestimates if *actions achieve many preconditions.*

Using heuristics in planning

We can go a little further by learning from *Constraint Satisfaction Problems* and adopting the *most constrained variable* heuristic:

- Prefer the precondition *satisfiable in the smallest number of ways.*

This can be computationally demanding but two special cases are helpful:

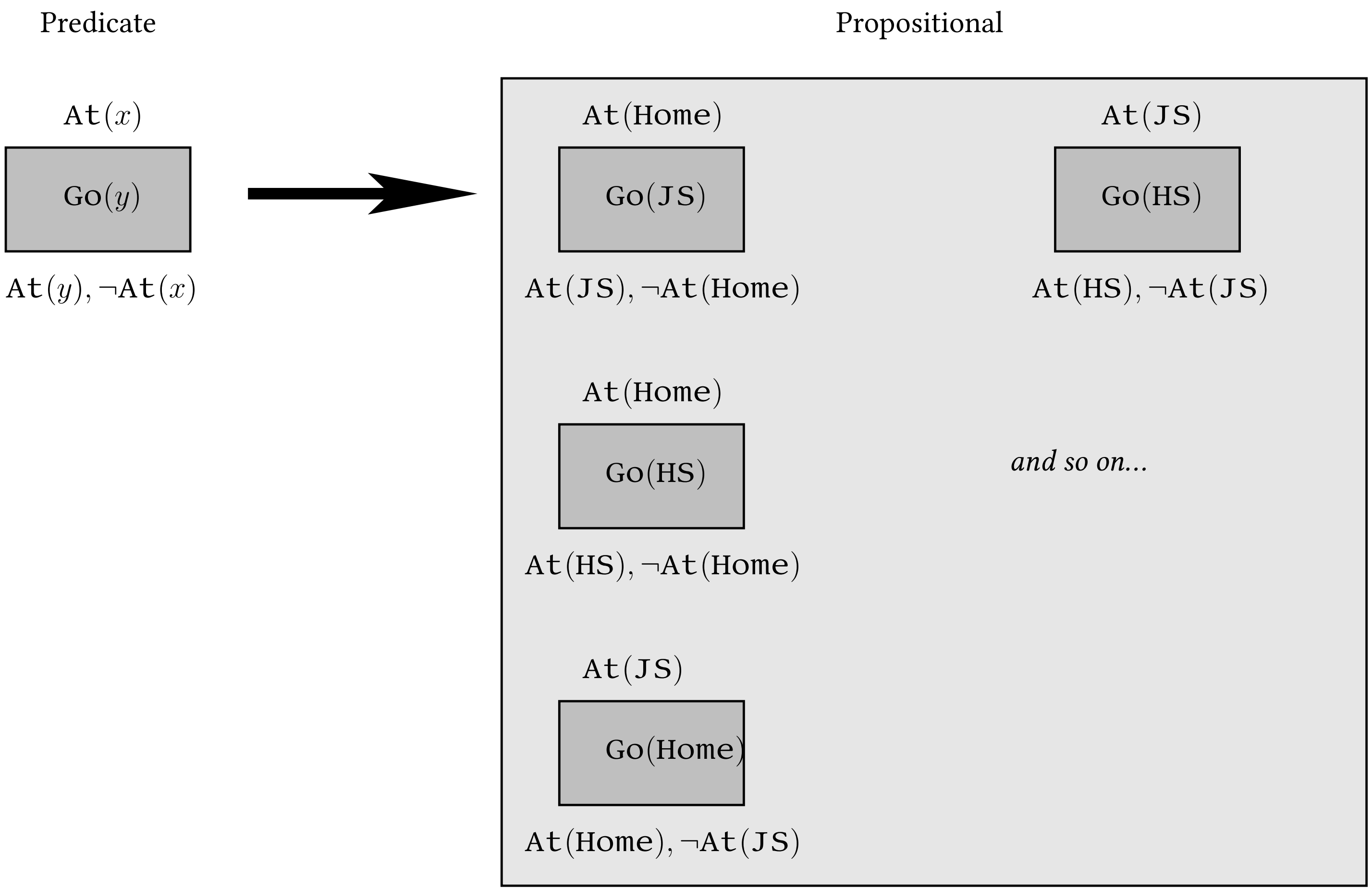
- Choose preconditions for which *no action will satisfy them.*
- Choose preconditions that *can only be satisfied in one way*.

But these still seem somewhat basic.

We can do better using *Planning Graphs*. These are *easy to construct* and can also be used to generate *entire plans.*

Planning graphs

Planning Graphs apply when it is possible to work entirely using *propositional* representations of plans. Luckily, STRIPS can always be propositionalized...



Planning graphs

A planning graph is constructed in levels:

- Level 0 corresponds to the *start state*.
- At each level we keep *approximate* track of all things that *could* be true at the corresponding time.
- At each level we keep *approximate* track of what actions *could* be applicable at the corresponding time.

The approximation is due to the fact that not all conflicts between actions are tracked. *So*:

- The graph can *underestimate* how long it might take for a particular proposition to appear, and therefore …
- …a heuristic can be extracted.

For example: the triumphant return of the gorilla-purchasing roof-climbers...

Planning graphs: a simple example

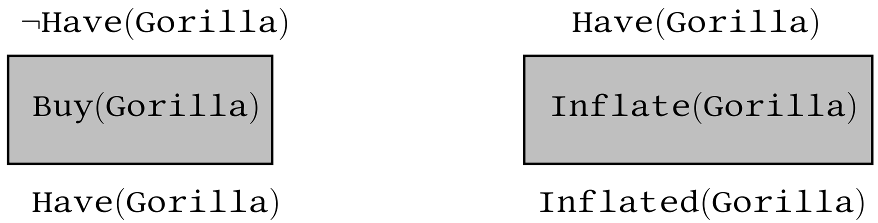
Our intrepid student adventurers will of course need to inflate their *gorilla* before attaching it to a *distinguished roof*. It has to be purchased before it can be inflated.

Start state: Empty.

We assume that anything not mentioned in a state is false. So the state is actually

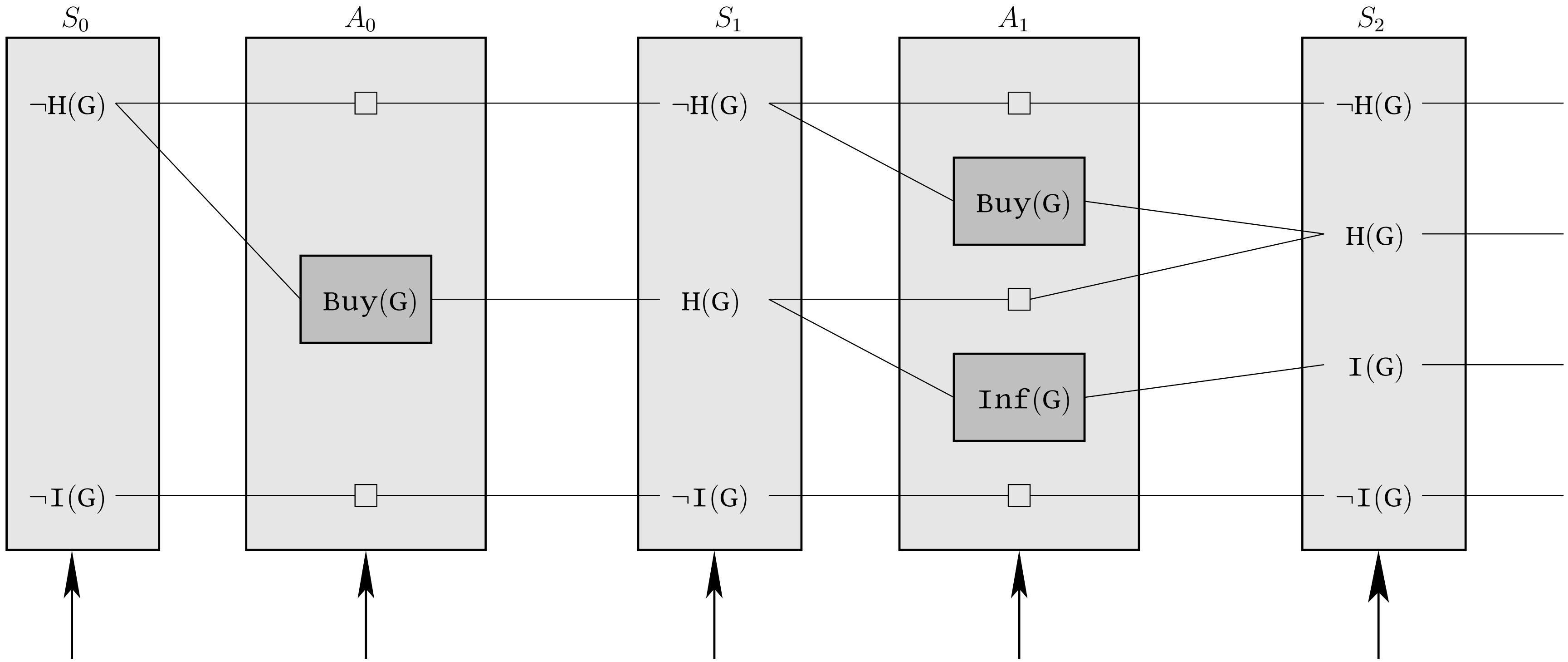
$$\neg\texttt{Have}(\texttt{Gorilla}) \text{ and } \neg\texttt{Inflated}(\texttt{Gorilla})$$

Actions:



Goal: $\texttt{Have}(\texttt{Gorilla})$ and $\texttt{Inflated}(\texttt{Gorilla})$.

Planning graphs



Describe start state.

All actions available in start state.

All possibilities for what might be the case at time 1.

All actions that might be available at time 1.

All possibilities for what might be the case at time 2.

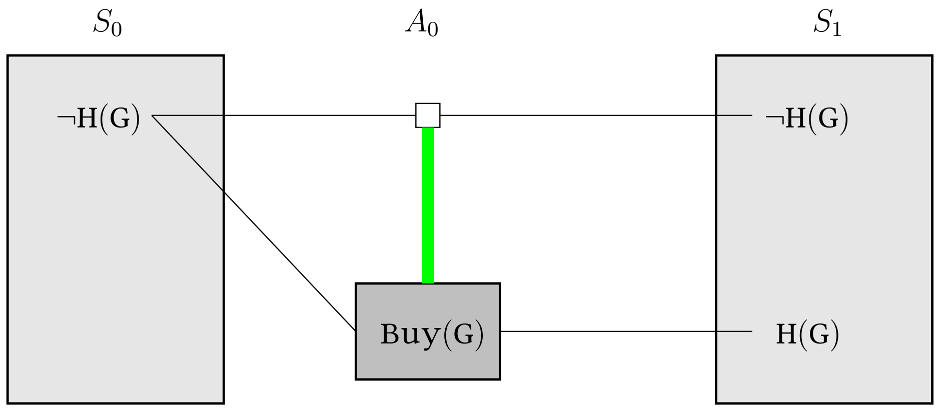
□ = a *persistence action*—what happens if no action is taken.

An action level A_i contains *all* actions that *could* happen given the propositions in S_i.

Mutex links

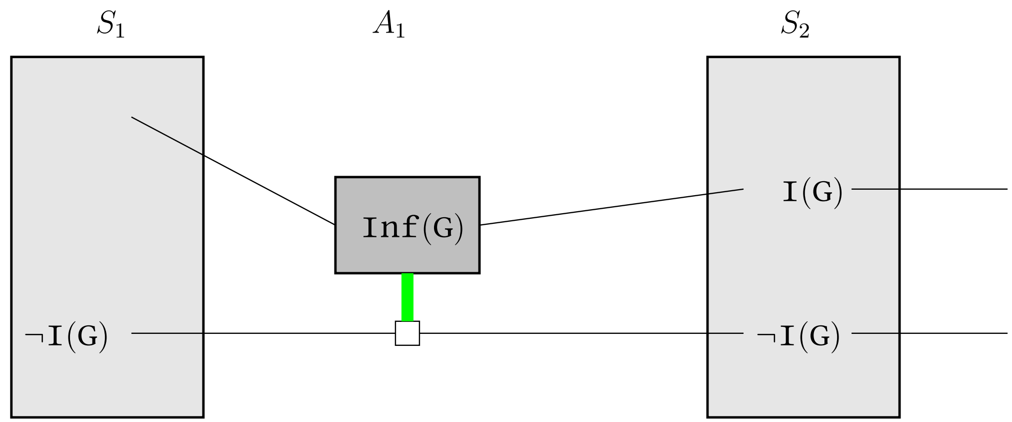
We also record, using *mutual exclusion (mutex) links* which pairs of actions could not occur together.

Mutex links 1: Effects are inconsistent.



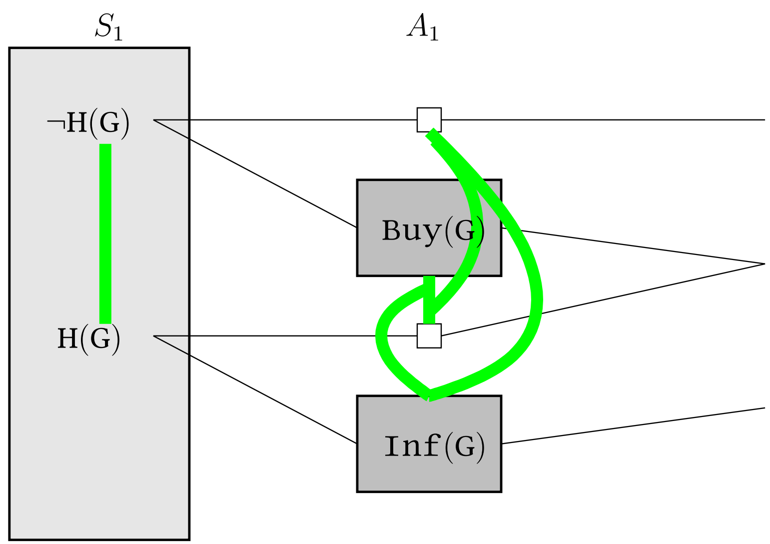
The effect of one action negates the effect of another.

Mutex links 2: The actions interfere.



The effect of an action negates the precondition of another.

Mutex links 3: Competing for preconditions.



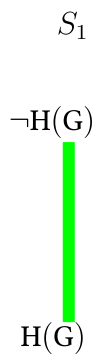
The precondition for an action is mutually exclusive with the precondition for another. (See next slide!)

Mutex links

A state level S_i contains *all* propositions that *could* be true, given the possible preceding actions.

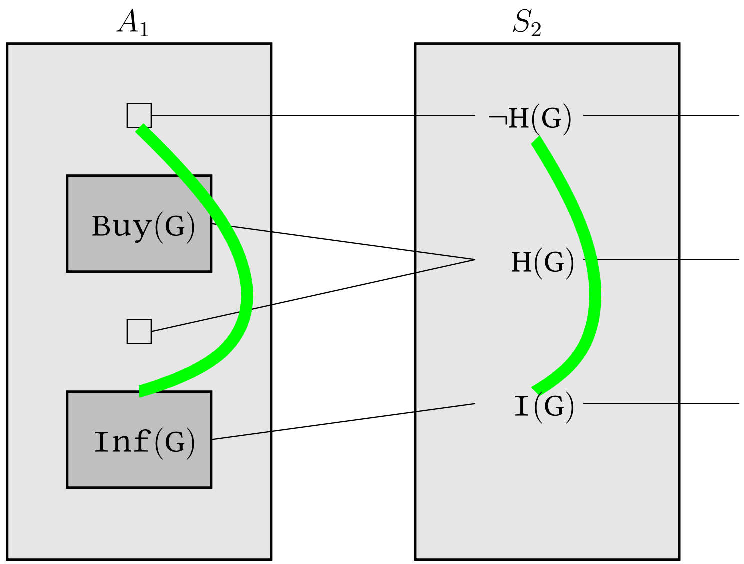
We also use mutex links to record pairs that can not be true simultaneously:

Possibility 1: pair consists of a proposition and its negation.



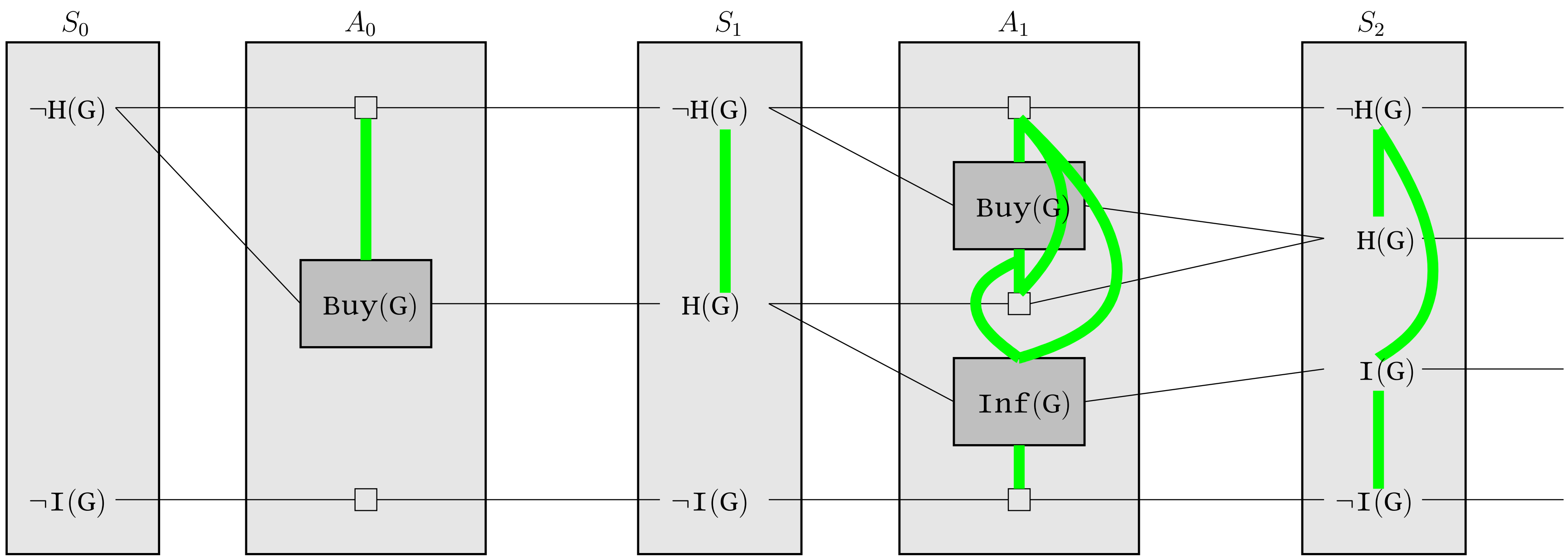
Mutex links

Possibility 2: all pairs of actions that could achieve the pair of propositions are mutex.



The construction of a planning graph is continued until two identical levels are obtained.

Planning graphs



Obtaining heuristics from a planning graph

To estimate the cost of reaching a single proposition:

- Any proposition not appearing in the final level has *infinite cost* and *can never be reached*.
- The *level cost* of a proposition is the level at which it first appears *but* this may be inaccurate as several actions can apply at each level and this cost does not count the *number of actions*. (It is however *admissible*.)
- A *serial planning graph* includes mutex links between all pairs of actions except persistence actions.

Level cost in serial planning graphs can be quite a good measurement.

Obtaining heuristics from a planning graph

How about estimating the cost to achieve a *collection* of propositions?

- *Max-level*: use the maximum level in the graph of any proposition in the set. Admissible but can be inaccurate.
- *Level-sum*: use the sum of the levels of the propositions. Inadmissible but sometimes quite accurate if goals tend to be decomposable.
- *Set-level*: use the level at which *all* propositions appear with none being mutex. Can be accurate if goals tend *not* to be decomposable.

Other points about planning graphs

A planning graph guarantees that:

1. *If* a proposition appears at some level, there *may* be a way of achieving it.
2. *If* a proposition does *not* appear, it can *not* be achieved.

The first point here is a loose guarantee because only *pairs* of items are linked by mutex links.

Looking at larger collections can strengthen the guarantee, but in practice the gains are outweighed by the increased computation.

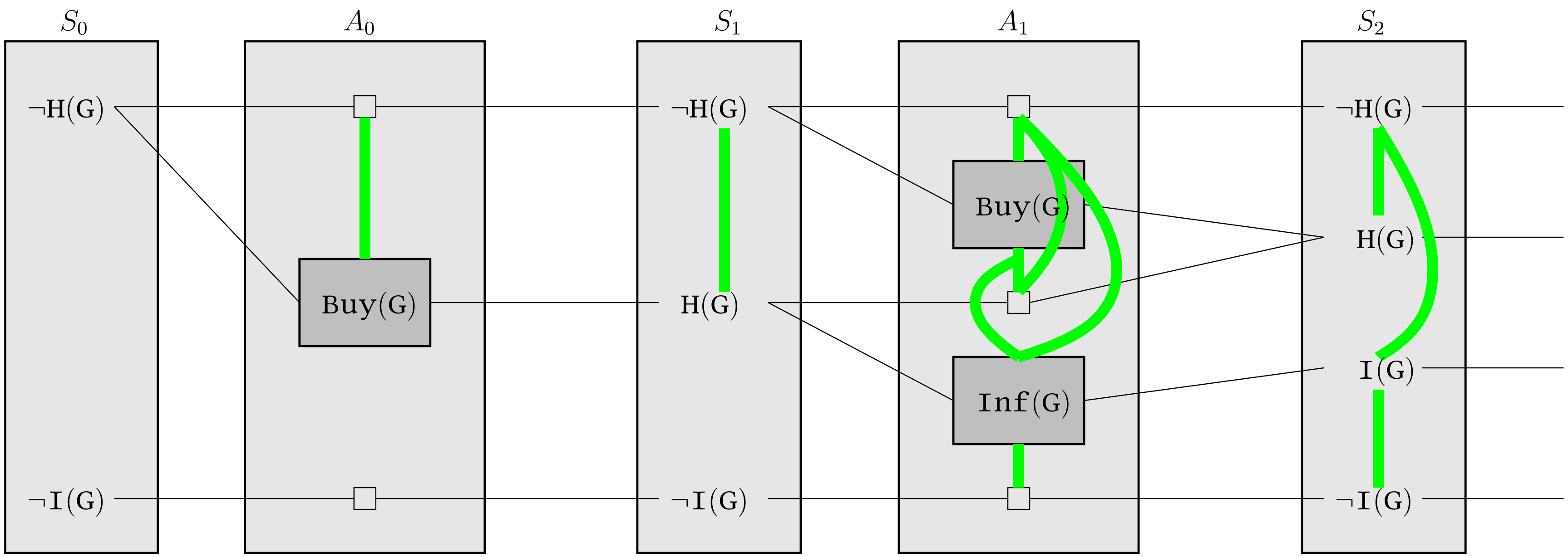
Graphplan

The *GraphPlan* algorithm goes beyond using the planning graph as a source of heuristics.

```
function GraphPlan()
    Start at level 0;
    while true do
        if All goal propositions appear in the current level AND no pair has a mutex link then
            Attempt to extract a plan;
            if A solution is obtained then
                return SOME solution;
            if Graph indicates there is no solution then
                return NONE;
        Expand the graph to the next level;
```

We *extract a plan* directly from the planning graph. Termination can be proved but will not be covered here.

Here, at levels S_0 and S_1 we do not have both H(G) and I(G) available with no mutex links, and so we expand first to S_1 and then to S_2.



At S_2 we try to extract a solution (plan).

Extracting a plan from the graph

Extraction of a plan can be formalised as a *search problem*.

States contain a *level*, and a collection of *unsatisfied goal propositions*.

Start state: the current final level of the graph, along with the relevant goal propositions.

Goal: a state at level S_0 containing the initial propositions.

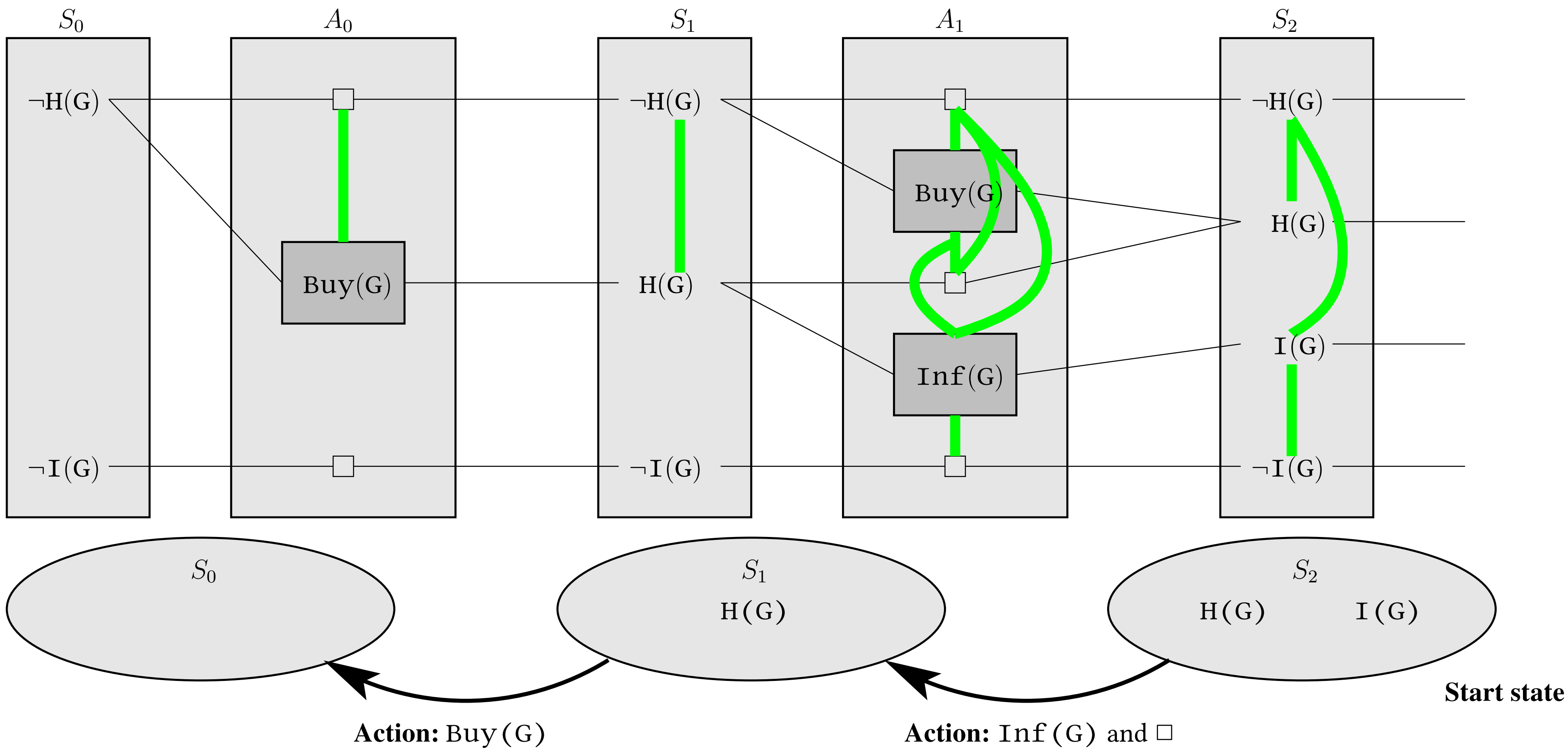
Actions: For a state S with level S_i, a valid action is to select any set X of actions in A_{i-1} such that:

1. no pair has a mutex link;
2. no pair of their preconditions has a mutex link;
3. the effects of the actions in X achieve the propositions in S.

The effect of such an action is a state having level S_{i-1}, and containing the preconditions for the actions in X.

Each action has a cost of 1.

Graphplan in action



Heuristics for plan extraction

We can of course also apply *heuristics* to this part of the process.

For example, when dealing with a *set of propositions*:

- Choose the proposition having *maximum level cost* first.
- For that proposition, attempt to achieve it using the action for which the *maximum/sum level cost of its preconditions is minimum.*